

An Investigation of Interpolation Methods Applied in Transmission Line Models for EMT Analysis

J. A. Gutierrez-Robles, L. A. Snider, J. L. Naredo, O. Ramos-Leaños

Abstract— Linear interpolation is the selected method to estimate intermediate values between samples for traveling waves determined by the principal line models in EMTP. As this is an order $O(\Delta t)$ method and the other numerical procedures in these models are order $O(\Delta t^2)$, an investigation was conducted to determine the effects of the linear interpolator on the accuracy of the line models. This investigation also focused on the improvement of the accuracy of the line models when the quadratic interpolator is adopted. Numerical performance of both interpolators, linear and quadratic, was also considered, and they were found to work well.

Keywords: transmission lines, traveling wave line models, EMTP, State-Space analysis, interpolations.

I. INTRODUCTION

COMMONLY used electromagnetic transient (EMT) simulation programs solve the nodal equations of a power system at sequential values of time that are separated by a fixed time-step Δt [1]. When an intermediate value (i.e., a value between samples) is needed, an interpolation process has to be applied [1]. This is usually the case when solving the transmission line equations.

The main transmission line models in EMTP are based on traveling wave principles by which an N -conductor line can involve up to N travel-times. The solution at one line terminal requires the knowledge of the solution at the other terminal at times corresponding to the various the line travel-times. In most cases, these delays usually are not multiples of the simulation time-step Δt , consequently an interpolation process must be applied.

The most common interpolation used with EMTP line models is a linear one [1]. This is an order one process, or $O(\Delta t)$. On the other hand, most other calculations inside line models are performed with the trapezoidal rule of integration which is order two, or $O(\Delta t^2)$. A basic principle in numerical analysis establishes that when combining numerical processes

with different orders of accuracy, the lowest one prevails. Thus, the question arises as to the extent the linear interpolation affects the accuracy of line models [2].

This paper presents an analysis of the errors introduced by the interpolation processes used within a traveling-wave line-model. The analysis includes numerical experiments and it also examines issues related to numerical efficiency. It is shown here that an order two interpolation is not necessarily much costlier in terms of computation time than the conventional linear interpolation.

II. LINE MODELING

Consider a multi-conductor transmission line, as the one depicted in Fig. 1, of length ℓ and with N independent phase-conductors. One of the line ends is located at $x=0$ and is called here the sending-end. The other end is at $x=\ell$ and is called the receiving-end. At the sending-end, V_0 represents the vector of the phase-conductor voltages and I_0 the vector of the currents being injected into the line. At the receiving end V_ℓ and I_ℓ are the respective vectors of conductor voltages and of injected currents. According to transmission line theory, the following expressions relate all terminal voltages and injected currents [3], [4]:

$$I_0 = Y_C V_0 - H [I_\ell + Y_C V_\ell] \quad (1a)$$

and

$$I_\ell = Y_C V_\ell - H [I_0 + Y_C V_0] \quad (1b)$$

where Y_C is characteristic admittance matrix of the line and H is its propagation matrix. Matrices Y_C and H are respectively given by the following expressions:

$$Y_C = Y^{-1} \sqrt{YZ} \quad (2)$$

and

$$H = \exp(-\sqrt{YZ} \ell). \quad (3)$$

Y and Z are the respective matrices of shunt admittance and of series impedance parameters of the line. The matrix of line propagation coefficients Γ is further defined as follows:

$$\Gamma = \sqrt{YZ} \quad (4)$$

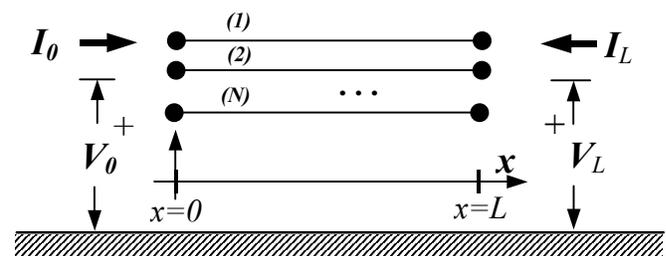


Fig. 1 Multi-conductor transmission line with end conditions.

