A Practical Steady-State Initialization Method for Electromagnetic Transient Simulations

Taku Noda and Kiyoshi Takenaka

***Abstract*— The purpose of most electromagnetic transient (EMT) simulations is to calculate disturbances occurring from steady-state conditions. Therefore, a steady-state initialization function is required in an EMT analysis program so as to enable a simulation which directly starts from a steady state. This avoids a long EMT computation to establish the steady state from a zero initial condition. This paper investigates the performance of a primitive but quite practical steady-state initialization method. The method is divided into three stages. The first stage carries out a conventional single-phase power-flow calculation that gives a balanced solution as a first approximation. Based on the solution, the second stage calculates the three-phase 50/60-Hz steady-state solution considering unbalanced circuit conditions. Finally, the third stage lets the EMT simulation run from the 50/60-Hz solution for a certain number of cycles to obtain harmonics in the solution. In this way, we can obtain a steady-state solution taking harmonics and unbalanced conditions into account by specifying positive-sequence power-flow constraints. The method is applied to the simulations of a distribution line with a STATCOM and a transmission line with a line-commuted HVDC converter.**

***Index Terms*— Electromagnetic transient analysis, Power-flow calculation, Power system harmonics, Steady-state initialization, and Unbalanced condition.**

I. Introduction

MOST electromagnetic transient (EMT) simulations are for calculating disturbances occurring from steady-state conditions. Therefore, a steady-state initialization function is required in an EMT analysis program so as to enable a simulation which directly starts from a steady state. This avoids a long EMT computation to establish the steady state from a zero initial condition. For steady-state initialization, the following points should be noted:

- A steady-state solution by nature includes harmonics generated by nonlinear loads and power-electronics devices.
- Due to unbalanced impedances of transmission lines and loads, voltages and currents are not perfectly balanced.
- It is convenient to calculate a steady-state solution by specifying power-flow constraints.
- The circuit to be analyzed includes distributed-parameter line models.

T. Noda is with Electric Power Engineering Research Laboratory, CRIEPI (Central Research Institute of Electric Power Industry), 2-6-1 Nagasaka, Yokosuka, Kanagawa 240-0196, Japan (e-mail: takunoda@ieee.org).

K. Takenaka is with System Engineering Research Laboratory, CRIEPI, 2-11-1 Iwado-kita, Komae, Tokyo 201-8511, Japan (e-mail: takenaka@criepi.denken.or.jp).

Paper submitted to the International Conference on Power Systems Transients (IPST2011) in Delft, the Netherlands, June 14–17, 2011

Xia and Heydt proposed a method to solve harmonic circuit equations together with power-flow constraints [1], [2]. In this pioneer work, however, only positive-sequence circuit equations are considered, and thus, unbalanced circuit conditions are not represented. The methods proposed in [3]–[5] take both harmonics and unbalanced conditions into account, but they calculate a steady-state solution by specifying the waveforms of the voltage and the current sources, not by power-flow constraints. The methods proposed in [6]–[8] can take all of harmonics, unbalanced conditions and power-flow constraints into account. A serious and intrinsic problem of these methods, when applied to steady-state initialization, is the fact that they are based on the shooting method [9]. Since the shooting method assumes that the circuit equations are described by a set of ODEs (ordinary differential equations), distributed-parameter line modes which are described by PDEs (partial differential equations) cannot be included [10].

Considering the above, this paper investigates the performance of a primitive but quite practical steady-state initialization method. The method consists of the three stages below, and thus, it is called a Three-Stage Method in this paper:

1) With specified power-flow constraints, a conventional single-phase power-flow calculation is carried out.
2) The generators and the upper-voltage substation busses are replaced with three-phase sinusoidal voltage sources obtained by the power-flow result. Then, the three-phase 50/60-Hz steady-state solution is calculated.
3) From the 50/60-Hz solution, the EMT simulation is kept running for a certain number of cycles.

The first stage gives a balanced 50/60-Hz solution as a first approximation. Then, the second stage takes into account unbalanced conditions at the power frequency. The third stage generates harmonics by actually calculating transients. In this way, we can obtain a steady-state solution considering harmonics and unbalanced conditions by specifying positive-sequence power-flow constraints. This is actually not a novel method. Some users of EMT analysis programs may manually carry out these stages for steady-state initializations in their routine simulations.

This paper first describes the Three-Stage Method, and then it is applied to the simulations of a distribution line with a STATCOM and a transmission line with a line-commuted HVDC converter for investigating the performance.

II. Three-Stage Method

Fig. 1 shows the flowchart of the Three-Stage Method. The procedure of each stage is described below.

A. Stage 1: Single-Phase Power-Flow Calculation

At Stage 1, a conventional single-phase (positive-sequence) power-flow calculation is carried out with power-flow constraints specified by the user. The algorithm used to calculate the power flow is a standard one based on the Newton-Raphson iteration, and it can be found in textbooks like [11]. Since the algorithm assumes a balanced circuit and considers the power frequency only, the solution obtained is a balanced 50/60-Hz approximation.

