# A Comparative Analysis of Transformer Models Available in the ATP Program for the Simulation of Ferroresonance

L. B. Viena, F. A. Moreira, N. R. Ferreira, N. C. de Jesus

***Abstract*-- Ferroresonance is a phenomenon of complex nature which should be carefully analyzed so that preventive measures could be taken before its appearance may cause damages to electric power installations. There are some configurations as well as winding connections in transformers that may lead to the appearance of ferroresonance. In particular, medium-voltage underground networks formed by insulated cables, supplying unloaded or slightly loaded transformers. In order to identify the phenomenon and the nature of overvoltages, simulation is normally performed in EMTP-type programs. The ATP (Alternative Transients Program) has been chosen in this work. Several transformer models have been implemented in ATP, in particular the SATURABLE, BCTRAN and Hybrid (XFMR) models. Simulations are performed with these models for typical configurations of distribution networks with different transformer power ratings and cable lengths and the results are compared among the different models. It is shown that peak voltage amplitudes and even the onset of ferroresonance may depend on the model chosen.**

***Keywords*: Distribution transformer, BCTRAN, SATURABLE, Hybrid model, ferroresonance, simulation, ATP.**

## I. Introduction

THE supply of electrical energy to a group of residential or commercial customers may lead to the occurrence of ferroresonance in certain distribution network configurations unless preventive measures are adopted. For example, ferroresonance may happen when customers are supplied in medium-voltage when distribution transformers are used. The connection point between the primary network and the distribution transformer is normally done through monopolar fuses. Monopolar switching either due to failures or maintenance procedures may result in the occurrence of ferroresonance which is a phenomenon of complex nature that requires the correct representation of the core saturation in transformers as well as the magnetic coupling between windings for its simulation. Due to this situation, the main purpose of this work is the simulation of conditions through which certain network configurations may produce sustained overvoltages capable of provoking damages to power system equipments due to ferroresonance. Transient simulation with the ATP (Alternative Transients Program) is performed, with each network component being modeled from its electric parameters which are obtained from tests or other specific programs. Among the power system components, the transformer is undoubtedly the equipment that requires the most detailed modeling for the analysis of ferroresonance. If the model is not sufficiently precise the simulation may not reproduce the real behavior of the system resulting in inaccurate or false results. The intrinsic difficulty in transformer modeling is due to several factors, among which the type of study to be performed. Ferroresonance is a phenomenon characterized by low frequencies. There are also characteristics that should be correctly represented such as core configuration, self and mutual inductances between windings, leakage fluxes and the magnetic core saturation.

## II. Characteristics of Ferroresonance

The first work about resonance in transformers was published in 1907 [1]. The term ferroresonance was used for the first time by Boucherot in 1920 with the purpose of describing a complex resonant oscillation in an RLC circuit with a non-linear inductance [2].

In simple terms, ferroresonance is a series resonance that involves a non-linear inductance and a capacitance. It typically involves the magnetizing saturation inductance of the transformer and the capacitance of a distribution cable or a transmission line connected to the transformer. Its occurrence is more frequent in the absence of adequate damping. The connection of three-phase transformers through underground cables is becoming more common in industrial, commercial and residential systems. Due to this situation, the possibility of establishing a series connection between the capacitance and the transformer non-linear inductance, favorable to the occurrence of ferroresonance, increases. Not only the cable capacitance and consequently its length are important, but also other elements are necessary for the appearance of this phenomenon. The main characteristic of ferroresonance is the possibility of existing more than one stable response in steady-state for the same network parameters set. The energization of

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transformers or loads and the occurrence or removal of faults may contribute to the onset of ferroresonance. The response may change immediately from normal steady-state (sinusoidal with the same source frequency) to other steady-state ferroresonant conditions, characterized by elevated overvoltages and elevated harmonic levels that may lead equipments, such as surge arresters, loads or even the transformer itself to serious damages [3].

### A. *Influencing factors*

The possibility of the onset of ferroresonance is based on the existence of a series connection between a capacitance and a non-linear inductance. However, some factors may have a significant influence on its occurrence, such as protection, operation and construction configurations. Some of these configurations are the operation of single-phase fuses, monopolar switching, primary winding connections, transformer core project, low loss transformer or lightly loaded or unloaded transformers. The capacitance may be due to several elements such as underground cables, transmission lines, shunt capacitors, stray capacitances in transformers and equalizer capacitors in circuit breakers [4,5].

