

# Impact of Grounding Systems Frequency Dependency on Lightning Arresters Transient Response

K. Sheshyekani, F. Rachidi, S. H. H. Sadeghi, R. Moini, and M. Paolone

**Abstract**—The paper presents a frequency-domain method for the analysis of lightning surge response of overhead transmission lines equipped with surge arresters. The model takes into account the frequency dependence of the grounding system. The transmission line is represented in frequency domain by making use of the BLT (Baum-Liu-Tesche) equations while the grounding system, to which the arrester station is connected, is simulated using general electromagnetic model solved by means of the Method of Moment (MoM) in frequency domain. The lightning-originated overvoltages in presence of the surge arrester are then calculated in frequency domain using the Arithmetic Operator Method (AOM). The validity of the proposed approach is tested by comparing its results with those obtained using the Electromagnetic Transient Program (EMTP), for a simple configuration. The efficiency of the proposed technique is demonstrated considering the case of a more complex grounding system.

**Keywords:** lightning overvoltages, grounding systems, surge arresters.

## I. INTRODUCTION

The protection performances of surge arresters against lightning overvoltages are considerably affected by the transient behavior of grounding systems to which the arresters are connected. For instance, in the case of high values of the transient grounding impedance, the overvoltages might exceed the Lightning Impulse Withstand Level (LIWL) of power system apparatuses [1].

An accurate analysis of the transmission line overvoltages requires that the grounding system to which the arrester is connected be adequately incorporated in the computations. However, this is not a straightforward task in the sense that one should consider both the nonlinearity of the arrester and the frequency dependence of the grounding system. Time domain techniques (e.g. EMTP) are well suited to treat nonlinear components such as surge arresters. Nevertheless,

their application is less straightforward when the frequency-dependency of grounding systems needs to be included in the analysis. An analysis has been conducted for the proper modeling of grounding systems based on the transmission line approach to be incorporated in EMTP [2]. However, this technique is based on some approximations that may not provide accurate results for high frequency excitations.

The purpose of this paper is to propose a pure frequency domain analysis approach allowing the simulation of the transient response of transmission lines equipped with surge arresters. This study is a continuation of an effort to develop a method to incorporate the effect of frequency dependence of grounding systems into the analysis of power system transients.

The method is based on the frequency domain solution of the governing Electric Field Integral Equation (EFIE) for the grounding system using the Method of Moment (MoM), while the transmission line is modeled in frequency domain by use of the BLT equations [3]. The nonlinear behavior of the lightning arrester is then treated by means of the Arithmetic Operator Method (AOM) which performs nonlinear analysis entirely in the frequency domain [4].

## II. THEORETICAL MODELING

Consider the single-phase transmission line above a lossy ground shown in Fig.1 in which the lightning surge is originated by a direct strike to the phase conductor. As shown in Fig.1, the considered line geometry assumes a surge arrester connected to the grounding system at the end of the transmission line ( $x=L$ ).

For such a configuration, the arrester could be considered in series with the transient impedance of the grounding system  $Z_g(f)$ . This transient impedance represents the frequency-dependence of the grounding system. It is determined by means of a rigorous electromagnetic model (EFIE numerically solved by means of the MoM) [5]-[8].

Fig 2 shows a circuit representation of the model in which two distinct parts can be identified, namely: i) the terminal non-linear load formed by the arrester and the frequency-dependent ground impedance, and ii) the transmission line which is assumed to be linear.

The transmission line is characterized by a length  $L$ , a propagation constant  $\gamma$ , and a characteristic impedance  $Z_c$ . As shown in Fig. 3, this linear portion of the model can be represented by an equivalent Norton equivalent circuit. In frequency domain, the input admittance of the circuit is given by  $Y_{in}=1/Z_{in}$ , where  $Z_{in}$  is the input impedance of the line. The short-circuit current is defined as  $I_{sc}=V_{oc}/Z_{in}$ , where  $V_{oc}$  is the

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open-circuit voltage [3]. For the determination of  $Y_{in}$  in Fig. 3, the line is supposed to be open-circuited at its end ( $x=L$ ) and matched at its beginning ( $x=0$ ). The use of the BLT equations allows to evaluate the load voltage  $V(L)$ . The ratio of this open-circuit voltage to the excitation current provides the input impedance:

$$Z_{in} = \frac{V(L)}{I_0} = Z_c \frac{1+\rho_1 e^{-2\gamma L}}{1-\rho_1 e^{-2\gamma L}} \quad (1)$$

in which the reflection coefficient  $\rho_1 = 0$ , hence the input impedance is identical to the characteristic impedance of the line.