B. Stage 2: Three-Phase AC Solution

The result of Stage 1 gives the voltages (the voltage magnitudes and their phase angles) of all busses in the circuit. With this information, we replace the generators and the upper-voltage busses in the circuit with three-phase sinusoidal voltage sources. Then, the three-phase 50/60-Hz (AC) steady-state solution of the circuit is calculated by solving the circuit equations in the $j\omega$ domain. The solution takes into account unbalanced circuit conditions at the power frequency due to untransposed transmission lines and so on.

Obtaining the three-phase AC solution is important not only for considering unbalanced conditions but also for modeling loads and transformers. It should be noted that all dynamic elements such as inductors, capacitors and distributed-parameter line models in the circuit have to be initialized for the subsequent EMT simulation at Stage 3. Initializing the dynamic elements requires their branch voltages or currents in the AC steady state.

In a power-flow calculation, a load is completely characterized and modeled by the two parameters P and Q, active and reactive power values. On the other hand, there are a variety of ways to model a load in EMT simulations. When the values of P and Q $(Q > 0)$ of a load are given, it can be realized either by a series R–L or a parallel R–L circuit. Or, an even more complicated model, which represents the ratio of induction motors to the total capacity or the skin effects inside the load for instance, can be used [12]. These different load models give different responses under transient conditions. In the case of an EMT simulation, therefore, the user will choose which load model to use depending on the purpose of the simulation, and the simulation program cannot know in advance how many dynamic elements are included in the load model and how they are connected to the rest of the model. Thus, the information obtained at Stage 1, the voltages of all busses, is not enough to initialize the dynamic elements arbitrary contained in the load model. Calculating the three-phase AC solution solves this problem. When the solution is calculated, the internal circuit of the load model is treated just as a part of the whole circuit. Thus, the solution gives the steady-state voltages across or the currents through the dynamic elements inside the load models, and with this information the dynamic elements can be initialized for the subsequent EMT simulation.

A similar situation applies to transformer models. The user decides what transformer model to use depending on the purpose of simulation, and the simulation program cannot know in advance how many and how dynamic elements are included in the transformer model. In addition to this, the following point also justifies obtaining the three-phase AC solution. Since the power-flow result obtained at Stage 1 is per-unit quantities, it does not directly give the voltages at both sides of a transformer. If nominal voltages of both sides of the transformer were specified, this problem could be solved. However, obtaining the three-phase AC solution directly gives the voltages at both sides, since the ideal transformer inside the transformer model automatically takes into account the turns ratio and thus solves this problem without specifying the nominal voltages. This is simpler in terms of both data input and programming.

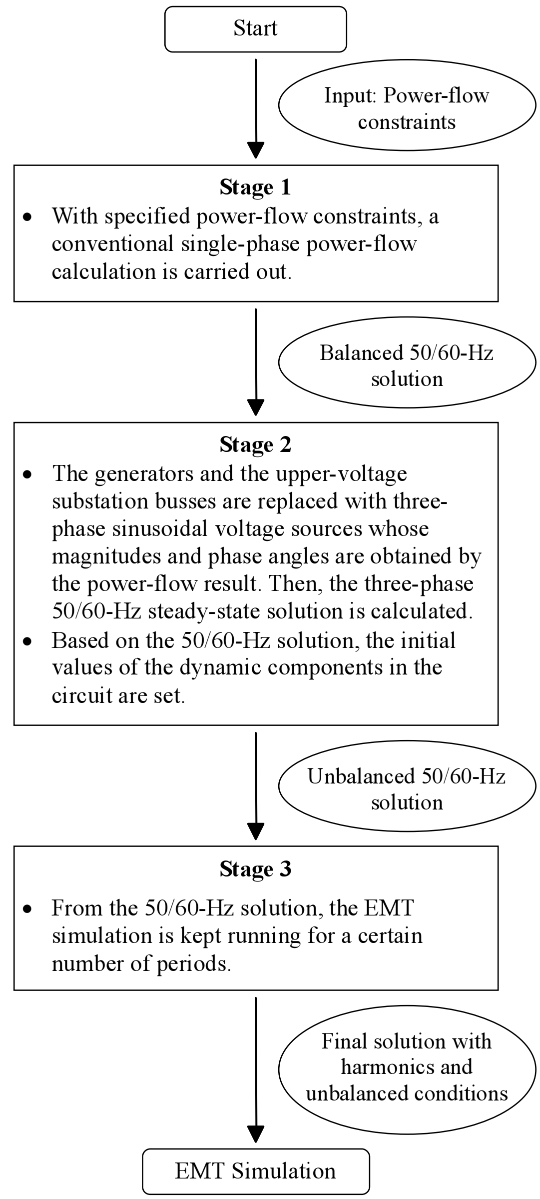

Fig. 1. Flowchart of the Three-Stage Method.

