Frequency domain modeling of transmission lines excited by nearby lightning strokes

Pablo Gómez, Juan C. Escamilla

Abstract-- In this paper, the frequency domain modeling of a multiconductor line illuminated by a nearby lightning stroke is described, considering also variations of the line parameters along its length (nonuniform line case). Modeling of the illuminated line is based on Taylor's formulation, while the incident electromagnetic field is described following Master and Uman's expressions. The numerical Laplace transform algorithm is used for the frequency-time transformation required. Comparison with an experimental result previously published is provided for an initial validation of the method. As a second application example, a test case is used to analyze the effect of the point of impact on the magnitude and waveshape of the transient overvoltages obtained. The effect of line nonuniformities is also discussed.

Keywords: Frequency domain analysis, illuminated line modeling, lightning transients, nonuniform line modeling, numerical Laplace transform.

I. INTRODUCTION

THE effect of lightning induced overvoltages in transmission and distribution lines can be classified, according to the point of impact, in direct and indirect lightning phenomena. Although, as expected, direct lightning strokes produce higher overvoltages, the incident electromagnetic field from nearby lightning strokes can still be of importance for the design of insulation and protection elements, particularly for the voltage level of distribution lines, being also a much more frequent phenomenon.

In a more general sense, a transmission line excited by an incident electromagnetic field from any source is known as an *illuminated line*. Several researchers have modeled and analyzed this problem for power and electronics applications [1]-[10]. Formulations proposed by Taylor [1], Agrawal [2] and Rachidi [3] are the most commonly applied to this date.

This work describes the modeling of an illuminated multiconductor line in the frequency domain, considering also variations of the line parameters along its length (nonuniform line case). The modeling is based on Taylor's formulations, in which the incident field is approximated by distributed sources connected along the line's length. Here, following the procedure described in [1], [10], the technique is reduced to lumped current sources connected only at the line ends. This leads to a substantial simplification of the analysis without losing accuracy of the results.

The incident electromagnetic field representing the nearby lightning stroke is computed from the formulation described by Master and Uman [4], which is basically dependent on variables related to the point of impact and the return stroke current. However, the usual integro-differential form of the equations in time domain is replaced by a more convenient algebraic form in the frequency domain. Once the electromagnetic field is obtained, the sources required by the model can be included. The numerical Laplace transform algorithm is used for the frequency-time transformation required [11].

Performance of the resulting method is initially validated by comparison with experimental results from a reduced-scale model reported in [9]. Then, the effect of the point of impact on the magnitude and waveshape of the transient overvoltages at the line ends is analyzed on a test case. The differences obtained when considering line nonuniformities are also discussed.

This work is intended as an extension of the capabilities of frequency domain analysis tools, which are believed to be a valuable addition to the well-known time domain analysis.

II. MODELING OF THE ILLUMINATED TRANSMISSION LINE

According to Taylor's formulation [1], [10], a transmission line excited by an incident electromagnetic field (illuminated line) can be described by means of the inclusion of distributed series voltages sources and shunt current sources along the line's length. For a multiconductor line, the Telegrapher equations are defined in Laplace domain as follows

$$\begin{aligned} \frac{d\mathbf{V}(z,s)}{dz} &= -\mathbf{Z}(z,s)\mathbf{I}(z,s) + \mathbf{V}_F(z,s) \\ \frac{d\mathbf{I}(z,s)}{dz} &= -\mathbf{Y}(z)\mathbf{V}(z,s) + \mathbf{I}_F(z,s) \end{aligned} \quad (1)$$

where s is the Laplace variable; $\mathbf{V}(z,s)$ and $\mathbf{I}(z,s)$ are the vectors of voltages and currents along the propagation axis z; $\mathbf{Z}(z,s)$ and $\mathbf{Y}(z)$ are the matrices of series impedances and shunt conductances per unit length, given by $\mathbf{Z}=\mathbf{R}+s\mathbf{L}$ and $\mathbf{Y}=\mathbf{G}+s\mathbf{C}$ respectively. It is noticed that, for the general nonuniform line case, these matrices are a function of z. Also,