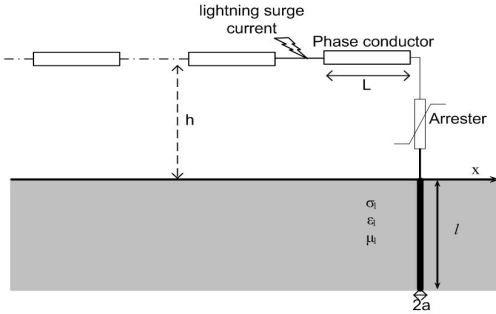


Fig. 1. Schematic diagram of the analyzed problem: a single conductor transmission line above a lossy ground with a grounded surge arrester placed at one line termination connected to a vertical grounding rod.

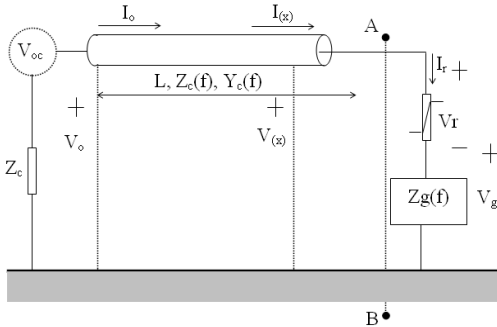


Fig. 2. Circuit representation of the problem shown in Fig. 1.

Similarly, the open-circuit voltage can be determined by calculating  $V(L)$  due to a single lumped voltage source at  $x=0$  and setting the reflection coefficient at the line end  $\rho_2 = 1$ :

$$V_{oc} = V_o \frac{e^{-\gamma L}(1-\rho_1)}{1-\rho_1 e^{-2\gamma L}} = V_o e^{-\gamma L} \quad (2)$$

By making reference to the simple model shown in Fig 1, the lightning current is divided in two equal parts traveling in both directions of the line. Before reflected surges reach the strike point, the injected voltage can be easily calculated by multiplying the characteristic impedance of the line by the injected lightning current. Thus the short-circuit current in Fig. 3, is given by,

$$I_{sc} = \frac{V_{oc}}{Z_{in}} = \frac{I_P Z_c e^{-\gamma L}}{Z_c} = I_P e^{-\gamma L} \quad (3)$$

where  $I_P$ , is the injected lightning impulse current.

Given the input impedance and the short circuit current, the Norton equivalent circuit is treated in frequency domain using the AOM.

With reference to Fig. 3, let  $\bar{V}_r$  and  $\bar{V}_g$  represent the arrester voltage (henceforth called arrester residual voltage) and the grounding system voltage, respectively.

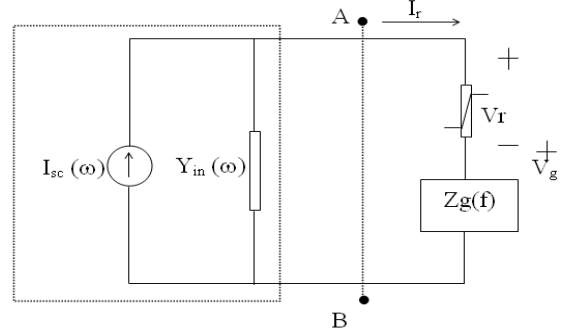


Fig 3. . Equivalent circuit model of the problem shown in Fig. 2.

Applying the Kirchhoff Current Law (KCL) to the surge arrester and the grounding system leads to

$$-\bar{I}_{sc} + \bar{Y}_{in}(\bar{V}_r + \bar{V}_g) + \bar{I}_r = 0 \quad (4)$$

where

$$\bar{V}_r = [V_{r,0}, J_{r,1}, J_{r,2}, \dots, J_{r,2P-1}, J_{r,2P}]^T \quad (5)$$

$$\bar{I}_{sc} = [I_0, J_{sc,1}, -I_{sc,2}, J_{sc,3}, -I_{sc,4}, \dots, J_{sc,n}, -I_{sc,n+1}, \dots, 0, 0]^T \quad (6)$$

$$\bar{I}_r = [I_{r,0}, J_{r,1}, J_{r,2}, \dots, J_{r,2P-1}, J_{r,2P}]^T \quad (7)$$

$$\bar{Y}_{in} = \begin{bmatrix} 0 & 0 & \dots & \dots & 0 & 0 \\ 0 & Y_{r1} & -Y_{i1} & & & \\ & Y_{i1} & Y_{r1} & & & \\ & & & Y_{r2} & -Y_{i2} & \\ & & & Y_{i2} & Y_{r2} & \\ 0 & & & & & \\ 0 & & & & & \end{bmatrix} \quad (8)$$

where

- $P$  is the number of frequencies in the output spectral vector, and  $n$  is the number of excitation frequencies.
- $Y_{rk}$  and  $Y_{ik}$  are the real and imaginary parts of  $Y_{in}(\omega_k) = Y_{rk} + jY_{ik}$ ,
- $V_{r,k}, I_{r,k}$  are the  $k$ -th components of arrester terminal voltage waveform,  $v_r(t)$ , and arrester current waveform,  $i_r(t)$  respectively.