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Expressions (1a) and (1b) are the basis for the principal transmission line models in EMTP; namely, Bergeron [1], FD-Line or J. Marti [3], and Wide-Band (WB) or Universal Line Model (ULM) [4]. This paper is concerned with the last two models.

For modeling purposes, it is convenient to express (1a) in the following general form:

$$\mathbf{I}_{IN} = \mathbf{I}_{SHUNT} - \mathbf{I}_{AUX} \quad (5a)$$

where

$$\mathbf{I}_{IN} = \mathbf{I}_0 \quad (5b)$$

$$\mathbf{I}_{SHUNT} = \mathbf{Y}_C \mathbf{V}_{IN} \quad (5c)$$

$$\mathbf{V}_{IN} = \mathbf{V}_0 \quad (5d)$$

$$\mathbf{I}_{AUX} = \mathbf{H} \mathbf{I}_{FAR} \quad (5e)$$

and

$$\mathbf{I}_{FAR} = \mathbf{I}_\ell + \mathbf{Y}_C \mathbf{V}_\ell \quad (5f)$$

Expressions (5a-f) further suggest the circuit representation in Fig. 2a for the line sending end.

It should be clear from the previous analysis and from (1b) that the receiving end of the line can be also represented by expressions analogous to (5a-f), as well as by the same circuit as in Fig. 2a. Moreover, the Fig. 2b circuit provides a complete representation for the transmission line. Note from this figure that the two end circuits are not connected directly. Their mutual influence is through the auxiliary sources of currents and this influence involves a delay due to the time for a wave to travel along the line.

The simulation of a transmission line transient requires solving twice the expressions (5a-f) sequentially; that is, one time per each line end. Note that the time domain forms of (5b), (5e) and (5f) involve convolutions. This paper focuses on the evaluation of (5e) in the time domain.

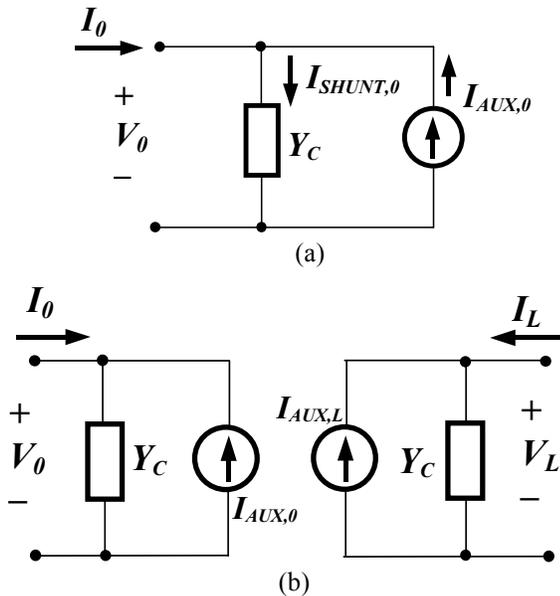


Fig. 2 a) Circuit representation of line sending end. b) Complete circuit representation of transmission line.

Consider the following time domain form of (5e):

$$\mathbf{i}_{AUX} = \mathbf{h} * \mathbf{i}_{FAR} \quad (6)$$

where the symbol $*$ represents the convolution operation which, for digital simulations, must be solved numerically. This is currently done through the following techniques: 1) Recursive Convolutions [5], 2) representation of \mathbf{H} through synthesized networks [3] and 3) State-Space Analysis in Discrete-Time (DTSS) [6]. Although these three methods are equivalent, for the purposes of this paper the latter one is considered the most convenient.