### B. *Ferroresonance in three-phase systems*

The occurrence of ferroresonance in three-phase systems may involve a large number of power transformers, or instrument transformers (CTs or VTs). The general requirements for ferroresonance in transformers are an applied or induced voltage source, a saturable magnetizing inductance in the transformer, a capacitance and low damping. Due to the several core and windings configurations in transformers, capacitance sources and non-linearities involved, the scenarios under which ferroresonance may occur are almost endless. Disturbances that may initiate ferroresonance are monopolar switching, the actuation of fuses, and the loss of grounding systems. Some typical configurations in distribution systems that may lead to ferroresonance in transformers are shown in Figs. 1 and 2. Therefore, the primary winding connected either in wye or in delta may present adequate conditions for the appearance of ferroresonance in unbalanced conditions, such as single-phase switching and unloaded transformer operation, as already mentioned [4,6].

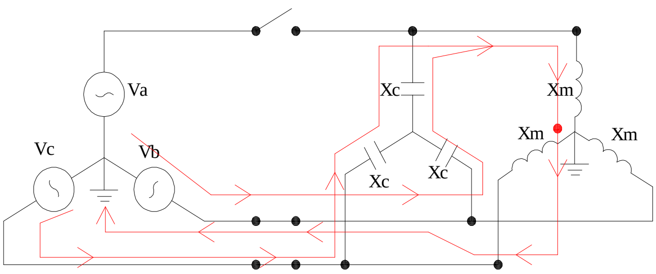


Fig. 1. Single-phase switching with primary connected in grounded-wye.

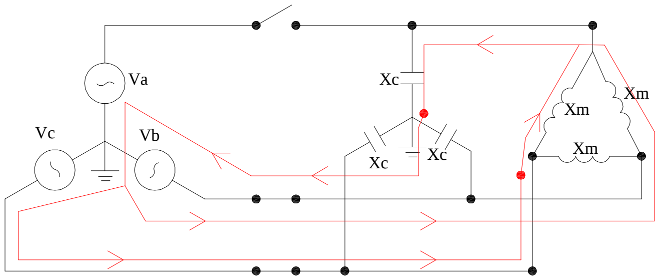


Fig. 2. Single-phase switching with primary connected in delta.

The capacitances shown in Figs. 1 and 2 may be due to the cable or line shunt capacitances or due to a capacitor bank. Each phase of the transformer is represented by its magnetizing reactance $X_{m}$. If a phase is opened and if the capacitor bank (in case there is one) or the transformer contains a grounded neutral, then a series path through the capacitance and the magnetizing reactance will exist and ferroresonance may occur. The opening of two phases may also lead to the existence of this path and the onset of ferroresonance. If both neutrals are grounded or isolated, then there will be no series path and there will be no apparent possibility for the occurrence of ferroresonance. For the delta winding configuration, the same principles are valid. This phenomenon is admissible for any type of core configuration, even for single-phase transformer banks.

Depending on the type of transformer core, ferroresonance may be possible even when a clear series path of voltage applied through the capacitance and magnetizing reactance does not exist. This is possible with certain three-phase core types that provide a direct coupling among phases where a voltage may be induced in the transformer open phase [4]. In order to study this phenomenon, three methods are essential:

1. Laboratory and field tests. Although these results may be realistic, there is a limitation in the quantity of tests that may be performed.
2. The use of mathematical models and analytical techniques. Allows flexibility concerning the types of scenarios that may be evaluated, although they are limited to single-phase transformers due to its complexity.
3. The use of digital simulation tools in order to simulate three-phase transformers and other components of the electric system.

## III. Transformer Modeling

A precise simulation of electromagnetic transients should be based in the adequate representation of the components that comprise the electric system. However, it is difficult to obtain a model that is valid for a wide range of frequencies resulting from electromagnetic transients. The behavior of a transformer, for example, is dominated by the magnetic coupling among windings and by the core saturation when a low or medium frequency transient occurs. On the other hand, when transients generated by high frequencies disturbances such as lightning discharges happen, the transformer behavior is affected mainly by stray capacitances and capacitances among windings [5].