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due to skin effect in conductors and in ground plane, the impedance matrix is frequency dependent. On the other hand, $\mathbf{V}_F$ and $\mathbf{I}_F$ represent the vectors of distributed sources, which are related to the incident electromagnetic field as [3]

$$\mathbf{V}_F(z,s) = s\begin{bmatrix} \vdots \\ \int_0^{h_i(z)} B_{x,i}(z,s)dy + E_{z,i}(0,s) \\ \vdots \end{bmatrix}$$
$$\mathbf{I}_F(z,s) = -\mathbf{Y}(z)\begin{bmatrix} \vdots \\ \int_0^{h_i(z)} E_{y,i}(z,s)dy \\ \vdots \end{bmatrix} \tag{2}$$

where $h_i(z)$ is the height of the i^{th} conductor; $E_{y,i}(z,s)$ and $B_{x,i}(z,s)$ are the vertical electric and transversal magnetic field components of the i^{th} conductor in the Laplace domain, respectively. These variables are also a function of z for the general case of a nonuniform line. Besides, $E_{z,i}(0,s)$ is the horizontal electric field at ground level, which is due to the finite ground conductivity [3].

From the concept of the matrix exponential and the application of modal decomposition, solution to (1) for a line segment Δz in terms of the chain matrix $\mathbf{\Phi}(\Delta z, s)$ is given by

$$\begin{bmatrix} \mathbf{V}(z+\Delta z,s) \\ \mathbf{I}(z+\Delta z,s) \end{bmatrix} = \mathbf{\Phi}(\Delta z,s)\begin{bmatrix} \mathbf{V}(z,s) \\ \mathbf{I}(z,s) \end{bmatrix} + \int_z^{z+\Delta z} \mathbf{\Phi}(z-\tau,s)\begin{bmatrix} \mathbf{V}_F(\tau,s) \\ \mathbf{I}_F(\tau,s) \end{bmatrix} d\tau \tag{3}$$

where

$$\mathbf{\Phi}(\Delta x,s) = \begin{bmatrix} \cosh(\mathbf{\Psi}\Delta x) & -\mathbf{Y}_0^{-1}\sinh(\mathbf{\Psi}\Delta x) \\ -\mathbf{Y}_0\sinh(\mathbf{\Psi}\Delta x) & \cosh(\mathbf{\Psi}\Delta x) \end{bmatrix} \tag{4}$$

$\mathbf{\Psi}$ is the phase domain constant propagation matrix of the line segment, defined as

$$\mathbf{\Psi} = \mathbf{M}\sqrt{\boldsymbol{\lambda}}\mathbf{M}^{-1} \tag{5}$$

$\mathbf{M}$ and $\boldsymbol{\lambda}$ are the eigenvector and eigenvalue matrices of the matrix product $\mathbf{Z}(z,s)\cdot\mathbf{Y}(z,s)$, respectively, and $\mathbf{Y}_0$ is the characteristic admittance matrix of the line segment, computed as follows:

$$\mathbf{Y}_0 = \mathbf{Z}(z,s)^{-1}\mathbf{\Psi} \tag{6}$$

Equation (3) relates voltages and currents from one end of the line segment with the same variables on the other end, considering the same direction of the current in both ends. If the segment is electrically small, the integral in (3) can be approximated as

$$\int_z^{z+\Delta z} \mathbf{\Phi}(z-\tau,s)\begin{bmatrix} \mathbf{V}_F(\tau,s) \\ \mathbf{I}_F(\tau,s) \end{bmatrix} d\tau \approx \begin{bmatrix} \mathbf{V}_F(z,s)\Delta z \\ \mathbf{I}_F(z,s)\Delta z \end{bmatrix} \tag{7}$$

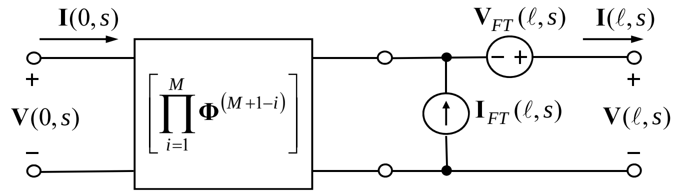


Fig. 1 Representation of the illuminated line by means of lumped equivalent sources [5]