State-Space methods are introduced at line transient analysis by assuming that \mathbf{H} of (5e) can be approximated accurately by a rational function. It is further assumed that \mathbf{H} is a matrix function of the frequency variable ω and, through Analytic Extension, of the Laplace s -variable as well. Before proceeding with the rational approximation, it is convenient to decompose \mathbf{H} as follows:

$$\mathbf{H} = \mathbf{H}_1 \exp(-s \tau_1) + \mathbf{H}_2 \exp(-s \tau_2) + \dots + \mathbf{H}_{Nd} \exp(-s \tau_{Nd}), \quad (7)$$

where $\mathbf{H}_1, \mathbf{H}_2, \dots$ and \mathbf{H}_{Nd} are minimum phase matrix functions, $\exp(-s \tau_1), \exp(-s \tau_2), \dots$ and $\exp(-s \tau_{Nd})$ are pure delay factors and Nd is the number of different travel times along the line owing to multimode propagation, $Nd \leq N$. Recall that a function is of minimum phase if its real and imaginary parts are uniquely related through the Hilbert Transform. The reason for decomposing \mathbf{H} as in (7) is that minimum phase functions are easy to fit rationally with high accuracy and low order, [3] and [4].

Let now each one of the matrices $\mathbf{H}_1, \mathbf{H}_2, \dots$ and \mathbf{H}_{Nd} be fitted rationally as follows:

$$\mathbf{H}_i = \sum_{k=1}^{K(i)} \frac{1}{s - p_{i,k}} \mathbf{R}_{i,k} \quad (8)$$

where $p_{i,k}$ are the fitting poles, $\mathbf{R}_{i,k}$ are the corresponding residue matrices and $K(i)$ is the order of the fit for \mathbf{H}_i . On replacing (8) in (7):

$$\mathbf{H} = \sum_{i=1}^{Nd} \left(\sum_{k=1}^{K(i)} \frac{1}{s - p_{i,k}} \mathbf{R}_{i,k} \right) \exp(-s \tau_i) \quad (9)$$

Expression (9) is then introduced in (5e) as follows:

$$\mathbf{I}_{AUX} = \sum_{i=1}^{Nd} \sum_{k=1}^{K(i)} \mathbf{W}_{i,k} \quad (10a)$$

with

$$\mathbf{W}_{i,k} = \frac{1}{s - p_{i,k}} \mathbf{R}_{i,k} \mathbf{I}_{FAR} \exp(-s \tau_i); \quad \begin{aligned} i &= 1, 2, \dots, Nd \\ k &= 1, 2, \dots, K(i) \end{aligned} \quad (10b)$$

Next, the continuous-time State Space forms equivalent to (6) are obtained as follows by applying the Inverse Laplace transform: to (10a) and (10b):

$$\mathbf{i}_{AUX} = \sum_{i=1}^{Nd} \sum_{k=1}^{K(i)} \mathbf{w}_{i,k} \quad (11a)$$

and

$$\frac{d\mathbf{w}_{i,k}}{dt} = \mathbf{p}_{i,k} \mathbf{w}_{i,k} + \mathbf{R}_{i,k} \mathbf{i}_{FAR}(t - \tau_i); \quad (11b)$$

$$i = 1, 2, \dots, Nd$$

$$k = 1, 2, \dots, K(i)$$

Finally, for the digital solution of (11a) and (11b), these two expressions must be converted to discrete time State–Space. This is usually done in EMTP by applying the mid–point differentiation rule in (11b), thus obtaining:

$$\mathbf{w}_{i,k} = \alpha_{i,k} \mathbf{w}'_{i,k} + \mathbf{B}_{i,k} [\mathbf{i}_{FAR}(t - \tau_i) + \mathbf{i}'_{FAR}(t - \tau_i)]; \quad (12a)$$

$$i = 1, 2, \dots, Nd$$

$$k = 1, 2, \dots, K(i)$$

where

$$\alpha_{i,k} = (2 + \Delta t p_{i,k}) / (2 - \Delta t p_{i,k}) \quad (12b)$$

$$\mathbf{B}_{i,k} = (\Delta t / (2 - \Delta t p_{i,k})) \mathbf{R}_{i,k} \quad (12c)$$

$$\mathbf{w}'_{i,k} = \mathbf{w}_{i,k}(t) = \mathbf{w}_{i,k}(t - \Delta t) \quad (12d)$$

$$\mathbf{i}'_{i,k}(t - \tau) = \mathbf{i}_{i,k}(t - \tau - \Delta t) \quad (12e)$$

and Δt is the integration time–step.