The representation of transformers may be very complicate due to several core configuration types and due to the fact that some parameters are both non-linear and frequency dependent. The physical characteristics of the transformer that should be correctly represented, depending on the frequency range of interest, are the core and winding configurations, self and mutual inductances, leakage fluxes, magnetic core saturation, stray current losses, core hysteresis, and capacitive effects [7]. The model used to represent the transformer may be divided in two parts: the first part (linear) refers to the representation of the windings and the second part refers to the representation of the core (non-linear). However, both are frequency dependent. Each one performs an important role, depending on the study for which the transformer model is being formulated. For example, the representation of the core saturation is very important for studies that involve ferroresonance, although it is negligible for short-circuit or load-flow studies that are usually performed in steady-state [8], or even in the case of electromagnetic transients generated by lightning discharges.

### *A. Matrix Representation*

The representation of single-phase transformers for any number of windings for transient and steady-state studies is performed in a direct way. However, for three-phase transformers, there are difficulties either with the utilization of network analyzers or with the representation in digital simulations. This representation is performed from the formulation of impedance and admittance matrices. The elements of these matrices are specified from short-circuit and positive and zero sequence excitation tests [9].

For three-phase transformers, each winding under analysis consists of three coils for the three phases or three core columns. This means that each element in the $[Z]$ matrix becomes a 3x3 matrix [9].

In a similar manner to other power system components, the self and mutual impedances are related to the positive and zero sequence impedances $Z_1$ and $Z_0$ as

$$Z_S = \frac{1}{3}(Z_0 + 2Z_1) \tag{1}$$

$$Z_M = \frac{1}{3}(Z_0 - Z_1) \tag{2}$$

where $Z_S$ is the self impedance of the phase or column and $Z_M$ is the mutual impedance among the three phases or columns.

The modeling of transformers through the impedance matrix has the disadvantage of not describing the short-circuit impedances, which give the transfer characteristics of a transformer [8]. For low excitation currents, the matrix self and mutual elements are very similar and the impedance matrix may become ill-conditioned. In order to avoid this problem, an alternative representation may be used in the form of the admittance matrix $[Y]$ as follows:

$$[I] = [Y][V] \tag{3}$$

The matrix $[Y]$ elements may be determined directly from short-circuit tests.

In the ATP program, several models for the representation of transformers have been implemented. A brief description of the main models is now presented

### *B. BCTRAN*

Supporting routine developed by Brandwajn *et al.* [9,10]. It may be used for determining the matrices [*A*]-[*R*] or [*R*]-[$\omega L$] for single or three-phase, shell or core type transformers and any number of windings. The parameters are determined from excitation and short-circuit tests at power frequency. Matrix [*A*] is the inverse of the inductance matrix [*L*], matrix [*R*] corresponds to the winding resistances matrix, and $\omega$ is the nominal angular frequency. The impedance and admittance matrices represent the linear behavior of the transformer quite reasonably for a frequency range until approximately 1 kHz.

The non-linear behavior may not be included directly in the BCTRAN model. This behavior (saturation and/or hysteresis) may be taken into account with the inclusion of an external magnetizing branch connected to the appropriate transformer terminals. This external connection of the magnetizing branch is not, in general, topologically correct [11], and, for this reason, may lead to incorrect results when simulating ferroresonance, for example.

### *C. Saturable Transformer Component (STC Model)*

This model is based on the representation of the transformer through single-phase circuits. Saturation and hysteresis effects may be modeled with the inclusion of a non-linear inductance in the wye point [7,8].

The STC model may be used for three-phase units by adding a zero-sequence reluctance parameter. The data that should be entered in the model consist in the resistance and inductance values of each branch, the transformation ratio, and information about the magnetizing branch. However, this model presents some limitations. The magnetizing inductance with $R_{mag}$ in parallel, similar to the BCTRAN model, may not be connected to the topologically correct point. The onset of numerical instability has been described when using this model for three-winding transformers.