From (3) and (7), an illuminated line can be described dividing the line in M electrically small segments of length Δz and including the sources defined in (2) between each of the segments. Moreover, applying the boundary conditions z=0 y $z=\ell$, where ℓ is the total line length, an equivalent representation can be obtained, in which the incident field is included by means of lumped sources connected only at the line's far end (Fig. 1). This representation is obtained adding the vector of distributed sources at each step of the cascaded connection of the M chain matrices; this is:

$$\begin{bmatrix} \mathbf{V}(\ell,s) \\ \mathbf{I}(\ell,s) \end{bmatrix} = \prod_{i=1}^{M} \mathbf{\Phi}^{(M+1-i)} \begin{bmatrix} \mathbf{V}(0,s) \\ \mathbf{I}(0,s) \end{bmatrix} + \begin{bmatrix} \mathbf{V}_{FT}(\ell,s) \\ \mathbf{I}_{FT}(\ell,s) \end{bmatrix} \tag{8}$$

where $\mathbf{\Phi}^{(i)}$ is the chain matrix of the i^{th} line segment. The first term on the right hand side of (8) corresponds to the cascaded connection of chain matrices of the unexcited line. Since each chain matrix can be different from the others, variation of the line's electrical parameters with its length can be directly considered (nonuniform line case). If the line is considered as uniform, this term can be replaced by the chain matrix of the complete line. On the other hand, the second term on the right hand side of (8) is defined as

$$\begin{bmatrix} \mathbf{V}_{FT}(\ell,s) \\ \mathbf{I}_{FT}(\ell,s) \end{bmatrix} = \sum_{i=1}^{M-1}\left\{\left[\prod_{n=1}^{M-i-1} \mathbf{\Phi}^{(M-n)}\right]\begin{bmatrix} \mathbf{V}_F(i\Delta z,s)\Delta z \\ \mathbf{I}_F(i\Delta z,s)\Delta z \end{bmatrix}\right\} \tag{9}$$

It can be noticed that, when $\Delta z \to 0$, eq. (9) can be expressed as a convolution in z between the vector of distributed sources and the chain matrix of the line; this is:

$$\begin{bmatrix} \mathbf{V}_{FT}(\ell,s) \\ \mathbf{I}_{FT}(\ell,s) \end{bmatrix} = \int_0^{\ell} \mathbf{\Phi}(\ell-z,s)\begin{bmatrix} \mathbf{V}_F(z.s) \\ \mathbf{I}_F(z.s) \end{bmatrix} dz \tag{10}$$

Further algebraic manipulation of (8) results in the equivalent nodal representation described in (11), where the direction of $\mathbf{I}(\ell,s)$ has been reversed (nodal analysis considers injected currents). In this case the incident electromagnetic field is represented by means of current sources connected at both ends of the line:

$$\begin{bmatrix} \mathbf{I}(0,s) \\ \mathbf{I}(\ell,s) \end{bmatrix} = \begin{bmatrix} \mathbf{Y}_{SS} & -\mathbf{Y}_{SR} \\ -\mathbf{Y}_{SR} & \mathbf{Y}_{RR} \end{bmatrix}\begin{bmatrix} \mathbf{V}(0,s) \\ \mathbf{V}(\ell,s) \end{bmatrix} + \begin{bmatrix} \mathbf{I}_{sc}(0,s) \\ \mathbf{I}_{sc}(\ell,s) \end{bmatrix} \tag{11}$$

Elements of the nodal admittance matrix are given by

$$\begin{aligned} \mathbf{Y}_{SS} &= -\mathbf{\Phi}_{12}^{-1}\mathbf{\Phi}_{11} \\ \mathbf{Y}_{SR} &= -\mathbf{\Phi}_{12}^{-1} = -\mathbf{\Phi}_{22}\mathbf{\Phi}_{12}^{-1}\mathbf{\Phi}_{11} + \mathbf{\Phi}_{21} \\ \mathbf{Y}_{RR} &= -\mathbf{\Phi}_{22}\mathbf{\Phi}_{12}^{-1} \end{aligned} \quad (12)$$