Notice in (12d) and (12e) that primed variables represent previous variable–values obtained one time–step Δt before. For the proper simulation of the distributed features of transmission lines it is required that the integration time–step Δt is fixed at a value smaller than any of the line travel–times τ_i . When this is the case, $\mathbf{i}_{FAR}(t - \tau_i)$ and $\mathbf{i}'_{FAR}(t - \tau_i)$ in (12a) can be determined from their previously obtained values, or from their initial conditions; then, the state variables $\mathbf{w}_{i,k}$ is updated by (12a) and, subsequently, \mathbf{i}_{AUX} is updated by (11a). Expressions (11a) and (12a) can therefore be consider proper DTSS forms to evaluate \mathbf{i}_{AUX} sequentially for a transmission line model.

Evaluation of $\mathbf{i}_{FAR}(t - \tau_i)$ and $\mathbf{i}'_{FAR}(t - \tau_i)$ must be done usually through interpolations; unless the ratio $\tau_i / \Delta t$ is an integer, say equal to m . In this last case, $\mathbf{i}_{FAR}(t - \tau_i) = \mathbf{i}_{FAR}(t - m\Delta t)$ simply corresponds to a value obtained at m Δt –steps before. Similarly, $\mathbf{i}'_{FAR}(t - \tau_i)$ is given by its value at the previous $m+1$ Δt –steps. Interpolations, however, are required for the vast majority of EMT studies. Multi–conductor lines involve several travel times and EMT studies usually include several lines. The possibility of finding a suitable value of Δt to avoid interpolations is thus highly improbable.

III. INTERPOLATION

EMTP simulations are performed sequentially with a time step of constant length that must be previously specified by the user. The independent variable of time becomes discrete assuming values of the form $t = \ell \Delta t$, with $\ell = 1, 2, \dots, etc$, and the simulated signals are sampled approximations of those in the system being simulated. The selection of a suitable time interval Δt usually relies on the user’s experience. A guideline

can be provided by the Nyquist–Shannon Sampling Theorem [7]. According to this, if the bandwidth (BW) of the transient phenomenon is known, the sampling interval Δt must be at most equal to the inverse of the bandwidth:

$$\Delta t \leq \frac{1}{BW} \quad (13)$$

where BW is in Hertz. The maximum sampling interval, corresponding to the equality in (13), is called the Nyquist limit. From practical experience, an appropriate time–step for an EMTP simulations often is within one fifth and one tenth of this Nyquist limit.

The Sampling Theorem refers to signals that are strictly band–limited, meaning by this that the Energy outside the bandwidth is zero. This Theorem also provides the following ideal interpolator for fully recovering the signal values between samples, provided the sampling interval complies with (13).

$$\phi(t) = \frac{\sin(\pi t / \Delta t)}{(\pi t / \Delta t)} \quad (14)$$

Figure 3 provides a plot of this interpolator.

In spite of the usefulness of the Sampling Theorem to select a simulation time–step, one has to bear in mind that physical signals are not strictly band–limited, that information is always lost at sampling processes and that the ideal interpolator and its approximations may not be the best practical choice to retrieve signal values between samples.

A signal that arises frequently in transient analysis is the step function. It often is caused by switching operations and reflections on transmission lines. Figure 4 illustrates a sampled step function and its retrieval through the ideal interpolator of (14). It is clear that interpolation by (14) or its approximations are inappropriate for transient analysis as the overshoot of the retrieved signal amounts to a 9 % error.

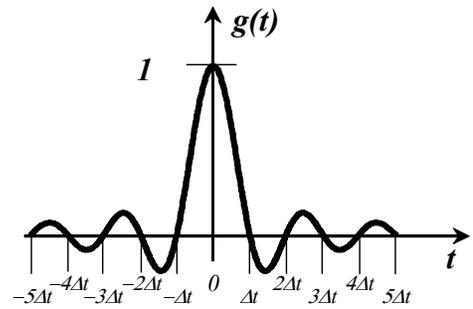


Fig. 3 Ideal interpolator.