The circuit equations to obtain the three-phase AC solution can be formulated by the traditional nodal analysis [13], the modified nodal analysis [14] or the sparse tableau approach [15]. The traditional nodal analysis has restrictions on handling voltage sources and current-controlled sources [16]. Except this point, any one of these methods can be used. In terms of programming, the same formulation method as that for the EMT solution should be chosen for the three-phase AC solution so as to simplify the code. Assume that

$$Fx = y \tag{1}$$

is the circuit equations for the EMT solution formulated by one of those formulation methods, where F is the coefficient matrix, x is the unknown vector and y is the vector of driving sources. F includes the conductance values of the circuit elements. If the conductance values of the dynamic elements in F are replaced with their corresponding admittance values at the power frequency, (1) readily becomes the circuit equations for the three-phase AC solution. Thus, the admittance values of the dynamic elements are shown below with ω_0 being the power frequency multiplied by 2π. The admittance of an inductor L and that of a capacitor C are given by

$$Y_L = \frac{1}{j\omega_0} L^{-1}, \quad Y_C = j\omega_0 C. \tag{2}$$

If L is considered as an inductance matrix, the former equation can be used for a multiphase inductor. The admittance matrix of a distributed-parameter line model with length l is given by

$$Y_{TL} = \begin{bmatrix} Y_0 \coth(\Gamma l) & -Y_0 \text{cosech}(\Gamma l) \\ -Y_0 \text{cosech}(\Gamma l) & Y_0 \coth(\Gamma l) \end{bmatrix}, \tag{3}$$

where Y_0 and Γ are the characteristic-admittance and the propagation-constant matrix at ω_0 respectively. When the line model is n-phase, the size of Y_{TL} is $2n$ by $2n$. For both constant-parameter and frequency-dependent line models, the above equation can be used.

Since the three-phase AC solution assumes that the circuit to be solved is linear, it cannot consider nonlinear elements. Thus, nonlinear elements are ignored or linearized in some way. Voltage and current sources with non-sinusoidal waveforms also cannot be dealt with. Such sources are ignored, or their power-frequency components are considered.

In this way, the circuit equations in the form (1) for the three-phase AC solution are formulated and solved. Then, the solution x, which takes into account unbalanced circuit conditions at the power frequency, is obtained. Using the solution, the initial currents of the inductors, the initial voltages of the capacitors, and the history arrays of the distributed-parameter line models are set for the subsequent EMT simulation.

C. Step 3: EMT Simulation To Obtain Harmonics

At Stage 3, the EMT simulation is started from the three-phase AC solution obtained at Stage 2, and it is kept running for a certain number of cycles. During this simulation, the circuit conditions and the states of the control systems are fixed to the steady-state operation condition. In the EMT simulation, the source of harmonics such as power-electronics converters and the source of unbalanced voltages and currents such as untransposed transmission lines are modeled as they are. Thus, they gradually generate harmonics and unbalanced voltages and currents during the EMT simulation. To judge if the circuit has converged to its final steady-state solution or not, we observe the voltage or the current waveform of an important point in the circuit. When the difference between the waveform of the last period and that of one-period before becomes negligible, the EMT simulation is terminated. At this point, the initial currents of the inductors, the initial voltages of the capacitors, and the history arrays of the distributed-parameter line models are all set to correct initial values. And thus, we can start an EMT simulation for the original purpose.

Here, we get a very important question: “How many cycles should we keep the EMT simulation running so as to reach the steady state?” Since the number of cycles required highly depends on the circuit to be analyzed, we cannot give a general answer. Therefore, this paper investigates this question using typical circuits.

D. Treatment of Converter Bridge Circuits, DC Initial Values and Internal States of Controllers

Circuit components which has to be treated in exceptional manners are described here.

Converter bridge circuits cannot be included in the AC calculation at Stage 2 in a straightforward manner, since they are power-electronics circuits which convert AC power to DC power and vice versa. However, if we note the fact that the AC side of a converter bridge is designed to function as a three-phase voltage or current source at the power frequency, it can be modeled by a three-phase voltage or current source. The magnitude and the phase angle are set to the values obtained as the output from its controller in the steady state. In this way, the 50/60-Hz components from the converter bridge circuits are considered in the three-phase AC solution. Note that this procedure is independent of the switching pattern used such as PWM (pulse width modulation) and six-pulse control.

In a power system, there exist parts whose steady-state voltages and currents are DC. The DC capacitor of a STATCOM and the DC inductor of an HVDC system are good examples. These parts have to be ignored in the three-phase AC solution at Stage 2. To start Stage 3, however, the DC initial values of these components should be set appropriately to avoid unwanted transients. In a steady-state operation condition, fortunately, these DC initial values are controlled by control systems. Thus, the DC initial values are readily available as the steady-state output of their controllers.

