FDTD Computations of Lightning-Induced Voltages in the Presence of Corona

T. H. Thang, Y. Baba, N. Nagaoka, A. Ametani, N. Itamoto, V. A. Rakov

Abstract**-- In this paper, a simplified model of corona discharge for the finite-difference time-domain (FDTD) computations has been applied to analysis of lightning induced voltages at different points along the 5-mm radius, 1-km long single overhead wire in the presence of corona. Both perfectly conducting and lossy ground cases were considered. FDTD calculations were performed using a 3D non-uniform grid. The progression of corona streamers from the wire is represented as the radial expansion of cylindrical conducting (40 μS/m) region around the wire. The critical electric field on the surface of the 5-mm-radius wire for corona initiation is set to E_0 = 2.9 MV/m. The critical background electric field for streamer propagation is set to E_{cn} = 1.5 MV/m. The magnitudes of FDTD-computed lightning-induced voltages in the presence of corona discharge are larger than those computed without considering corona. The observed trend is in agreement with that reported by *Nucci et al*. and by *Dragan et al*., although the increase predicted by our full-wave model (up to 5%) is less significant than in their studies based on the distributed-circuit model with sources specified using electromagnetic field theory (up to a factor of 2).**

Keywords**: lightning, corona discharge, finite-difference time-domain method, induced voltage.**

I. Introduction

Nucci et al. [1] and *Dragan et al.* [2] have computed lightning-induced voltages on a single overhead wire in the presence of corona discharge, using a distributed-circuit model with distributed sources simulating electromagnetic coupling between the lightning channel and the wire. In their simulations, a 5-mm radius, 1-km long horizontal wire, located 7.5 m above ground was employed. Corona was taken into account by means of dynamic capacitance based on an assumed charge-voltage (*q*-V) diagram. It has been found that corona serves to increase the magnitude of lighning-induced voltages up to a factor of 2.

In this paper, we apply a simplified (engineering) model of corona discharge [3][4] developed for the finite-difference time-domain (FDTD) computations [5] to analyze lightning-induced voltages on a single wire above perfectly-conducting and lossy ground, which simulates the configuration employed in [1], [2]. In the corona model, the progression of corona streamers from the wire is represented as the radial expansion of cylindrical conducting region around the wire. The validity of this model has been tested in [3] and [4] against experimental data found in [6]-[8]. Specifically, it has been shown in [3] that the waveform of radial current, and the relation between the total charge (charge deposited on the wire and emanated corona charge) and applied voltage (*q*-V curve), computed using the FDTD method including the corona model for 22 m and 44 m long horizontal wires, agree reasonably well with the corresponding measured ones. Futher, it has been shown in [3] that the computed increase of coupling between the energized wire and another wire nearby due to corona discharge agrees well with the corresponding measured one. Finally, it has been shown in [4] that computed waveforms (including wavefront distortion and attenuation at later times) of fast-front surge voltages at different distances from the energized end of 1.4-km and 2.2-km long overhead wires agree reasonably well with the corresponding measured waveforms.

II. Methodology

We describe here the configuration, which represents the simulations carried out by *Nucci et al.* [1] and *Dragan et al.* [2], in order to analyze lightning induced voltages on an overhead horizontal wire with corona discharge.

Fig. 1 (a) and (b) show the plan (*xy*-plane) and side (*yz*-plane) views of a 5-mm radius, 1-km-long overhead horizontal perfectly conducting wire located 7.5-m above ground that was assumed to be either perfectly conducting or lossy with conductivity values of 0.01 and 0.001 mS/m. These values are employed because they were used in [1] and [2], respectively. Lightning channel is represented by a 600-m long, vertical phased ideal current source array [9]. The array simulates a current pulse that propagates upward at speed 130 m/μs. Lightning is assumed to terminate on ground at two locations, A (mid-point of the wire) and B (close to one of the line terminations). For stroke location A, both ends of the wire are connected to the ground via 480-Ω matching resistors. For stroke location B, one end of the wire is open-circuited, and the far end is connected to the ground via a 480-Ω matching resistor. Note that *x*, *y*, and *z* coordinates are defined here so

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that the wire is parallel to the y-axis and the ground surface is parallel to both x and y axes (and therefore perpendicular to the z-axis).