### *D. Hybrid Transformer Model*

This model has been recently developed and it is also known as XFMR [12,13]. The model combines the matrix representation from BCTRAN for the winding modeling and the duality principle for the representation of the topologically correct core magnetization in legs and yokes. The model supports three-phase, two and three winding transformers, autotransformers, and wye and delta couplings. Although it is an extremely powerful model, it is still seldom used by the scientific community in part due to its complexity and in part due to the quantity of data that needs to be informed. At this moment, as far as the authors are concerned, the model supports three and five columns stacked cores. The necessary information for representing the transformer with this model may come from three different sources: design project (geometry, and the type of material of the windings and core), manufacturer tests (similar to BCTRAN, except for the core modeling), and typical data based on nominal power and voltage. The main difficulty in this model is the adjustment of the parameters of the Frolich equation which models the behavior of the *B x H* curve of the material that constitutes the magnetic core.

Only two parameters of the core material are necessary for the coefficients of the Frolich equation: $\mu_m$ which is the core maximum relative permeability and $B_{sat}$ which is the flux density for maximum saturation [14].

The variable core permeability is the slope of the *B x H* curve in a specific point and can be written as

$$\mu = \frac{dB}{dH} \tag{4}$$

The Frolich equation may be written as

$$B = \frac{H}{c + bH} \tag{5}$$

where $b$ and $c$ are constants determined from the core material. The core permeability is calculated as follows:

$$\mu = \frac{dB}{dH} = \frac{(1 - bB)^2}{c} \tag{6}$$

When the flux density reaches its saturated value, the core relative permeability reaches unity. This concept supplies a condition to define how constants $b$ and $c$ are related. For many types of silicon steel, the saturation density is in the range of 1.8 T to 2.2 T.

Making $B$ = 0 in (6) and solving for constant $c$:

$$c = \frac{1}{\mu_0 \mu_m} \tag{7}$$

Using the value of constant $c$ and making $B = B_{sat}$, constant $b$ may be obtained as

$$b = \frac{1 - \left( 1 / \sqrt{\mu_m} \right)}{B_{sat}} \tag{8}$$

With constants $b$ and $c$ calculated as described, the Frolich equation represents the *B x H* curve between $B = 0$ and $B = B_{sat}$.

## IV. Modeling and Simulation of the Circuit

The circuit to be analyzed represents some typical configurations adopted by electric power utilities for the supply of electrical energy in medium voltage for residential and commercial installations. These customers are usually supplied from derivations from the electric utility network, which has been represented with a short-circuit power of 154 MVA. The transformers are connected to the network through fuses and underground cables. The overvoltages caused by ferroresonance will be verified through the fuse opening of only one phase, which may be attributed to a single-phase to ground short-circuit or some kind of switching. For the transformer modeling the SATURABLE, BCTRAN, and XFMR models will be used. The simplified single-phase diagram of the circuit for the study of the ferroresonance phenomenon is shown in Fig. 3.

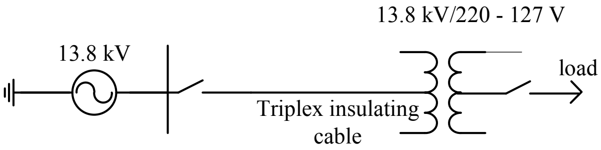


Fig. 3. Single-phase diagram of the underground circuit.

### A. *Characteristics of the System Components*

#### 1) *Insulated Cables*

The underground primary circuit is composed of XLPE / PVC triplex cables with cross-section of 35 mm$^2$ and voltage class of 8.7/15 kV. The cable electrical parameters are:

1. Positive sequence resistance: 0.6726 Ω/km
2. Positive sequence reactance: 0.1793 Ω/km
3. Zero sequence resistance: 1.6793 Ω/km
4. Zero sequence reactance: 0.6332 Ω/km
5. Capacitance: 0.2240 µF/km

#### 2) *75 kVA Distribution Transformer*

The 75 kVA distribution transformer is supplied through the underground network. It is a delta-wye connected, three-column, core type transformer (13.8 kV / 220-127 V). The information obtained from no-load and short-circuit tests, as well as the main information about the physical characteristics of the transformer, are:

*a) No-load test (low-voltage)*

1. Voltage: 220 V
2. Current: 1.6 %
3. Losses: 287 W

*b) Short-circuit test (high-voltage)*

1. Losses: 1140 W
2. Percentage impedance: 3.5 %

*c) Core data*

1. Window height: 280 mm
2. Window width: 115 mm
3. Cross section area: 105.22 cm$^2$
4. Core material: M4 (0.27 mm)
5. Diameter: 126 mm
6. Leg length: 0.52 m
7. Yoke length: 0.6 m