Current sources at the line ends are defined as follows:

$$\begin{aligned} \mathbf{I}_{SC}(0,s) &= -\mathbf{\Phi}_{12}^{-1}\mathbf{V}_{FT}(\ell,s) \\ \mathbf{I}_{SC}(\ell,s) &= \mathbf{\Phi}_{22}\mathbf{\Phi}_{12}^{-1}\mathbf{V}_{FT}(\ell,s) - \mathbf{I}_{FT}(\ell,s) \end{aligned} \quad (13)$$

where $\mathbf{\Phi}_{11}$, $\mathbf{\Phi}_{12}$, $\mathbf{\Phi}_{21}$ and $\mathbf{\Phi}_{22}$ are the sub-matrices of the chain matrix for the complete line:

$$\left[\prod_{i=1}^{M}\mathbf{\Phi}^{(M+1-i)}\right] = \begin{bmatrix} \mathbf{\Phi}_{11} & \mathbf{\Phi}_{12} \\ \mathbf{\Phi}_{21} & \mathbf{\Phi}_{22} \end{bmatrix} \quad (14)$$

Nodal form given by (11) defines the illuminated line model by means of the nodal admittance matrix of the line without excitation, and the connection of current sources at the line ends representing the incident electromagnetic field due to the nearby lightning stroke. Therefore, there are 3 fundamental aspects for the correct application of the model:

1. *Computation of the line's electrical parameters, from which the nodal admittance matrix is obtained*: Since the analysis is performed in the frequency domain, skin effects in conductors and in ground plane can be directly considered when computing **Z**. For this purpose, the concept of complex penetration depth is applied [14]. In the case of the ground impedance, the Sunde approximation is considered, which has shown better results for low ground conductivity [15].
2. *Computation of the incident electromagnetic field from the relevant variables of the lightning stroke*: Formulation by Master and Uman is applied [4], considering also de Curray-Rubenstain correction for ground of finite conductivity [15]. The integro-differential set of equations defined in time domain is replaced by a more straightforward algebraic form in the frequency domain. The technique is described in Section III.
3. *Frequency-time transformation of the solution*: The numerical Laplace transform, which has been previously used in several works with good results (see for instance [10]-[13]), is applied in this work.

III. Computation of Incident Electromagnetic Field

Fig. 2 shows the geometrical configuration of a transmission line excited by an incident electromagnetic field due to a nearby lightning stroke.

Assuming ground as a perfect conductor, Master and Uman defined the components of electric and magnetic field produced by a differential segment of the lightning channel as [4]:

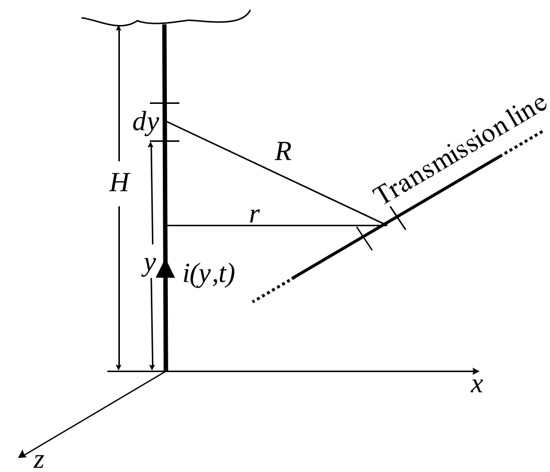


Fig. 2 Geometrical configuration of the transmission line-lightning channel arrangement

$$dE_r(r,y,t) = \frac{dy}{4\pi\varepsilon_0}\left[\begin{array}{l} \frac{3r(h-y)}{R^5}\int_0^t i(y,\tau-R/c)\,d\tau \\ \quad + \frac{3r(h-y)}{cR^4} i(y,t-R/c) \\ \quad + \frac{r(h-y)}{c^2R^3}\frac{\partial i(y,t-R/c)}{\partial t} \end{array}\right] \quad (15a)$$

