Parallel-in-Space-and-Time Scheme for Implicitly Coupled Electromechanical and Electromagnetic Transients Simulation

Shrirang Abhyankar, Alexander J. Flueck

***Abstract*—In this paper, we describe a parallel implementation of the implicitly coupled solution approach for combined electromechanical and electromagnetic transients simulation. In the proposed parallel-in-space-and-time scheme, the equations for transient stability (TS) are partitioned in space and the coupled-in-time electromagnetic transients equations (EMT) are partitioned in time. Thus each processor is assigned equations for a part of the TS subnetwork along with a subset of coupled-in-time EMT equations. We present the implementation details of this parallel in space-and-time scheme, including parallel in space-and-time partitioning, equations for each processor, and experimentation with different linear solvers. We also discuss the scalability of the proposed scheme on different test systems.**

***Keywords*—Hybrid simulator, Implicitly-coupled solution approach, Transient stability, Electromagnetic transients.**

I. Introduction

The dynamic behavior of electrical power systems is simulated by using transient stability (TS) simulators and electromagnetic transient (EMT) simulators. A TS simulator, running at large time steps, is used for studying relatively slower dynamics (e.g., electromechanical interactions among generators) and can be used for simulating large-scale power systems. In contrast, an EMT simulator models the same components in finer detail and uses a smaller time step for studying fast dynamics (e.g. electromagnetic interactions among power devices). Because of small step size, simulating large-scale power systems with an EMT simulator is computationally inefficient. A hybrid simulator attempts to interface the TS and EMT simulators, which are running at different time steps. By modeling the bulk of the large-scale power system in a TS simulator and a small portion of the system in an EMT simulator, the fast dynamics of the smaller area can be studied in detail, while providing a global picture of the slower dynamics for the rest of power system. In the existing hybrid simulation interaction protocols, the two simulators run independently, exchanging solutions at regular intervals.

In [1], the authors proposed an implicitly coupled solution approach for the combined transient stability and electromagnetic transient simulation. This approach combines the two sets of equations with their different time steps by a simultaneous calculation of several time steps of the EMT equations while computing a single time step of the TS equations.

In this paper we present an implementation of the implicitly-coupled electromechanical and electromagnetic transient simulation based on a novel parallel approach in space and time. In the proposed approach, the equations for TS are partitioned spatially while the coupled-in-time EMT equations are partitioned in time. We illustrate the scalability of this parallel implementation on several test systems using two different linear solution schemes.

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II. Hybrid simulators

The idea of combined TS-EMT simulation was first proposed by Heffernan et al. [10] to simulate combined HVAC-HVDC systems. They modeled a HVDC link in detail within a stability-based AC system framework, thus exploiting the advantages of both EMT and TS. Specifically, they executed TS and EMT alternately with periodic coordination of the results. Reeve and Adapa [14] proposed that the boundary of the interface should be extended further into the AC network in order to take into consideration the effect of harmonics generated by power electronics on the AC network. Anderson et al. [6] presented another approach to take the harmonics into account. In their approach, the network equivalent for the TS network is represented by a frequency-dependent equivalent, instead of a simple fundamental frequency equivalent circuit. Sultan et. al., [19] basically adopted the approaches described above, extending the interface location into the AC network to some extent, and at the same time having a frequency-dependent TS network equivalent. Kasztenny et al.[12] have also discussed a general method for linking different modeling techniques such as waveform-type, phasor-type, and algebraic-type simulation techniques into one complete model. Over the years, many researchers have further explored the combined TS-EMT simulation in terms of both modeling and algorithm. *Hybrid simulator* has become a common term to refer to a combined TS-EMT simulator.