For FDTD computations, this conductor system is accommodated in a working volume of 400 m × 1200 m × 750 m, which is divided non-uniformly into rectangular cells and is surrounded by six planes of Liao's second-order absorbing boundary condition [10] to minimize unwanted reflections there. Cell sides along x, y and z axes are not constant: 2.2 cm in the vicinity (1.0 m × 1.0 m) of the horizontal and vertical conductors, and increasing gradually to 10 and 200 cm beyond that region, as shown in Fig. 2. The equivalent radius [11] of the horizontal wire is $r_0 \approx 5$ mm ($=0.23\Delta x = 0.23\Delta z = 0.23 \times 2.2$ cm).

Corona discharge is assumed to occur only on the horizontal wire.

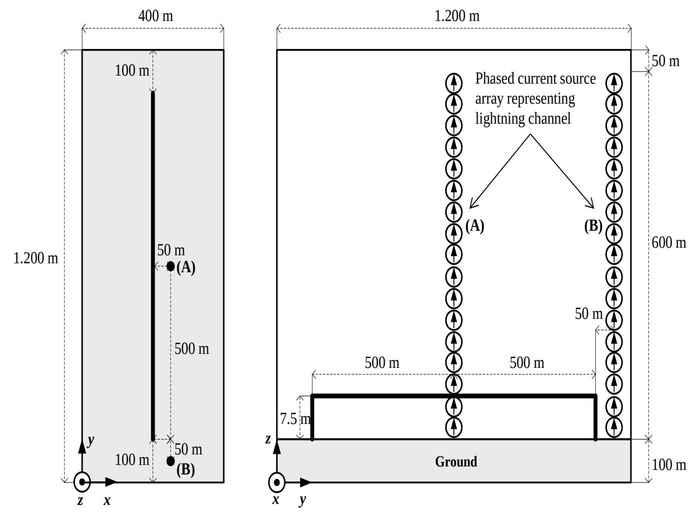


Fig. 1. (a) Plan (xy plane) and (b) side views (yz plane) of 5-mm radius, 1-km long overhead horizontal wire located 7.5 m above ground. Corona discharge is assumed to occur only on the horizontal wire.

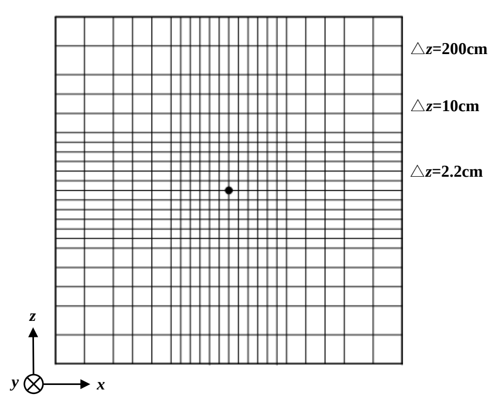


Fig. 2. Cross-sectional (zx-plane) view of the discretized space around horizontal conductor used in the FDTD computation.

We assumed that the critical electric field E_0 on the surface of a cylindrical wire of radius r_0 for initiation of corona discharge is given by equation of *Hartmann* [12], which is reproduced below:

$$E_0 = m \cdot 2.594 \times 10^6 \left(1 + \frac{0.1269}{r_0^{0.4346}}\right) \quad [\text{V/m}] \qquad (1)$$

where m is a coefficient depending on the wire surface conditions. We assumed that m=0.5. When r_0=5 mm, E_0 is 2.9 MV/m for m=0.5, which is the same as the corona threshold field that was used in the model of *Nucci et al.* [1].

We set the critical background electric field necessary for streamer propagation [13] to E_{cn}=1.5 MV [14]. The corona ionization process is simulated by expanding the conducting region of constant conductivity (σ_{cor} = 40 μS/m) to the corona radius r_c. The corona radius r_c is obtained, using analytical expression (2) based on E_{cp} (0.5 MV/m) and the FDTD-computed charge per unit length (q). Then, the conductivity of the cells located within r_c is set to σ_{cor} = 40 μS/m.

$$E_c = \frac{q}{2\pi\varepsilon_0 r_c} + \frac{q}{2\pi\varepsilon_0 (2h - r_c)} \quad [\text{V/m}] \qquad (2)$$

Note that, in our model [3], [4], the corona radius for each meter along the overhead wire is calculated at each time step. As a result, the corona radius has a nonuniform distribution along the wire.