To start Stage 3 without unwanted transients, the internal states of controllers have to be set to steady-state values. This can be done by inspection of control blocks automatically by the simulation program or by hand.

III. Performance Investigation

For performance investigation, the Three-Stage Method is applied to the steady-state initialization of a distribution line with a STATCOM and a transmission line with a line-commuted HVDC converter.

A. Distribution Line with a STATCOM

Fig. 2 (a) shows the one-line diagram of a simple 6.6-kV distribution line, which has no branches and connects a distribution substation to a load at the remote end. The line length is 5 km, and Fig. 2 (b) shows the conductor arrangement to be used for the line constants calculation. The voltage at the substation is 1 p.u. The load is inductive, and the active and the reactive power consumed are 3,500 kW and 1,300 kVar respectively. In order to compensate the voltage drop in the distribution line, the load is equipped with a STATCOM whose control is in the constant voltage mode. It provides the reactive power Q_s to the line, and the voltage at the load end is regulated to 1 p.u.

Stage 1 is carried out using the positive-sequence circuit shown in Fig. 2 (a). The distribution substation is specified as the swing node with $|V_1| = 1$, and the load end as a PV node with $P_2 = 0.35$ and $|V_2| = 1$.

Fig. 3 (a) shows the three-phase circuit for Stage 2, and the three-phase AC steady-state solution at 60 Hz is calculated. The series impedance of the distribution line is calculated considering the skin effects of both the wires and the ground soil at 60 Hz [17], and the line is modeled by its impedance matrix $[R] + j\omega[L]$. Since it is assumed to be untransposed, the impedance matrix is unbalanced resulting in unbalanced voltages and currents. Fig. 3 (b) shows the circuit of the STATCOM, but its bridge circuit is replaced by a three-phase voltage source as explained in Section II-D. Since the STATCOM is a voltage source converter, the magnitude and the phase angle of the voltage source are calculated by the operating condition. The initial voltages of the DC capacitors C_{dc1} and C_{dc2} are also calculated in the same way and set.

At Stage 3, the circuit shown in Fig. 3 (a), now together with the bridge circuit shown Fig. 3 (b) as it is, is used to carry out the EMT simulation. The IGBT and the free-wheeling diode of each arm are modeled by ideal switches whose on- and off-resistance values are 10 mΩ and 1 MΩ respectively. The PWM pattern that gives the voltage magnitude and the phase angle used at Stage 2 is continuously sent to the IGBT switches during the EMT simulation. The carrier frequency and the dead time used for the PWM pattern generation are 1620 Hz and 3 μs. To assess the effects of harmonics, no harmonics filter is equipped. The EMT simulation of Stage 3 is continued for 200 ms (12 cycles) with a time step of 1 μs. For the EMT simulation, the program XTAP, which uses the 2-stage diagonally-implicit Runge-Kutta method for the numerical integration [18], was used. To compare with this result, an EMT simulation of the same circuit started from a zero initial condition is also carried out (this case is hereafter referred to as a "zero-start case"). In the zero-start case, the currents of all inductors and the voltages of all capacitors are set to zero at $t = 0$, and all voltage sources and control systems continuously give their steady-state outputs from $t = 0$. The calculated results of the currents through the coupling resctors are shown in Fig. 4. The waveforms obtained by the Three-Stage Method are very close to the steady state from the first cycle. On the other hand, the waveforms of the zero-start case require about 6 cycles to settle into the steady state due to transients. To evaluate the deviation of the present cycle from the previous one, the following index is calculated for each cycle of the result.

$$e_n = \frac{|i(nT) - i((n-1)T)|}{i_{\text{rms}}}, \tag{4}$$

where $i(nT)$ is the current value at the end of the nth cycle and i_{rms} denotes the root-mean-square value of the 12th (last)