$$v_r(t) = V_{r,0} + \sum_{k=1}^P \{V_{r,2k-1} \cos \omega_k t + V_{r,2k} \sin \omega_k t\} \quad (9-a)$$

$$i_r(t) = I_{r,0} + \sum_{k=1}^P \{I_{r,2k-1} \cos \omega_k t + I_{r,2k} \sin \omega_k t\} \quad (9-b)$$

$-I_{sc,k}$  ( $k=1,2,\dots,n$ ) is the  $k$ -th component of the short-circuit current waveform  $i_{sc}(t)$ :

$$i_{sc}(t) = I_1 \cos \omega_1 t + I_2 \cos \omega_2 t + \dots + I_K \cos \omega_K t \quad (9-c)$$

Note that  $I_{sc}$  is calculated only at the excitation frequencies. To take into account the surge arrester  $V$ - $I$  characteristic ( $i=f_n(vr)$ ), Equation (4) can be written as:

$$\bar{I}_{sc} = \bar{I}_r + \bar{Y}_{in} \bar{Z}_g \bar{I}_r + \bar{Y}_{in} \bar{V}_r \quad (10)$$

where  $\bar{Z}_g$  is the matrix form of the grounding system transient impedance.

$$\bar{Z}_g = \begin{bmatrix} 0 & 0 & & & & 0 & 0 \\ 0 & Z_{r1} & -Z_{i1} & & & & \\ & Z_{i1} & Z_{r1} & & & & \\ & & & Z_{r2} & -Z_{i2} & & \\ & & & Z_{i2} & Z_{r2} & & \\ 0 & & & & & & \\ 0 & & & & & & \end{bmatrix} \quad (11)$$

$Z_{rk}$  and  $Z_{ik}$  are the real and imaginary parts of  $Z_g(\omega_k) = Z_{rk} + jZ_{ik}$ .

It should be noted that for given excitation frequencies and the maximum order of arrester nonlinearity (of order 7), the so-called Basic Intermodulation Product Description (BIPD) table defines the basis for the spectral vectors by determining all of the non-negative combinations or weightings of these frequencies up to the maximum order of arrester nonlinearity. A detailed description of the BIPD table is found in [9].

To solve (10) for  $\bar{V}_r$ , we need to expand  $\bar{I}_r$  in terms of  $\bar{V}_r$ . This is done by converting the arrester  $V$ - $I$  characteristic in frequency domain. Recall from the theory of the Fourier transform that repeated multiplication of time-domain functions corresponds to repeated convolution in the frequency-domain, *i.e.*,

$$[v(t)]^n \xrightarrow{F} \underbrace{V(f) * V(f) * \dots * V(f)}_{n \text{ times}} \quad (12)$$

We can now use the Arithmetic Operator Method (AOM) to describe the convolution operations as matrix vector operations. The AOM uses basic arithmetic operations on signal spectra in the frequency-domain [9]. The use of AOM for calculating  $V_r(\omega)$  at each harmonic frequency is described in [10].

### III. NUMERICAL ANALYSIS

We consider the case of a lightning strike to the mid-span of a transmission line terminated with an arrester (Fig. 1). The conductor diameter is  $d=5.62$  cm and the vertical height at mid-span is  $h=22$  m. The conductor is lossy and its parameters (resistance, inductance and capacitance) are considered to be frequency dependent. The arrester is connected to a vertical grounding rod of circular cross-section of radius  $r=12.5$  mm, buried in a soil with a conductivity  $\sigma_1=0.01$  S/m and a relative permittivity  $\epsilon_r=10$ . Fig. 4 shows the input impedance

(henceforth called the harmonic impedance to ground) of the vertical grounding rod for various rod lengths, computed using the general electromagnetic approach.

The proposed approach is validated by comparing its results to those obtained using the Electromagnetic Transient Program (EMTP-RV [11-13]) in which the grounding system is properly modeled using an equivalent RLC circuit [14].