III. Analysis and Discussion

Fig. 3 shows two waveforms of negative lightning return-stroke current (positive charge moving up) at the channel base that were employed in the FDTD simulations. The waveform shown in Fig. 3 (a), which was used for stroke location A, has a peak of 35 kA and a maximum time derivative of 42 kA/μs. The waveform shown in Fig. 3 (b), which was used for stroke location B, has peak of lightning current is 55 kA and the maximum time derivative of 66 kA/μs.

Figs. 4 and 5 illustrate induced voltages at different points along the overhead wire with corona: at d=0, 250, and 500 m from the either end (due to symmetry) of the horizontal wire above perfectly conducting ground and lossy ground whose conductivity is σ_{gr} = 0.01 and 0.001 S/m, computed using the FDTD method. Fig. 4 is for stroke location A (35-kA current), and Fig. 5 is for stroke location B (55-kA current). Note that in Fig. 4 the maximum radius of corona region around the wire is 19.8 cm at d=500 m, and in Fig. 5 it is 13.2 cm at d=0 m. The FDTD-computed waveforms of induced voltages without considering corona are also shown in the figures.

For stroke location A, peak values of lightning-induced voltages at d=500 m computed using the FDTD method without considering corona, are about 140, 185, and 300 kV for ground conductivity of infinity, 0.01, and 0.001 S/m,

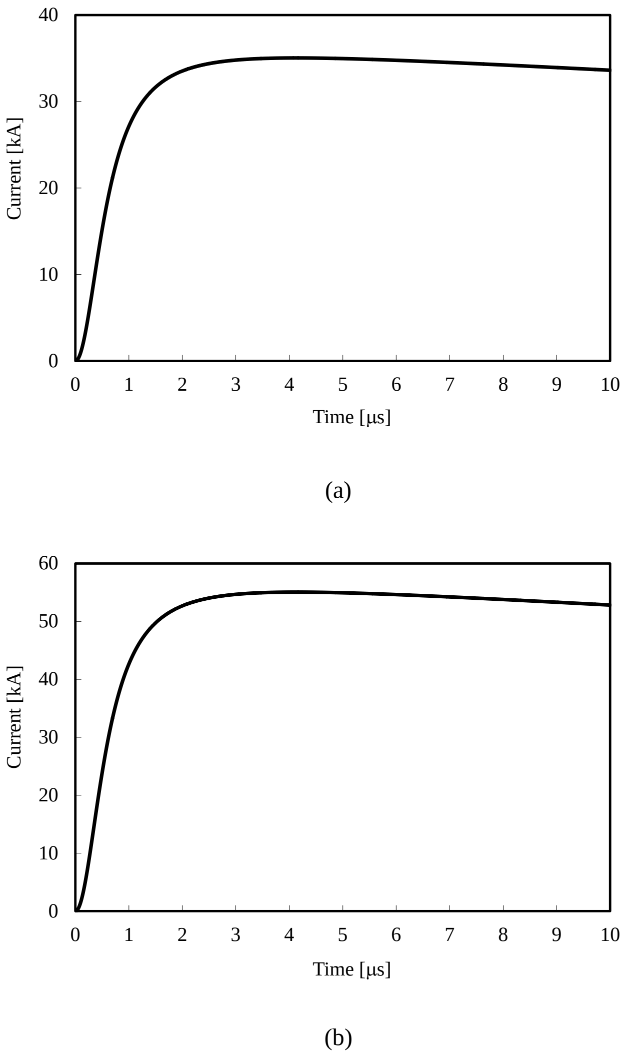


Fig. 3. Waveforms of injected negative lightning return-stroke current (positive charge moving up). The peak of the injected current is (a) 35 kA for stroke location A, and (b) 55 kA for stroke location B.

respectively. For stroke location B, FDTD-computed peak values of lightning-induced voltages at d=0 m without considering corona, are about 135, 195, and 335 kV for ground conductivity of infinity, 0.01, and 0.001 S/m, respectively. These results agree reasonably well with the corresponding computed results in [1], [2]: about 130, 160, 250 kV for stroke location A, and about 130, 200, 400 kV for stroke location B.