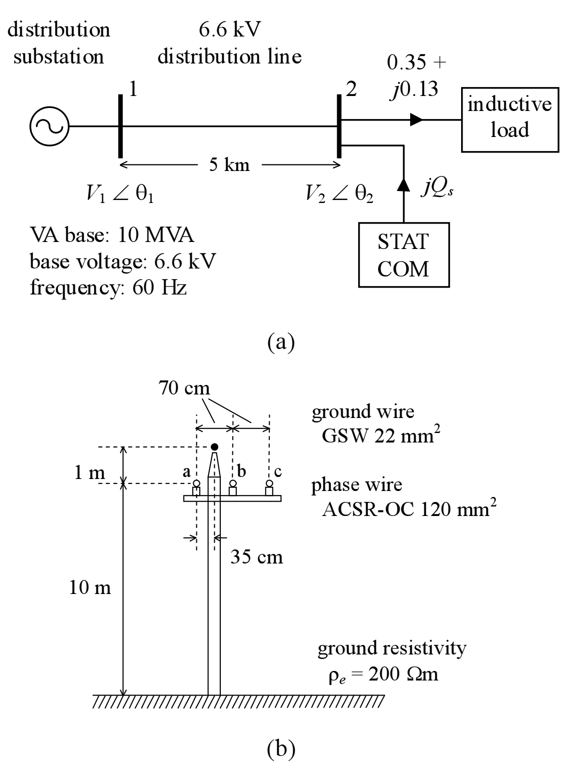

Fig. 2. Distribution line with a STATCOM. (a) shows the one-line diagram for Stage 1, and (b) the conductor arrangement.

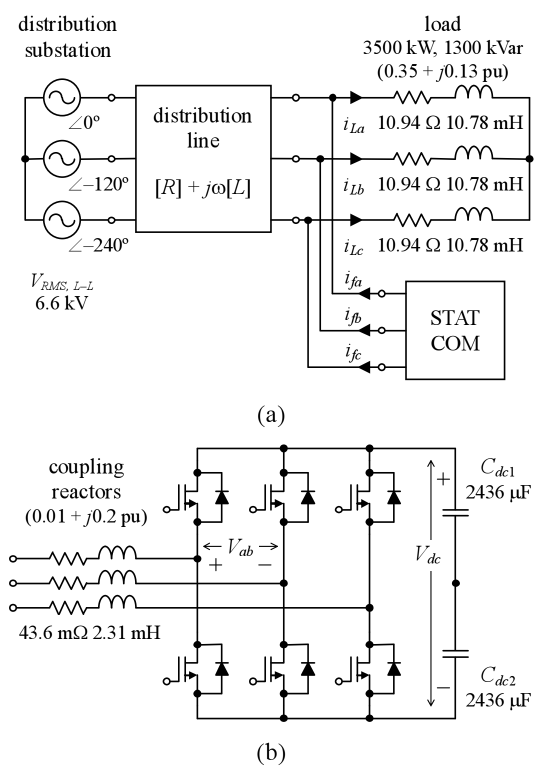

Fig. 3. Three-phase circuit of the distribution line case. (a) shows the circuit for Stages 2 and 3, and (b) the internal circuit of the STATCOM.

cycle. As shown in Fig. 5, the deviation e_n of the Three-Stage Method becomes smaller than 1 % in the second cycle, which indicates that the circuit is practically in the final steady state. The oscillation observed in e_n with respect to n is due to the fact that the period is not a multiple of the time step used and a linear interpolation was used to calculate e_n.

B. Transmission Line with an HVDC Converter

Fig. 6 shows the one-line diagram of a 500-kV transmission line which connects a substation to an HVDC converter station. The conductor arrangement is a typical double-circuit one used for 500-kV lines in Japan (shown in Fig. 6 of [5]), and a ground-resistivity of 200 Ωm is assumed. The line length is 150 km. The HVDC converter station uses a thyristor-based line-commuted converter, and it is in the REC mode transmitting 700 MW of power. The DC side of the converter is terminated by a resistor of 700 MW.

Stage 1 is carried out using the positive-sequence circuit shown in Fig. 6. The substation is specified as the swing node, and the HVDC converter station as a PV node. It is assumed that the voltage at the converter station is regulated to 1 p.u. by its shunt capacitor.

Fig. 7 (a) shows the three-phase circuit for Stage 2. Using this circuit, the three-phase AC steady-state solution at 60 Hz is calculated. The constants of the transmission line are calculated at 60 Hz considering the skin effects, and it is represented by a π-equivalent model. Fig. 7 (b) shows the circuit of the HVDC converter station, where SC is the shunt capacitor consisting of delta-connected capacitors with series reactors and ACF is the AC filter with four RLC filter branches. Three of them are for 5th, 11th and 13th harmonics, and the remaining one is for higher-order harmonics. The transformers are represented by fundamental equivalent circuits with winding and leakage impedance. In the three-phase AC solution, the AC sides of the converter bridges are replaced by three-phase voltage sources as explained in Section II-D. The initial current of DCL is set to its steady-state value.