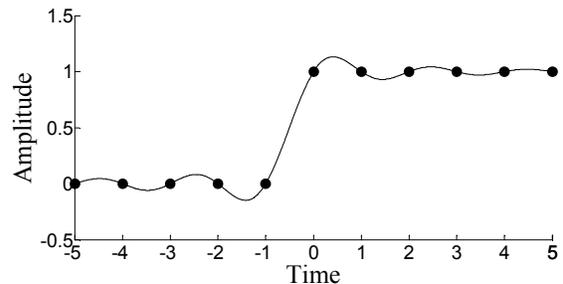


Fig. 4 Sampled step function and its retrieval by an ideal interpolator.

Line models of EMTP resort to the linear interpolator for evaluating $\mathbf{i}_{FAR}(t-\tau)$ [1]. This is a simple and effective process that requires a very small amount of computation. The linear interpolator is an order one $O(\Delta t)$ numerical process; that is, as the time step Δt approaches zero, the error becomes proportional to $1/\Delta t$. Most other processes inside EMTP and in line models are order two $O(\Delta t^2)$ with an error in the estimate becoming proportional to $1/(\Delta t^2)$. An example of this is the DTSS form (12a) to evaluate convolutions that are obtained from the mid-point rule, an order two process. A general rule of Numerical Analysis is to avoid combining processes of different orders as the lowest order will prevail. This issue motivates the following comparison between linear and quadratic interpolations.

A. Linear Interpolation

Consider two given values for a line travel-time τ and a simulation time step Δt , then

$$\tau = m\Delta t + \varepsilon_1 \quad (15a)$$

and

$$\varepsilon_2 = \Delta t - \varepsilon_1 \quad (15b)$$

where m is the integer part of $\tau/\Delta t$ and ε_1 is the remainder, smaller than Δt . Suppose that the current time at an ongoing simulation is $t = n\Delta t$; then

$$\mathbf{i}_{FAR}(t-\tau) = \mathbf{i}_{FAR}((n-m)\Delta t - \varepsilon_1).$$

On assuming linear variation between $\mathbf{i}_{FAR}((n-m-1)\Delta t)$ and $\mathbf{i}_{FAR}((n-m)\Delta t)$, as illustrated by Fig.5:

$$\mathbf{i}_{FAR}(t-\tau) = a_0\mathbf{I}_0 + a_1\mathbf{I}_{-1} \quad (16a)$$

where

$$a_0 = \varepsilon_2/\Delta t \quad (16b)$$

$$a_1 = \varepsilon_1/\Delta t \quad (16c)$$

$$\mathbf{I}_0 = \mathbf{i}_{FAR}((n-m)\Delta t) \quad (16d)$$

and

$$\mathbf{I}_{-1} = \mathbf{i}_{FAR}((n-m-1)\Delta t) \quad (16e)$$

Note that implementing linear interpolator (16a) amounts to performing two scalar-to-vector multiplications and one two-vector addition. As a side note about the computer implementation of the line model, assuming that the delay τ being considered is the largest one at an ongoing EMT study, a circular memory buffer of size $m+1$ must be created to hold the most recent values of \mathbf{i}_{FAR} and the retrieval of values \mathbf{I}_0 and \mathbf{I}_{-1} of (16d) and (16e) is done considering that operations $(n-m)$ and $(n-m-1)$ are in modulo $m+1$ arithmetic.

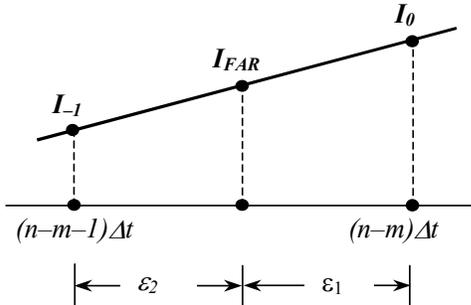


Fig. 5 Linear interpolation scheme.