*d) Low-voltage winding*

1. Internal diameter: 131 mm

2. External diameter: 154 mm
3. Coil conductor (copper): two axial and one radial 2x (7.5 mm x 5 mm)
4. Winding height: 254 mm

*e) High-voltage winding*

1. Internal diameter: 176 mm
2. External diameter: 218 mm
3. Coil conductor: 0.65 $mm^2$
4. Winding height: 238 mm

*f) Saturation curve points*

Considering that from 0 to 0.6 pu the saturation curve has a linear behavior, the values used for the determination of the saturated region start from 0.7 pu and are shown in Table I.

TABLE I: SATURATION CURVE POINTS FOR THE 75 KVA TRANSFORMER

| Voltage (pu) | Current (%) | Losses (W) |
|---|---|---|
| 0.701 | 0.22 | 85 |
| 0.81 | 0.448 | 131 |
| 0.907 | 0.843 | 193 |
| 1.00 | 1.6 | 287 |
| 1.10 | 2.979 | 420 |
| 1.209 | 6.061 | 650 |

### 3) *500 kVA Distribution Transformer*

Besides the 75 kVA transformer, simulations will also be performed with a 500 kVA distribution transformer, which is also supplied through the underground network. It is a delta-wye connected, three-column, core-type transformer (13.8 kV / 220-127 V). The physical data for this transformer are not available. The information obtained from no-load and short-circuit tests are:

*a) No-load test (low-voltage)*

1. Voltage: 220 V
2. Current: 0.9 %
3. Losses: 840 W

*b) Short-circuit test (high-voltage)*

3. Losses: 9140 W
4. Percentage impedance: 4.5 %

*c) Saturation curve points*

Considering that from 0 to 0.6 pu the saturation curve has a linear behavior, the values used for the determination of the saturated region start from 0.7 pu and are shown in Table II.

TABLE II: SATURATION CURVE POINTS FOR THE 500 KVA TRANSFORMER

| Voltage (pu) | Current (%) | Losses (W) |
|---|---|---|
| 0.70 | 0.127 | 353 |
| 0.80 | 0.258 | 496 |
| 0.90 | 0.484 | 671 |
| 1.00 | 0.9 | 840 |
| 1.10 | 1.711 | 1229 |
| 1.20 | 3.255 | 1671 |

## B. *Simulation Results*

Simulations were performed with the two different transformers in order to verify the occurrence of ferroresonant overvoltages and its magnitudes due to single-phase switching. However, due to the lack of physical information from the 500 kVA transformer, only the 75 kVA transformer has been modeled with the XFMR model.

### 1) *Single-Phase Switching (Phase A) - BCTRAN Model – 75 kVA Transformer*

In this case, phases B and C were kept closed and phase A was opened at 20 ms, considering that the transformer is operating at no-load. Fig. 4 shows the resulting voltages at phase A for two different cable lengths: 100 m (red curve) and 300 m (green curve). The occurrence of sustained overvoltages at the opened phase has been detected only for a cable length of 300 m, with a maximum peak value of approximately 4.4 pu.

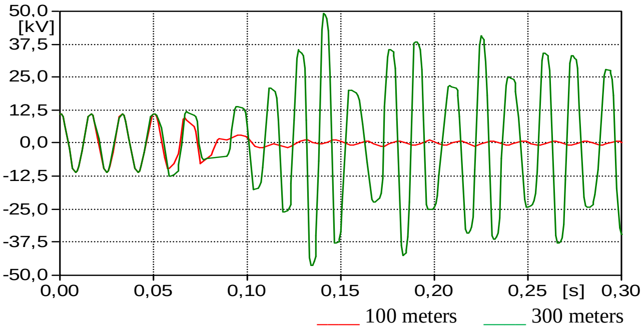


Fig. 4. Phase A voltage in the 75 kVA transformer primary for different cable lengths (BCTRAN model).

### 2) *Single-Phase Switching (Phase A) – XFMR Model – 75 kVA Transformer*

Fig. 5 presents the phase A voltages (opened phase) with the transformer modeled with the XFMR model, with phases B and C kept closed. Once again two different cable lengths have been considered: 100 m (red curve) and 300 m (green curve). With the XFMR model, ferroresonant overvoltages have been detected for both cable lengths. Another difference observed with this model is that for a cable length of 300 m the voltage reached a higher peak (approximately 5.0 pu).