At Stage 3, the thyristor bridges are modeled as shown

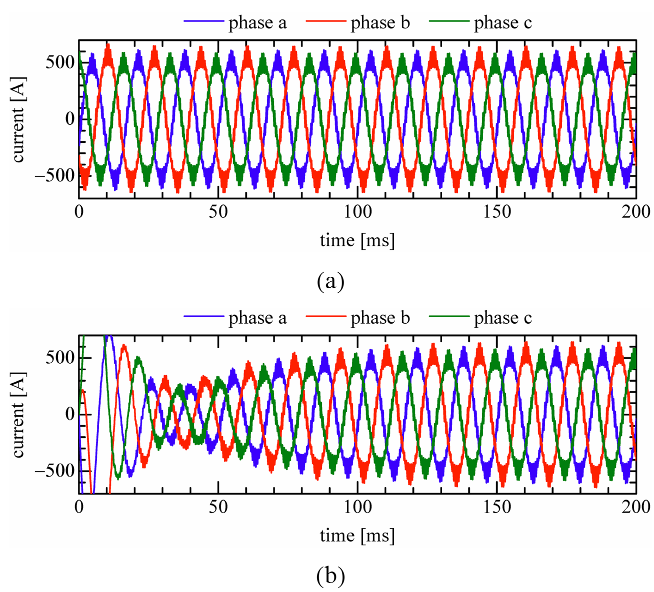

Fig. 4. Calculated waveforms of the currents i_{fa}, i_{fb} and i_{fc} through the coupling reactors of the STATCOM. (a) was obtained by the Three-Stage Method, and (b) is the corresponding zero-start case.

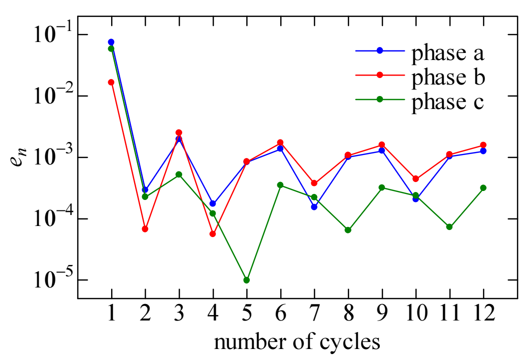

Fig. 5. Deviation e_n of the Three-Stage Method result with respect to the number of cycles for the distribution-line case.

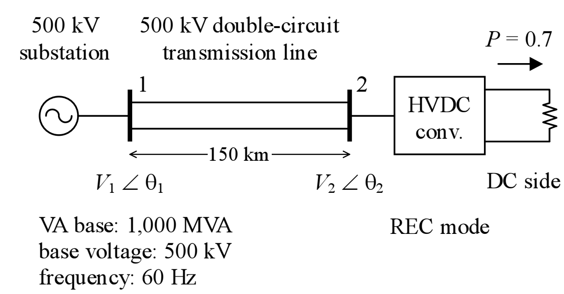

Fig. 6. One-line diagram of a 500-kV transmission line with an HVDC converter station.

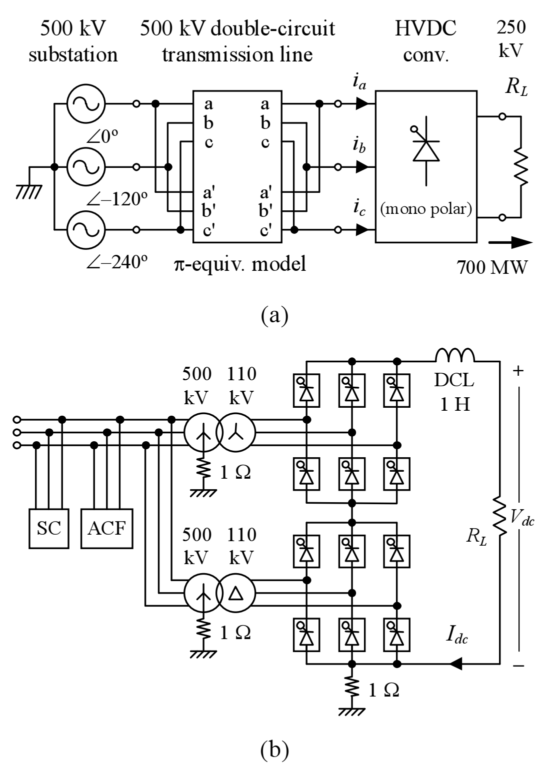

Fig. 7. Three-phase circuit of the transmission line case. (a) shows the circuit for Stages 2 and 3, and (b) the internal circuit of the HVDC converter station.

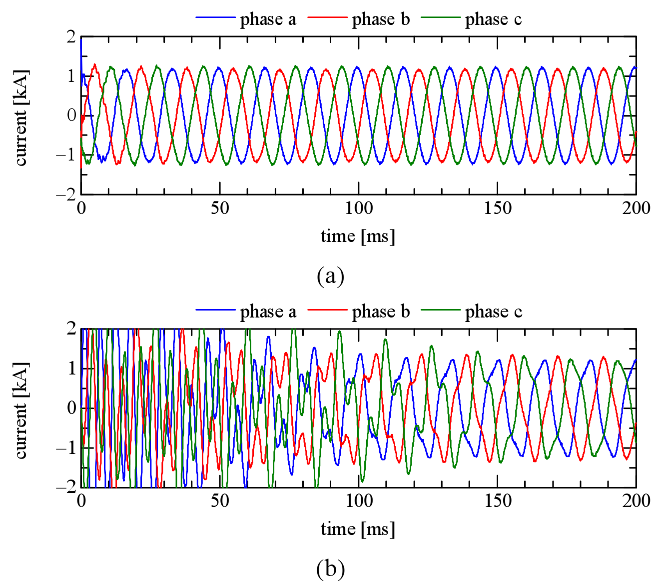

Fig. 8. Calculated waveforms of the currents i_a, i_b and i_c flowing into the HVDC converter station. (a) was obtained by the Three-Stage Method, and (b) is the corresponding zero-start case.