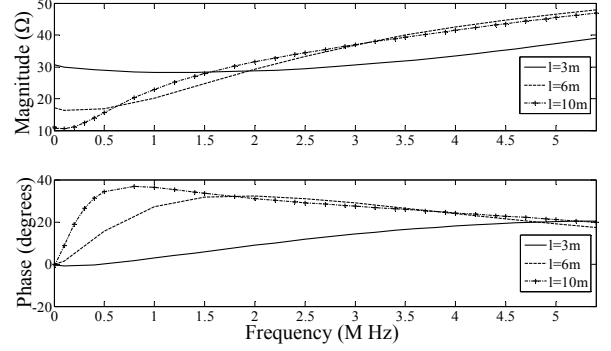


Fig. 4. Harmonic impedance to ground for a vertical grounding rod of various length  $l=3$  m, 6m, 10m and circular cross-section of radius  $r=12.5$  mm, buried in a soil with a conductivity  $\sigma_1=0.01$  mho/m and a relative permittivity  $\epsilon_r=10$ .

A 20-kA, 2/20  $\mu$ s lightning current is supposed to directly hit the transmission line at a point far from the both ends of the line; hence, the line could be considered with a match load at one propagation direction. Fig. 5 shows the transmission line overvoltages (for a line length of 400 m) considering two different grounding rod lengths, 3 and 6 m. Also shown in this figure are the overvoltages obtained by EMTP. It can be seen that the results provided with the proposed model are in good agreement with the ones provided by the EMTP model.

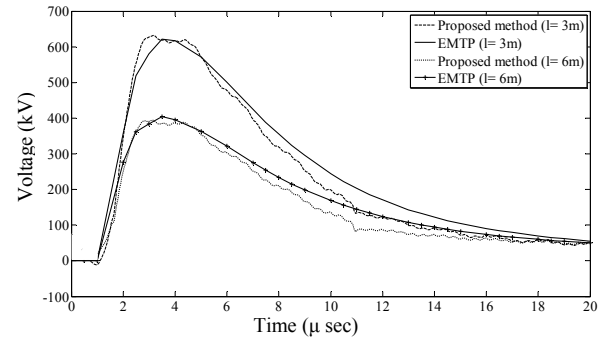


Fig. 5. Lightning overvoltage at the end of the transmission line of length  $L=400$  m considering two different grounding rod lengths, 3 m and 6 m and circular cross-section of radius  $r=12.5$  mm, buried in a soil with a conductivity  $\sigma_1=0.01$  mho/m and a relative permittivity  $\epsilon_r=10$ .

To demonstrate the ability of the proposed method to treat complex grounding systems, Fig. 6 shows the results when the arrester is connected to a grounding grid. The considered grounding grid is an equally-spaced 20m $\times$ 20m square. The depth of the grid is 1m and the conductors are of radius  $r=7$ mm. Soil conductivity and permittivity are respectively,  $\sigma_1=0.01$  mho/m and  $\epsilon_r=10$ . The results are compared with those obtained adopting a static model for the grounding system and show that it is important to take into consideration

the frequency dependence of the grounding system. It is seen that the inclusion of the frequency dependence of the grounding system affects primarily the early-time response of the overvoltages, namely their risetime and peak value. These parameters play an important role in the insulation coordination study and the selection of lightning arresters.

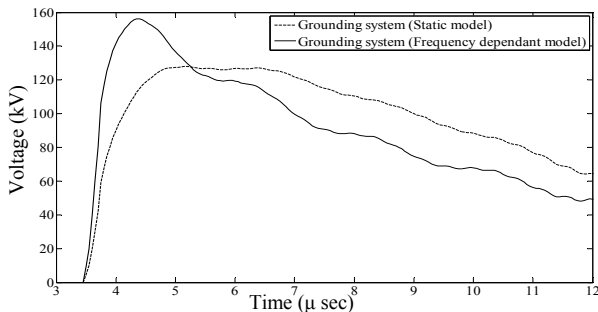


Fig.6. Overvoltages at the end of the transmission line. Arrester connected to a grounding grid.

#### IV. CONCLUSION

We presented a frequency-domain method for the analysis of lightning surge response of overhead transmission lines equipped with surge arresters. The model takes into account the frequency dependence of the grounding system. The transmission line is represented in frequency domain by making use of the BLT equations while the grounding system, to which the arrester station is connected, is simulated using general electromagnetic model solved by means of the Method of Moment in frequency domain. The lightning-originated overvoltages in presence of the surge arrester are then calculated in frequency domain using the Arithmetic Operator Method (AOM).

The validity of the proposed approach was first tested by comparing its results with those obtained using the Electromagnetic Transient Program, considering a simple configuration. Then, the efficiency of the proposed technique was demonstrated considering the case of a more complex grounding system.

It is worth noting that the theoretical foundation of the method is suited to be used in the case of more than one nonlinear load.

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