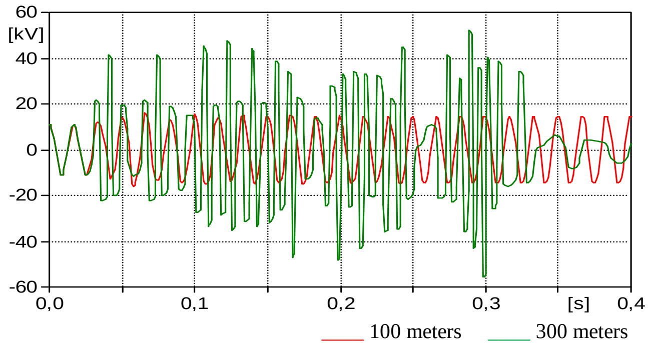


Fig. 5. Phase A voltage in the 75 kVA transformer primary for different cable lengths (XFMR model).

### 3) *Single-Phase Switching (Phase A) – SATURABLE Model- 75 kVA Transformer*

Fig. 6 shows the results obtained when the transformer is modeled with the SATURABLE model for the same cable lengths previously adopted. In this case sustained overvoltages have been detected for both line lengths: 100 m (red curve) and 300 m (green curve). The maximum peak value was obtained for the longer line length (approximately 4.4 pu).

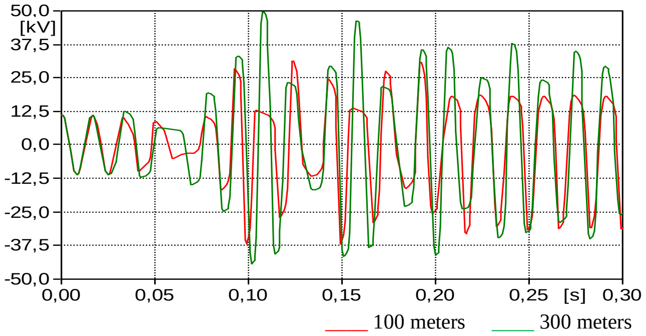


Fig. 6. Phase A voltage in the 75 kVA transformer primary for different cable lengths (SATURABLE model).

*4) Single Phase Switching (Phase A) – BCTRAN Model -500 kVA Transformer*

Fig. 7 shows the phase A voltage when opening this phase at 20 ms and keeping phases B and C closed for the 500 kVA transformer using the BCTRAN model. The same cable lengths have been considered: 100 m (red curve) and 300 m (green curve). The occurrence of ferroresonance is verified only for the 100 m cable length and the maximum peak value is approximately 2.5 pu.

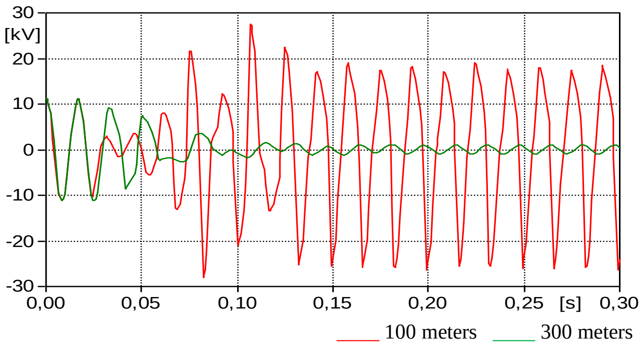


Fig.7. Phase A voltage in the 500 kVA transformer primary for different cable lengths (BCTRAN model).

*5) Single-Phase Switching (Phase A) – SATURABLE Model – 500 kVA Transformer*

Fig. 8 shows the phase A voltage when opening this phase at 20 ms and keeping phases B and C closed for the 500 kVA transformer using the SATURABLE model. Once again, the same cable lengths have been considered: 100 m (red curve) and 300 m (green curve). With the SATURABLE model, for the 500 kVA transformer, ferroresonance is only detected again for the 100 m cable length, exactly as observed with the BCTRAN model. The maximum peak value is approximately 2.5 pu as well.