B. Quadratic Interpolation

To implement the quadratic interpolator assume a parabolic variation for \mathbf{I}_{FAR} among the values $\mathbf{i}_{FAR}((n-m-2)\Delta t)$, $\mathbf{i}_{FAR}((n-m-1)\Delta t)$ and $\mathbf{i}_{FAR}((n-m)\Delta t)$ as illustrated in Fig. 6. Thus, the following estimate is obtained:

$$\mathbf{i}_{FAR}(t-\tau) = b_0\mathbf{I}_0 + b_1\mathbf{I}_{-1} + b_2\mathbf{I}_{-2} \quad (17a)$$

with

$$b_0 = \varepsilon_2(\varepsilon_2 + \Delta t)/(2\Delta t^2), \quad (17b)$$

$$b_1 = \varepsilon_1(\varepsilon_2 + \Delta t)/\Delta t^2 \quad (17c)$$

and

$$b_2 = -\varepsilon_1\varepsilon_2/(2\Delta t^2). \quad (17d)$$

It should be noticed at (12a) that \mathbf{i}_{FAR} appears added to \mathbf{i}'_{FAR} . The latter term can be estimated using the same values as \mathbf{i}_{FAR} at (17a). This is illustrated also by Fig. 6 and the order two interpolator is:

$$\mathbf{i}'_{FAR}(t-\tau) = c_0\mathbf{I}_0 + c_1\mathbf{I}_{-1} + c_2\mathbf{I}_{-2}, \quad (18a)$$

where

$$c_0 = -\varepsilon_1\varepsilon_2/(2\Delta t^2), \quad (18b)$$

$$c_1 = \varepsilon_2(\varepsilon_1 + \Delta t)/\Delta t^2 \quad (18c)$$

and

$$c_2 = \varepsilon_1(\varepsilon_1 + \Delta t)/(2\Delta t^2). \quad (18d)$$

On adding (17a) and (18a):

$$\mathbf{i}_{FAR} + \mathbf{i}'_{FAR} = d_0\mathbf{I}_0 + d_1\mathbf{I}_{-1} + d_2\mathbf{I}_{-2}, \quad (19a)$$

where

$$d_0 = \varepsilon_2^2/\Delta t^2, \quad (19b)$$

$$d_1 = (2\varepsilon_1\varepsilon_2 + \Delta t^2)/\Delta t^2 \quad (19c)$$

and

$$d_2 = \varepsilon_1^2/\Delta t^2. \quad (19d)$$

Expression (19a) is the recommended interpolator of order two to be implemented along with (12a). It takes three scalar-to-vector multiplications and two two-vector additions.

IV. TEST CASES.

To test the previously described interpolators a simulation was performed consisting of the energizing of a 150 km long single-phase transmission line with a 1 A step of current. The current source has an admittance of 1/600 S and the line receiving-end is open. Figure 7 provides a diagram of the line connections and Fig. 8 provides the transversal geometry and ground resistivity of the line.

The linear and the quadratic interpolators are used to obtain voltage-wave responses at both line ends, the sending and at the receiving ones. The sending-end waveform is

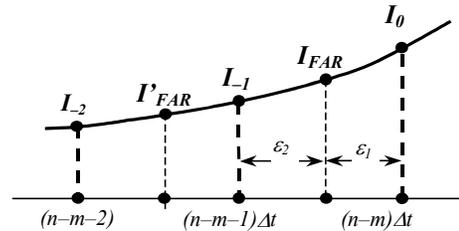


Fig. 6 Quadratic interpolation scheme.

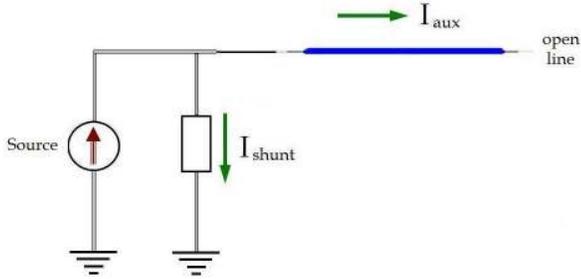


Fig. 7 Connection diagram of test-case line.