$$dE_y(r,y,t) = \frac{dy}{4\pi\varepsilon_0}\left[\begin{array}{l} \frac{2(h-y)-r^2}{R^5}\int_0^t i(y,\tau-R/c)\,d\tau \\ \quad + \frac{3(h-y)-r^2}{cR^4} i(y,t-R/c) \\ \quad - \frac{r^2}{c^2R^3}\frac{\partial i(y,t-R/c)}{\partial t} \end{array}\right] \quad (15b)$$

$$dB(r,y,t) = \frac{\mu_0 dy}{4\pi}\left[\frac{r}{R^3} i(y,t-R/c) + \frac{r}{cR^2}\frac{\partial i(y,t-R/c)}{\partial t}\right] \quad (15c)$$

where h is the line's height, r is the horizontal distance between a point of the line along the z axis and the lightning channel and c is the velocity of light in free space. The lightning channel current propagating towards the cloud, $i(y,t)$, is defined from the MTLE model as [5]:

$$i(y,t) = \exp(-\alpha y)\, i(0, t - y/\upsilon) \quad (16)$$

$i(0,t)$ is the channel current at ground level (initial current), α is the attenuation constant of the current as it propagates in vertical direction (towards the cloud), and υ is the velocity of the return current. Transforming (15) to the Laplace domain and integration along the lightning channel and its image, it yields:

$$E_r(r,y,s) = \frac{\exp(-Rs/c)}{4\pi\varepsilon_0}\int_{-H}^{H} I(y,s)\left[\begin{array}{l} \frac{3r(h-y)}{R^5 s} \\ + \frac{3r(h-y)}{cR^4} + \frac{r(h-y)s}{c^2R^3} \end{array}\right] dy \quad (17a)$$

$$E_y(r,y,s)=\frac{\exp(-Rs/c)}{4\pi\varepsilon_0}\int_{-H}^{H} I(y,s)dy\left[\begin{array}{c}\frac{2(h-y)^2-r^2}{R^5 s}\\ +\frac{2(h-y)^2-r^2}{cR^4}-\frac{r^2 s}{c^2R^3}\end{array}\right]dy \quad (17b)$$

$$B(r,y,s)=\frac{\mu_0\exp(-Rs/c)}{4\pi}\int_{-H}^{H} I(y,s)\left[\frac{r}{R^3}+\frac{r}{cR^2}\right]dy \quad (17c)$$

where H is the cloud's height and $I(y,s)$ is the Laplace domain image of the lightning channel current, given by:

$$I(y,s)=\exp(-\alpha y)\exp(-ys/\upsilon)I(0,s) \quad (18)$$

Integrals defined in (17) are evaluated by means of an algorithm of numerical integration. However, until now ground has been considered a perfect conductor. In order to consider ground of finite conductivity, Cooray-Rubinstein expression is applied [15]:

$$\tilde{E}_r(r,y,s)=E_r(r,y,s)-\frac{cB(r,0,s)}{\sqrt{\varepsilon_{rg}+1/(\varepsilon_0\rho_g s)}} \quad (19)$$

where $\tilde{E}_r(r,y,s)$ is the horizontal electric field modified by considering the ground resistivity ρ_g, ε_{rg} is the relative ground permittivity, and $B(r,0,s)$ is the magnetic field at ground level for perfectly conducting ground. Vertical electric field is modified in a similar manner.

IV. Application Examples

A. Comparison with an experimental result

As a first example (and a means of validation), the method described in this paper is applied to reproduce the result from an experimental measurement on a reduce-scale model reported in [9]. The experimental setup consists of a simple 2 cm diameter, single conductor, 10 m high overhead line. The line is excited by the field of a return stroke current model located 70 m away from the line and approximately equidistant from the line terminations. The current is approximated at ground level by a triangular waveform with front time of 2 µs and time to half-value of 85 µs. For the example a magnitude of 34 kA is considered.