In the hybrid simulator, the power system network is partitioned into two subnetworks: a large network (TS domain of operation) and a smaller network run with EMT. The large network has been called the external system [14], [6], [17], electromechanical transient network [16], or TS-program subsystem [9], while the smaller system has been called the detailed system [14], [6], [17], EMT network [16], or instantaneous network [20]. In this paper, the larger network

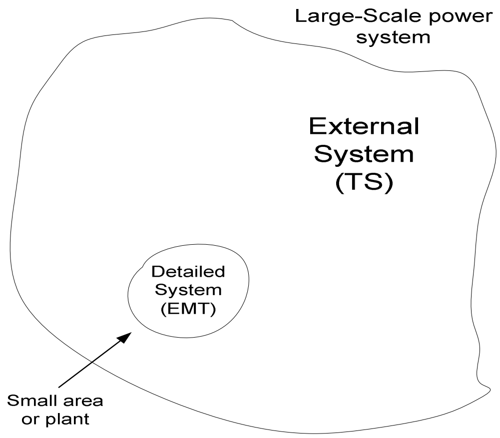

Fig. 1. Detailed and external system

will be called the external system and the smaller system will the detailed system (see Figure 1).

Since the TS and EMT run at different time steps, synchronization of these simulators is required for data exchange. This synchronization is done through predefined sequential actions that coordinate the data exchange between TS and EMT simulators [13]. Both serial [10], [6] and parallel [17], [18] interaction protocols have been proposed so far. In serial protocols, only one simulator, either TS or EMT, runs while the other is idle. In parallel protocols, both simulators run at the same time. A comprehensive overview of the state of the art in hybrid simulators is given in [13].

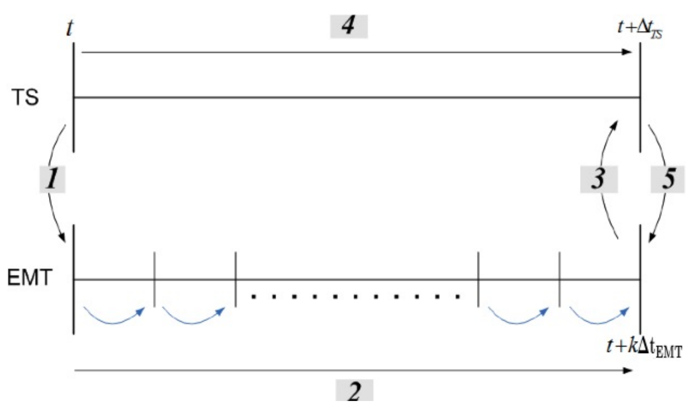

Fig. 2. Serial interaction protocol for one TS time step

Figure 2 describes the data exchange between the TS and EMT simulators, for one TS time step, in a serial interaction protocol.

We note here that the external system equivalent is not updated when EMT is running; it is held constant for all the EMT time steps within a TS time step. This equivalent can be also derived from some extrapolated history data, but either way, it may not accurately predict the conditions at the next TS time step. While such an approach would be sufficient if the TS system was evolving slowly (i.e., there is a small difference between the voltages and currents at two consecutive time steps), for large changes this approach may not be suitable.

Another point to note here is that no iterations are done between TS and EMT to check whether the solutions at each TS and EMT boundary are consistent. Having no iterations is probably sufficient when the external system equivalent does not change much, and it may be adequate for the gradually changing external system voltage profile. However, for large changes in voltages between consecutive TS time steps, iterations would be needed to update the external system equivalent repeatedly. Because of the explicit coupling, more iterations would be required, and the solution still might diverge [2].

III. Implicitly coupled Electromechanical and Electromagnetic Simulation (TSEMT)

Instead of coupling TS and EMT at the application level, one could couple these two at the equation solution level [2]. To combine the two sets of equations with their different time steps and ensure that the TS and EMT solutions are consistent, the equations for TS and coupled-in-time EMT equations are solved simultaneously in a single large system of equations. While computing a single time step of the TS equations, a simultaneous calculation of several time steps of the EMT equations is undertaken. For the remainder of this paper, this implicitly coupled combined TS and EMT simulator will be referred to as TSEMT.

One of the major assumptions in TS is that the transmission network is always balanced. Hence a positive sequence network suffices for the analysis. For the hybrid simulators, such an assumption results in using only a balanced external system equivalent for EMT. Instead, we use a full three-phase phasor model of the external system. The details of this three-phase phasor transient stability modeling can be found in [3].