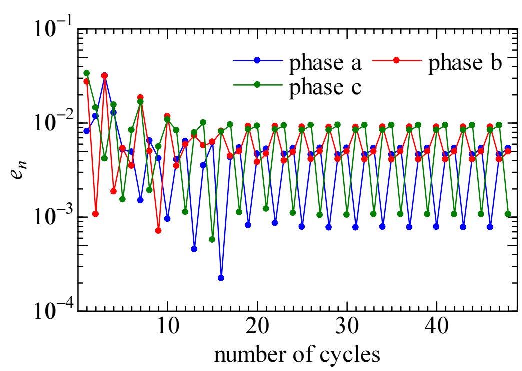

Fig. 9. Deviation e_n of the Three-Stage Method result with respect to the number of cycles for the transmission-line case.

in Fig. 7 (b), and the EMT simulation is continued for 800 ms (48 cycles) with a time step of 50 μs using XTAP. The thyristors are represented by ideal switches whose on- and off-resistance values are 1 mΩ and 100 kΩ, and each of them is equipped with a series reactor and a snubber circuit. A zero-start case is also carried out for this simulation. The calculated results of the currents flowing into the HVDC converter station are shown in Fig. 8 (first 12 cycles are shown). Although small transients are observed in the first few cycles, the result obtained by the Three-Stage Method settles into the steady state after that. The result of the zero-start case, on the other hand, shows intense transients and does not settle into the steady state in the 12 cycles. The deviation e_n of the Three-Stage Method, calculated by (4), is plotted in Fig. 9. It is confirmed that the deviation becomes smaller then 1 % in the first 8 cycles.

IV. Discussion and Conclusion

Comparing the results obtained by the Three-Stage Method with their corresponding zero-start cases (see Figs. 4 and 8), it is obvious that the Three-Stage Method is able to establish a steady state taking harmonics and unbalanced conditions into account by continuing an EMT simulation for a short duration. And, this steady-state initialization process can be started by specifying positive-sequence power-flow constraints.

Another point to note is the fact that the distribution-line case that includes a STATCOM requires only a few cycles to reach its steady state, while the transmission-line case with a line-commuted converter requires more cycles (see Figs. 5 and 9). Since the STATCOM uses a PWM for its switching pattern, it generates a relatively large amount of higher-order harmonics but a little lower-order ones. The line-commuted converter, on the other hand, uses a six-pulse switching pattern and thus generates a large amount of lower-order harmonics but a little higher-order ones. Because transients generated by higher-order harmonics die out in a shorter time, it is understood that a simulation case with a PWM-based converter is able to reach its steady state in a shorter duration compared with that with a six-pulse-based converter.

As mentioned in Sec. II-B, the circuit equations for Stage 2 and those for Stage 3 are in the same form that is shown in (1). The circuit equations for Stage 2 are complex-valued, while those for Stage 3 are real-valued. To solve a system of linear equations, the most time-consuming operation is multiplication. A multiplication of two complex numbers requires a four-times longer CPU time compared with a multiplication of two real numbers. Therefore, the time required to solve the circuit equations for Stage 2 is roughly four-times longer than that for one iteration step of one time step at Stage 3. To carry out Stage 3, an EMT simulation has to be performed for a certain number of cycles, which consist of hundreds or thousands of time steps. This means that the time required to carry out Stage 2 is negligible compared with the time for Stage 3. Meanwhile, the size of power-flow equations, which are real-valued, at Stage 1 is smaller than the circuit equations for Stage 3. A power-flow calculation at Stage 1 requires an iterative process to obtain a solution, while an EMT simulation at Stage 3 has to obtain solutions at successive time steps and each time step requires an iterative process. Thus, the time to carry out Stage 1 is also negligible compared with the time for Stage 3. As a result, the computation time of the Three-Stage Method is dominated by, or roughly the same as, the computation time for Stage 3.