Interconnected aluminum plates were used to form the ground plane, therefore it is assumed as a perfect conductor.

Fig. 3 shows the waveform obtained with the frequency domain model presented in this work, and its comparison with the experimental measurement. A high agreement between the waveforms can be noticed.

B. Test case: Variation of the point of impact on a 3-phase line considering sagging between towers

In this example, transient overvoltages due to a nearby lightning stroke on a 3-phase line with horizontal configuration are analyzed. Ground wires are not considered in the tower arrangement. Main characteristics of the line are listed in Table I.

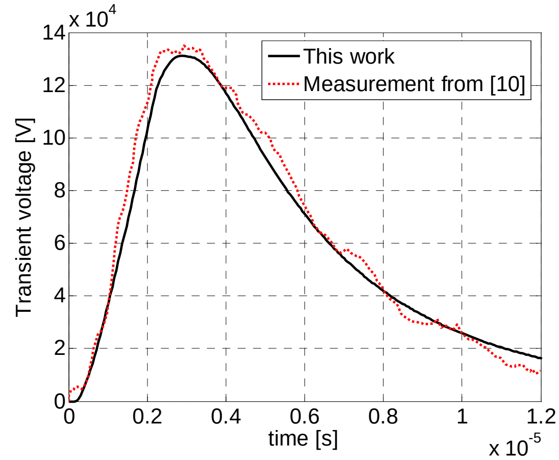


Fig.3 Comparison between the results from frequency domain analysis and the measurement reported in [10].

Table I. Transmission line data for example B

Data	*Value*
conductor radius	7.5 mm
height at towers	10 m
height at midspan	6.5 m
distance between towers	200 m
conductor resistivity	3.21×10^{-8} Ω-m
ground resistivity	100 Ω-m
distance between phases	2 m

The line is matched at both ends to avoid reflections. Electric and magnetic field components are computed from (17), considering also the modification given in (19) for finite ground conductivity.

The channel current at ground level considered for this example is a superposition of 2 Heidler functions, which has shown good agreement against field measurements. Parameters of the waveform can be found in ref. [5].

Simulations were performed for 3 different values of the point of impact: $P(25,50)$, $P(150,70)$ and $P(100,100)$, according to the arrangement shown in Fig. 4, in which x_p is the distance between the lightning stroke channel and the middle conductor (phase b). Coordinate $P(0,0)$ corresponds to node S of the line.

Figs. 5(a), (b) and (c) show the overvoltages at both line ends for the coordinates aforementioned. The differences in time delays and maximum overvoltages at nodes S and R of the line are consistent with the differences in the points of impact considered for the simulations.

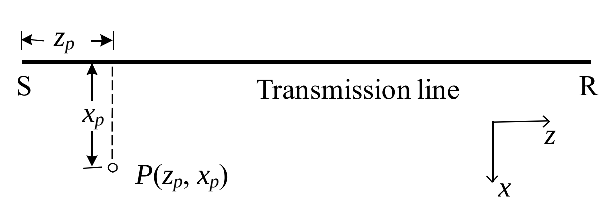
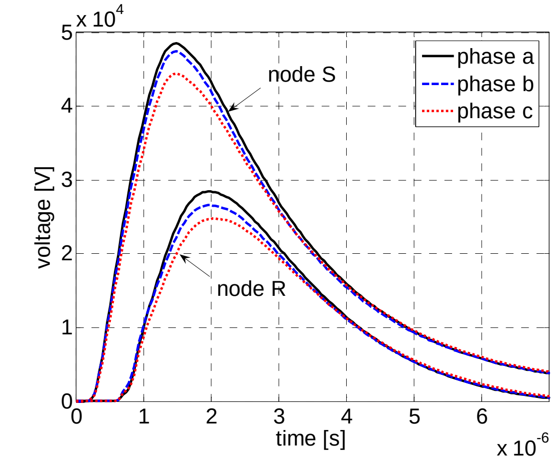
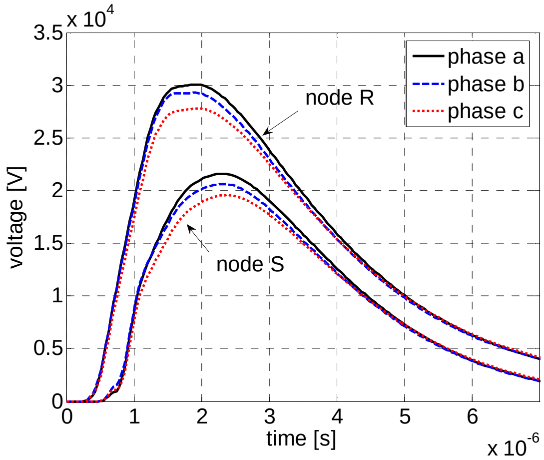