A. Network equivalents and waveform conversion

Network equivalents and waveform conversion form the coupling between TS3ph and EMT in the proposed TSEMT simulator. The equations for these are included in the overall system of equations and form the implicit coupling between the TS and EMT equations. We use a Thevenin equivalent and fundamental frequency phasor current source injection as the network equivalents for the external and detailed system respectively. The Thevenin equivalent connects the EMT to TS3ph and the fundamental frequency phasor current source injection connects TS3ph to EMT as shown in Fig. 3. The Thevenin impedance is kept constant unless there is a topological change in the external system. Only the Thevenin voltage V_{thev} needs to be updated at each time step. Since EMT uses instantaneous voltages, the phasor voltage V_{thev} needs to be converted to instantaneous waveform v_{thev}. This conversion is done by a fundamental fixed-frequency sine wave generator.

The inclusion of the external system equivalent in EMT introduces an additional set of differential equations,

$$L_{thev}\frac{di_{bdry}}{dt} = v_{thev} - R_{thev}i_{bdry} - v_{bdry}, \tag{1}$$

where v_{thev} is the instantaneous Thevenin voltage, i_{bdry} is the current flowing out through the detailed system boundary buses, and v_{bdry} is the instantaneous boundary bus voltage.

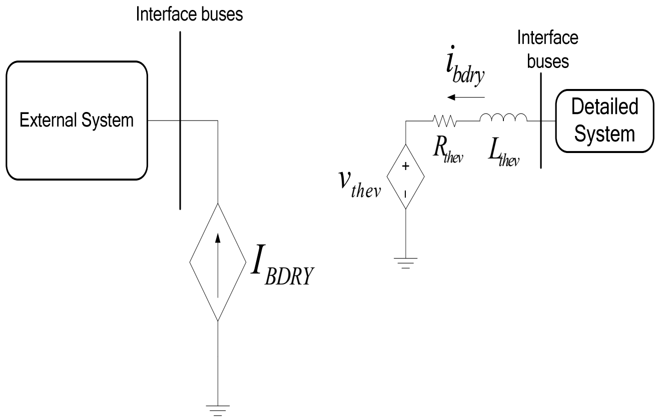

Fig. 3. Equivalent networks for detailed and external system

The detailed system equivalent used by the TS portion is a fundamental frequency phasor current injection at the boundary buses. This phasor current injection is computed by using a Fourier analysis of EMT boundary bus currents i_{bdry} over a running window of one cycle of fundamental frequency. The phasor current injections, I_{BDRY}, at the external system boundary buses expressed in rectangular form can be related to the instantaneous current, i_{bdry}, flowing through the detailed system boundary buses by Fourier analysis as follows:

$$\begin{aligned} I_{BDRY,D}(t+\Delta t_{TS}) &= \frac{2}{T}\int_{\tau=t}^{t+\Delta t_{TS}} i_{bdry}(\tau)\sin(\omega\tau)d\tau \\ I_{BDRY,Q}(t+\Delta t_{TS}) &= \frac{2}{T}\int_{\tau=t}^{t+\Delta t_{TS}} i_{bdry}(\tau)\cos(\omega\tau)d\tau. \end{aligned} \tag{2}$$

B. Implicitly coupled solution approach

In compact form, the TS system differential-algebraic model (DAE) equations are

$$\begin{aligned} \frac{dX_{TS}}{dt} &= F(X_{TS}, V_{TS}) \\ 0 &= G(X_{TS}, V_{TS}). \end{aligned} \tag{3}$$

In (3), X_{TS} represents the dynamic variables for the synchronous generators and the associated control circuitry (i.e., exciters, voltage regulators, turbine governors etc.) while V_{TS} are the network phasor bus voltages. The differential equations for EMT are described by (4).

$$\frac{dx_{EMT}}{dt} = f(x_{EMT}) \tag{4}$$

Note here that the differential model for EMT is due to using a state variable analysis scheme and the transmission lines modeled as equivalent π models. If distributed-parameter transmission line models and a numerical integration substitution solution scheme was used, then the EMT model would be described by algebraic equations with history terms instead.