It follows from Figs. 4 and 5, that the induced voltage magnitudes are larger and the risetimes are longer in the presence of corona discharge on the horizontal wire. This trend agrees with that reported in [1], [2], although the increase predicted by our full-wave model is less significant than in their studies based on the circuit-theory approach.

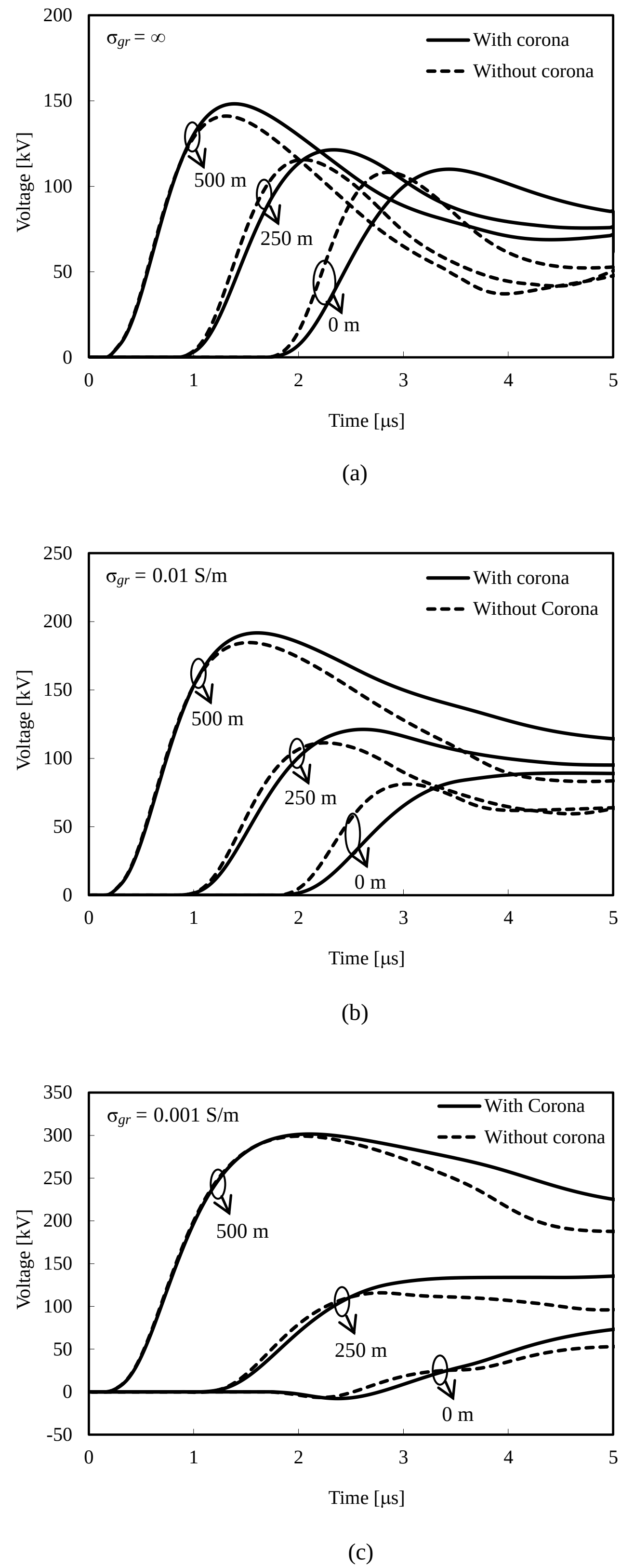
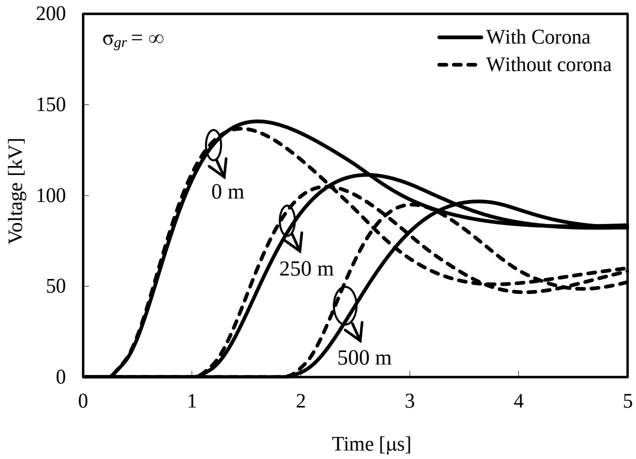
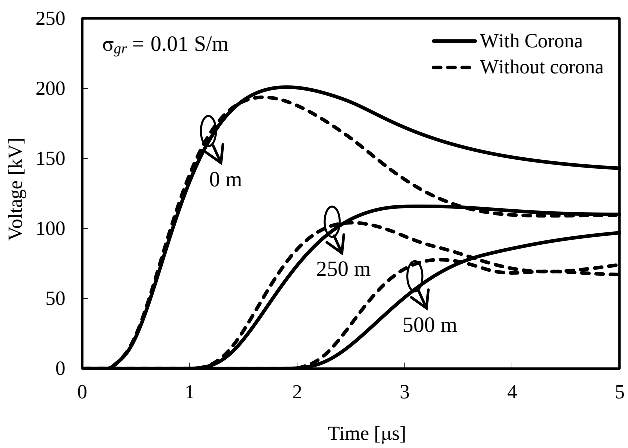