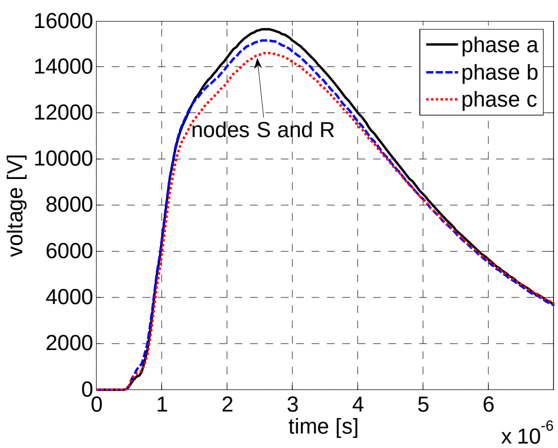
Fig.4 Coordinates of the point of impact.



(a)



(b)



(c)

Fig.5 Overvoltages obtained at the line ends for different points of impact of the lightning stroke: (a) *P*(25,50), (b) *P*(150,70), (c) *P*(100,100).

In addition, the effect of considering sagging between towers for this example is examined. Simulation for coordinates *P*(25,50) is repeated considering the line as completely uniform (height at towers = height at midspan = 10 m). Fig. 6 shows of the comparison for node S. An important difference in magnitude can be noticed. The maximum overvoltage at all phases assuming a uniform line is approximately 18% higher than considering the sagging between towers.

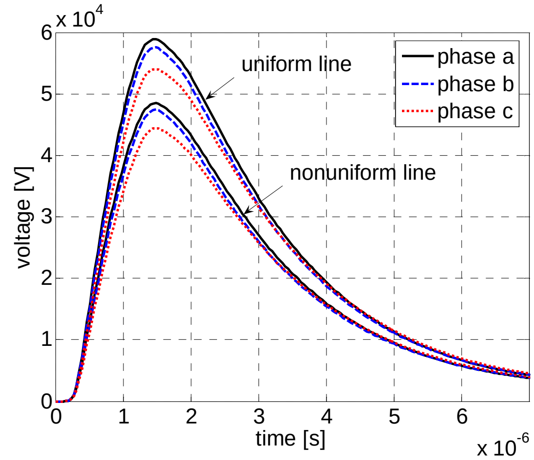


Fig.6 Overvoltages obtained at node S for point of impact *P*(25,50),

V. Conclusions

In this paper, a nonuniform multiconductor model of an illuminated transmission line has been described, applying a frequency domain approach. The model allows the inclusion of an incident electromagnetic field by means of lumped current sources connected only at the line ends. The particular case of a line excited by a field produced by a nearby lightning stroke is analyzed. For that purpose, a technique in the frequency domain to compute such field was implemented.

The model was initially validated by comparison with an experimental measurement on a reduced-scale model previously published, achieving high agreement between waveforms.

Then, the model was applied on a test case consisting of a 3-phase line with horizontal tower configuration. Different values of the point of impact of the lightning stroke were considered, in order to observe the differences in time delays and magnitudes of the overvoltages at the line ends. Besides, an additional simulation showed that neglecting the sagging between towers can result in an important overestimation of the maximum overvoltages.

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