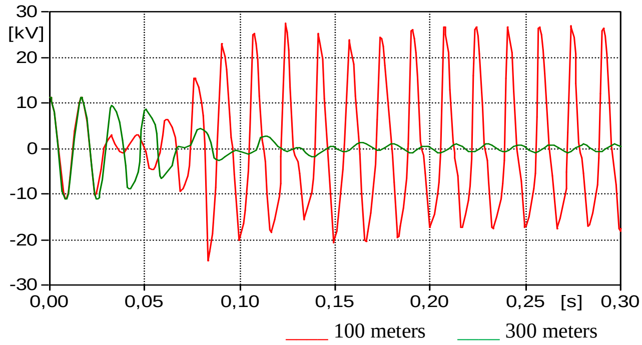


Fig. 8. Phase A voltage in the 500 kVA transformer primary for different cable lengths (SATURABLE model).

Tables III and IV summarize the results obtained in terms of voltage peak values observed for the 75 kVA and 500 kVA transformers and the different transformer models used, as well as the different cable lengths.

TABLE III: PEAK VOLTAGE VALUES FOR THE 75 KVA TRANSFORMER

| Model | 100 m | 300 m |
|---|---|---|
| BCTRAN | No Ferroresonance | 4.4 pu |
| XFMR | 1.4 pu | 5.0 pu |
| SATURABLE | 3.3 pu | 4.4 pu |

TABLE IV: PEAK VOLTAGE VALUES FOR THE 500 KVA TRANSFORMER

| Model | 100 m | 300 m |
|---|---|---|
| BCTRAN | 2.5 pu | No Ferroresonance |
| SATURABLE | 2.5 pu | No Ferroresonance |

## V. CONCLUSIONS

In this paper several simulations with the purpose of evaluating the phenomenon of ferroresonance in distribution transformers have been performed. An underground electric distribution system typically used by electric utilities has been modeled. The 75 kVA transformer has been modeled using three different models available in the ATP program: the SATURABLE, BCTRAN and XFMR models. The 500 kVA transformer has been modeled with the SATURABLE and BCTRAN models. Two different cable lengths have been adopted: 100 m and 300 m. Single-phase switching only has been considered. For the 75 kVA transformer, ferroresonance has been observed for both line lengths when modeling the transformer with the SATURABLE and with the XFMR model. When using the BCTRAN model, ferroresonance has only been observed for the 300 m cable length. It should be mentioned that in the BCTRAN model, the magnetizing branch has been added externally to the model, what may not be topologically correct. The XFMR model represents the core using the principle of duality and therefore it is a topologically correct model. For the 500 kVA transformer, ferroresonance has been verified with the BCTRAN and the SATURABLE models only for the 100 m cable length. The 500 kVA transformer has not been simulated with the XFMR model due to the lack of geometrical information. It should be noted, however, that the XFMR model may also be used when only test reports are available or even through the adoption of estimated values. In future work, the authors intend to compare simulations with the XFMR model using test reports only with those obtained when geometrical information is used.

The results obtained with the 75kVA transformer show that the identification of ferroresonance may depend on the model being used as well as the correct determination of the maximum peak voltage value.

It is believed that an important contribution of this paper is the application of the XFMR transformer model for the study of ferroresonance. The XFMR model has been recently implemented in the ATP and, although very powerful, it is still relatively unknown to the scientific community. In order to guarantee the most accurate results, the XFMR model needs several data from the transformer, including information regarding core geometry and material used for the core and windings. The authors have performed some simulations changing the core material of the 75 kVA transformer. This results in changes to the constants $b$ and $c$ as defined in (7) and (8), making the model very sensitive to the core material. However, if this information is not available, it is always possible to supply only data obtained through test reports for core saturation.

It is expected that in short time several researchers will start using the XFMR model more frequently by realizing that the results obtained with this model may be very reliable.

For future work the authors plan on performing some real measurements in low voltage transformers subject to ferroresonance with the purpose of comparing the real measurements with simulated ones.

It should be noted that ferroresonance is an important topic to be considered in designing electric distribution systems since it has been responsible for the damage of power system components under unbalanced conditions, especially surge arresters as described in [6]. Therefore, it should be thoroughly investigated for identification and proposition of recommendations.

## VI. Acknowledgement

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