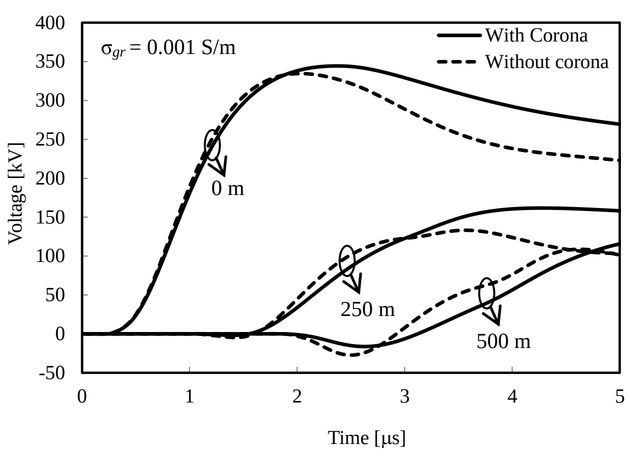
Fig. 4. Stroke location A: FDTD-computed (for σ_{cor} = 40 μS/m, E_0=2.9 MV/m, and E_{cn}=1.5 MV/m) waveforms of induced voltages at d =0, 250, and 500 m from either end of the 5-mm radius, 1.0-km-long horizontal wire. The computations were performed for (a) perfectly conducting ground (σ_{gr} = ∞), (b) σ_{gr} = 0.01 S/m, and (c) σ_{gr} = 0.001 S/m.



(a)



(b)



(c)

Fig. 5. Stroke location B: FDTD-computed (for σ_{cor} = 40 μS/m, E_0=2.9 MV/m, and E_{cn}=1.5 MV/m) waveforms of induced voltages at d =0, 250, and 500 m from the near end of the 5-mm radius, 1-km-long horizontal wire. The computations were performed for (a) perfectly conducting ground (σ_{gr} = ∞), (b) σ_{gr} = 0.01 S/m, and (c) σ_{gr} = 0.001 S/m.

IV. Conclusions

In this paper, a simplified (engineering) model of corona discharge for the FDTD computations has been applied to analysis of lightning induced voltages at different points along the 5-mm radius, 1-km long single overhead wire in the presence of corona. Both perfectly conducting and lossy ground cases were considered. FDTD calculations were performed using a 3D non-uniform grid. The progression of corona streamers from the wire is represented as the radial expansion of cylindrical conducting (40 μS/m) region around the wire. The critical electric field on the surface of the 5-mm-radius wire for corona initiation is set to E_0 = 2.9 MV/m. The critical background electric field for streamer propagation is set to E_{cn} = 1.5 MV/m. The magnitudes of FDTD-computed lightning-induced voltages in the presence of corona discharge are larger than those computed without considering corona. The observed trend is in agreement with that reported by *Nucci et al.* and *Dragan et al.*, although the increase predicted by our full-wave model (up to 5%) is less significant than in their studies based on the distributed-circuit model with sources specified using electromagnetic field theory (up to a factor of 2).

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