The simulation results of the two test cases shown in Sec. III indicate that the Three-Stage Method is able to obtain the final steady-state solution at Stage 3 within about ten cycles at most. This means that the computation time of the Three-Stage Method is less than the time required for an EMT simulation for ten cycles. This is quite small. The existing methods that can take all of harmonics, unbalanced conditions and power-flow constraints into account are based on the shooting method [6]–[8]. For one iteration step in the shooting method, $n + 1$ EMT simulations have to be carried out to evaluate the Jacobian matrix and then to proceed with one cycle. In the STATCOM test case, for example, there exist 10 dynamic elements (inductors and capacitors) in the circuit. Thus, just to proceed with one iteration step, the shooting method requires to carry out 11 EMT simulations for one cycle. On the other hand, Stage 3 of the Three-Stage Method requires only two cycles to reach the final solution. From this comparison, it is understood that the Three-Stage Method is efficient.

V. Acknowledgment

Finally, the authors would like to thank Mr. Itaru Watanabe of CRIEPI for his support in the simulation work.

References

[1] D. Xia and G. T. Heydt, "Harmonic power flow studies Part I – Formulation and solution," *IEEE Trans., Power Apparatus and Systems*, vol. 101, no. 6, pp. 1257–1265, June 1982.

[2] D. Xia and G. T. Heydt, "Harmonic power flow stud-ies Part II – Implementation and practical application," *IEEE Trans., Power Apparatus and Systems*, vol. 101, no. 6, pp. 1266–1270, June 1982.

[3] A. Semlyen and A. Medina, "Computation of the periodic steady state in systems with nonlinear components using a hybrid time and frequency domain methodology," *IEEE Trans., Power Systems*, vol. 10, no. 3, pp.1498–1504, Aug. 1995.

[4] E. Acha and M. Madrigal, *Power Systems Harmonics: Computer Modelling and Analysis*, John Wiley & Sons, Ltd, 1st edition, June 4, 2001.

[5] T. Noda, A. Semlyen, and R. Iravani, "Entirely harmonic domain calculation of multiphase nonsinusoidal steady state", *IEEE Trans., Power Delivery*, vol. 19, no. 3, pp. 1368–1377, July 2004.

[6] A. Semlyen and M. Shlash, "Principles of modular harmonic power flow methodology," *IEE Proceedings– Generation, Transmission and Distribution*, vol. 147, no. 1, pp. 1-6, Jan. 2000.

[7] N. Garcia and E. Acha, "Periodic steady-state analysis of large-scale electric systems using Poincare map and parallel processing," *IEEE Trans., Power Systems*, vol. 19, no. 4, pp. 1784–1793, Nov. 2004.

[8] K. L. Lian and T. Noda, "A time-domain harmonic power-flow algorithm for obtaining nonsinusoidal steady-state solutions," *IEEE Trans., Power Delivery*, vol. 25, no. 3, pp. 1888–1898, July 2010.

[9] T. J. Aprille and T. N. Trick, "Steady state analysis of nonlinear circuits with periodic inputs," *IEEE Proc.*, vol. 60, no. 1, pp. 108–114, Jan. 1972.

[10] K. S. Kundert, "Steady-state methods for simulating analog circuits," Ph.D. Thesis, University of California, Berkley, Technical Report No. UCB/ERL M89/63, 1989.

[11] A. R. Bergen, *Power Systems Analysis,* Prentice-Hall, Inc., Englewood Cliffs, NJ, 1986.

[12] Task Force on Harmonic Modeling and Simulation (T&D Comm., IEEE PES), "Impact of aggregate linear load modeling on harmonic analysis: a comparison of common practice and analytical models," *IEEE Trans., Power Delivery*, vol. 18, no. 2, pp. 625–630, Apr. 2003.

[13] H. W. Dommel, "Digital computer solution of electromagnetic transients in single- and multi-phase networks," *IEEE Trans., Power Apparatus and Systems*, vol. PAS–88, no. 4, pp. 388–399, Apr. 1969.

[14] C.-W. Ho, A. E. Ruehli, and P. A. Brennan, "The modified nodal approach to network analysis," *IEEE Trans., Circuit and Systems*, vol. CAS-22, no. 6, pp. 504-509, Jun. 1975.

[15] G. D. Hachtel, R. K. Brayton and F. G. Gustavson, "The sparse tableau approach to network analysis and design," *IEEE Trans., Circuit Theory*, vol. CT–18, no. 1, pp. 101–113, Jan. 1971.

[16] W. J. McCalla, *Fundamentals of Computer-Aided Circuit Simulation* (third printing), Kluwer Academic Publishers, Boston, 1995.

[17] T. Noda, "Numerical techniques for accurate evaluation of overhead line and underground cable constants," *IEEJ Trans., Electrical and Electronic Engineering*, vol. 3, Issue 5, pp. 549–559, 2008.

[18] T. Noda, K. Takenaka, and T. Inoue, "Numerical integration by the 2-stage diagonally implicit Runge-Kutta method for electromagnetic transient simula-tions", *IEEE Trans., Power Delivery*, vol. 24, no. 1, pp. 390–399, Jan. 2009.