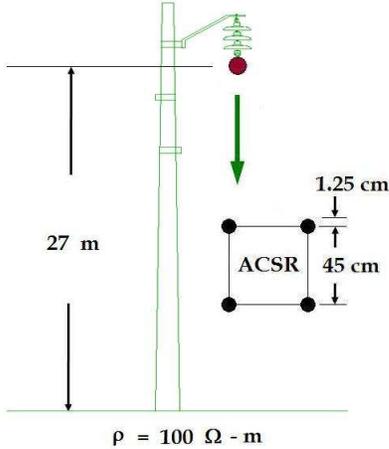


Fig. 8 Transversal line geometry and earth resistivity for test-case line.

shown in Figs. 9 and 10 and that for the receiving-end is shown in Figs. 11 and 12. Note that Fig. 10 is a close-up of Fig. 9 and Fig. 12 is a close-up of Fig. 11. The simulations are performed with a $32 \mu\text{s}$ time-step. Note that Figs. 9 to 12 include waveforms obtained without applying an interpolator, as well as those calculated with the Numerical Laplace Transform (NLT) technique as described in [8]. For the non-interpolated results, the closest sample is assigned to the required intermediate value. The NLT waveforms are obtained with a relative error-level specification of 10^{-4} .

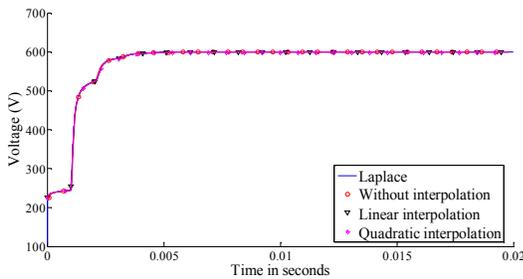


Fig. 9 Sending-end voltage response.

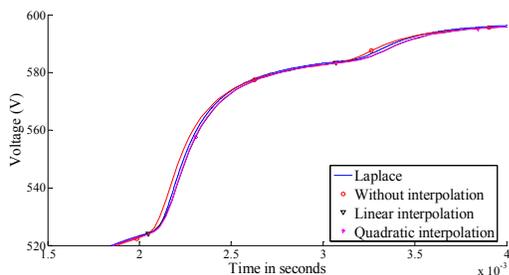


Fig. 10 Sending-end, voltage response close-up.

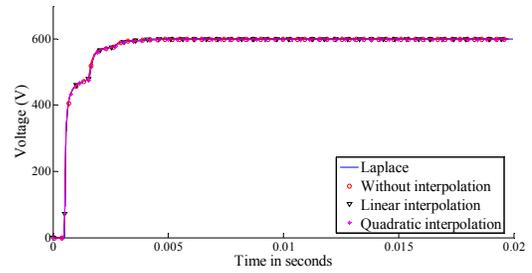


Fig. 11 Receiving-end voltage response.

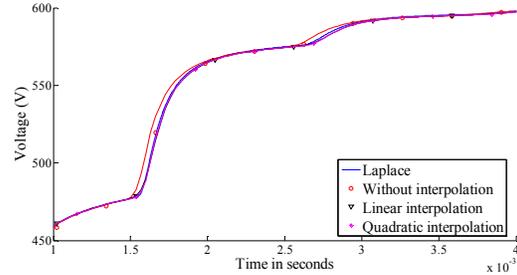


Fig. 12 Receiving-end, voltage response close-up.

Figure 13 provides the plots of percent errors for the waves at the sending-end, while Fig. 14 provides the same plots for the receiving-end voltages. It is observed in all these results that the linear interpolator is considerably accurate, given its simplicity. It is also observed there that the quadratic interpolator brings relatively small increases in the accuracy levels. At the sending-end the linear interpolator presents a maximum error of 1.72 %, while with the quadratic one this is 1.04 %. At the receiving-end the corresponding maximum errors are 10.7 % for the linear interpolator and 9.33 % for the quadratic one. In general, the accuracy improvement at maximum error peaks with the quadratic interpolator is about 1 %.

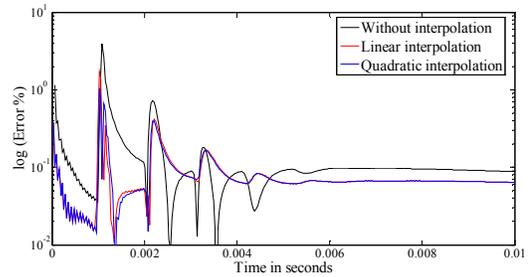


Fig. 13 Sending-end percent errors.

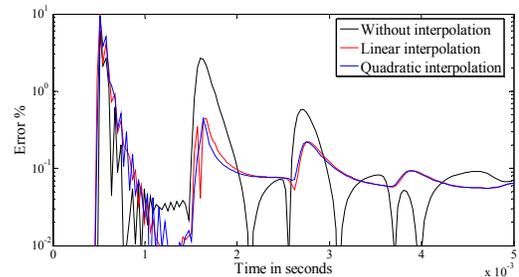


Fig. 14 Receiving-end percent errors.

V. CONCLUSIONS

It has been pointed out in this paper that most numerical processes involved in the simulation of transients by EMTP are of second order, and that one notable exception is the linear interpolation method used by most line models for estimating intermediate values between samples of traveling waves. It has been pointed out also that a generally accepted rule in Numerical Analysis is to avoid the mixing of processes with different orders, the reason being that the lower order methods tend to degrade the accuracy of the higher order ones.

A question has thus arisen as to the extent at which those linear interpolations degrade transient analysis results and an investigation has been conducted to address this issue. The main results of this investigation are summarized as follows. It has been found that the accuracy of the linear interpolator being employed is quite good, given its simplicity and economy in terms of computations.

It has been also found that the quadratic interpolator offers a reasonable increase of accuracy for a small increase in computational cost. Nevertheless, mention should be made here to the fact that this investigation have been focused so far on switching transients. Future work should consider fast and ultra-fast transients.

VI. REFERENCES

- [1] Dommel, H.W., EMTP Theory Book, Microtran Power System Analysis Corporation, 4689 W. 12th. Avenue, Vancouver, B.C. V6R 2R7, Canada, 2nd. edition, May 1992.
- [2] T. De Rybel, J. Marti and M. Hodgson, "Analytical validation of time-step interpolation in transient insular nodal analysis", Proc. Acoustics 2008, Paris, 3733-3738 (2008).
- [3] J. R. Marti, "Accurate Modeling of Frequency-Dependent Transmission Lines in Electromagnetic Transient Simulations." IEEE Transactions on Power Apparatus and Systems, Vol. PAS-101, pp. 147-157, 1982.
- [4] Atef Morched, Bjørn Gustavsen,, Manoocher. Tartibi, "A universal model for accurate calculation of electromagnetic transients on overhead lines and underground cables", IEEE Trans. on Power Delivery, Vol. 14, No. 3, pp. 1032-1038, July 1999.
- [5] A. Semlyen, Dabuleanu, "Fast and Accurate Switching Transient Calculations on Transmission Lines with Ground Return using Recursive Convolutions", *Trans. On Power Apparatus and Systems*, vol. PAS-94, No.2, pp. 561-571, 1975
- [6] Adam Semlyen and Mansour H. Abdel-Rahman, "A State Variable Approach for the Calculation of Switching Transients on a Power Transmission Line", *IEEE Transactions on Circuits and Systems*, Vol. CAS-29, No. 9, pp. 624-633, September 1982.
- [7] J. G. Proakis and D. G. Manolakis, *Digital Signal Processing. Principles, Algorithms and Applications*, Prentice-Hall, 4th Edition, 2007.
- [8] J. L. Naredo, J. Mahseredjian, Ilhan Kocar, J. A. Gutiérrez-Robles, J. A. Martínez-Velasco, "Frequency Domain Aspects of Electromagnetic Transient Analysis of Power Systems.", IEEE/PES Tutorial on Electromagnetic Transients in Power Systems. Solution Techniques, Applications, Simulation Tool Development, Editor: J. A. Martínez-Velasco, Chapter 3, pp. 20-36, **IEEE Code Number in Process**, July 2010.