Adding the coupling, the equations for TS and EMT in compact form are

$$\begin{aligned} \frac{dX_{TS}}{dt} &= F(X_{TS}, V_{TS}) \\ 0 &= G(X_{TS}, V_{TS}, I_{BDRY}) \\ \frac{dx_{EMT}}{dt} &= f_1(x_{EMT}, i_{bdry}) \\ \frac{di_{bdry}}{dt} &= f_2(x_{EMT}, i_{bdry}, v_{thev}) \end{aligned} \tag{5}$$

Discretizing the TS equations with the TS time step, Δt_{TS}, and EMT equations with EMT time step, Δt_{EMT}, and using an implicit trapezoidal integration scheme, one can represent the complete set of equations to solve at each TS time step (6)-(13). Equations (6) and (7) represent the equations for the external system for one TS time step, while (8)-(13) are the coupled-in-time EMT equations. Equations (6)-(13) are solved simultaneously by using Newton's method at each TS time step:

$$X_{TS}(t_{N+1}) - X_{TS}(t_N) - \frac{\Delta t_{TS}}{2}(F(t_{N+1}) + F(t_N)) = 0 \tag{6}$$

$$G(t_{N+1}) = 0 \tag{7}$$

$$x_{EMT}(t_{n+1}) - x_{EMT}(t_n) - \frac{\Delta t_{EMT}}{2}(f_1(t_{n+1}) + f_1(t_n)) = 0 \tag{8}$$

$$i_{bdry}(t_{n+1}) - i_{bdry}(t_n) - \frac{\Delta t_{EMT}}{2}(f_2(t_{n+1}) + f_2(t_n)) = 0 \tag{9}$$

$$x_{EMT}(t_{n+2}) - x_{EMT}(t_{n+1}) - \frac{\Delta t_{EMT}}{2}(f_1(t_{n+2}) + f_1(t_{n+1})) = 0 \tag{10}$$

$$i_{bdry}(t_{n+2}) - i_{bdry}(t_{n+1}) - \frac{\Delta t_{EMT}}{2}(f_2(t_{n+2}) + f_2(t_{n+1})) = 0 \tag{11}$$

$$\vdots$$

$$\vdots$$

$$x_{EMT}(t_{n+k}) - x_{EMT}(t_{n+k-1}) - \frac{\Delta t_{EMT}}{2}(f_1(t_{n+k}) + f_1(t_{n+k-1})) = 0 \tag{12}$$

$$i_{bdry}(t_{n+k}) - i_{bdry}(t_{n+k-1}) - \frac{\Delta t_{EMT}}{2}(f_2(t_{n+k}) + f_2(t_{n+k-1})) = 0 \tag{13}$$

where

$$\begin{aligned} I_{BDRY}(t_{N+1}) &= h_{EMT->TS3ph}(i_{bdry}(t_{n+1}), \\ &\quad i_{bdry}(t_{n+2}), \ldots, i_{bdry}(t_{n+k})) \\ (v_{thev}(t_{n+1}), v_{thev}(t_{n+2}), &\ldots, v_{thev}(t_{n+k})) \\ &= h_{TS3ph->EMT}(V_{thev,TS}(t_N), V_{thev,TS}(t_{N+1})) \end{aligned}$$

represents the coupling between TS and EMT.

IV. Parallel-in-space-and-time TSEMT

For parallel TSEMT equations, we propose a strategy of doing a spatial decomposition (i.e., parallel in space) for the external system equations and a temporal decomposition (i.e., parallel-in-time) for the coupled-in-time detailed system equations as shown in Figure 4. The proposed parallel strategy

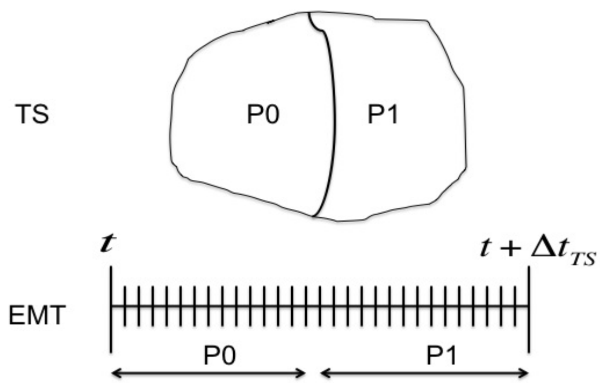

Fig. 4. Parallel in space and time partitioning for TSEMT

was chosen from the following observations:

- The external system is larger compared than the detailed system for large-scale power systems.
- The detailed system has coupled-in-time equations.
- The biggest part in the TS subsystem is the transmission network.
- Generators and loads are incident at transmission nodes, that is, their equations are local.
- Communication between processors should be minimum, and the work load for each processor should be balanced.

A. External system spatial decomposition

For the external system, we adopt a domain decomposition approach that partitions the external system into several subnetworks. Figure 4 shows an example of the partitioning of an external system on two processors. Each external system subnetwork is the domain of operation of a processor; in other words, each processor is assigned the DAE equations for the subnetwork. Equation (14) represents the equations for each processor, where the superscript p represents the variables for the current processor and the superscript c represents the variables needed from other processors to compute the function on the current processor.

$$\begin{aligned} \frac{dX_{TS}^p}{dt} &= F(X_{TS}^p, V_{TS}^p) \\ 0 &= G(X_{TS}^p, V_{TS}^p, V_{TS}^c, I_{BDRY}) \end{aligned} \tag{14}$$

Note that the differential equations (i.e., the electromechanical equations for machines) are naturally decoupled because they are incident at a bus only, whereas the algebraic network equations require communication with other processors to compute their local function. Hence, we partition the external system based only on the network topology. This partitioning of the network can be done by hand by judicious topology scanning or by graph partitioning techniques. We use the ParMetis package [11] for network partitioning. ParMetis is a parallel graph partitioning package that is used for partitioning of unstructured graphs. It tries to partition a given graph with the objective of minimizing the edge cuts while having balanced partitions, that is, balancing computational load for each processor. Along with the network topology information, larger weights were assigned to the vertices having generators to account for the extra computation involved for the generator differential and algebraic equations. With vertex weights, ParMetis tries to minimize the edge cuts and also have the same number of vertex weights in each subdomain.

B. Detailed system temporal decomposition

The detailed system equations consist of coupled-in-time EMT equations, and hence we use a parallel-in-time decomposition where all the EMT equations within one TS time step are split among different processors. Thus, each processor is assigned equations k/np EMT time steps where k is the number of coupled-in-time EMT time steps and np is the number of processors. An example that splits the equations on two processors is shown in Figure 4. Here, the first k/np are assigned to processor 0 and the remainder to processor 1. Equations (15)-(20) describe the detailed system equations for a processor with starting time t_m and p coupled-in-time EMT steps.

$$x_{EMT}(t_{m+1}) - x_{EMT}(t_m) - \frac{\Delta t_{EMT}}{2}(f_1(t_{m+1}) + f_1(t_m)) = 0 \tag{15}$$

$$i_{bdry}(t_{m+1}) - i_{bdry}(t_m) - \frac{\Delta t_{EMT}}{2}(f_2(t_{m+1}) + f_2(t_m)) = 0 \tag{16}$$

$$x_{EMT}(t_{m+2}) - x_{EMT}(t_{m+1}) - \frac{\Delta t_{EMT}}{2}(f_1(t_{m+2}) + f_1(t_{m+1})) = 0 \tag{17}$$

$$i_{bdry}(t_{m+2}) - i_{bdry}(t_{m+1}) - \frac{\Delta t_{EMT}}{2}(f_2(t_{m+2}) + f_2(t_{m+1})) = 0 \tag{18}$$

$$\vdots$$

$$\vdots$$

$$x_{EMT}(t_{m+p}) - x_{EMT}(t_{m+p-1}) - \frac{\Delta t_{EMT}}{2}(f_1(t_{m+p}) + f_1(t_{m+p-1})) = 0 \tag{19}$$

$$i_{bdry}(t_{m+p}) - i_{bdry}(t_{m+p-1}) - \frac{\Delta t_{EMT}}{2}(f_2(t_{m+p}) + f_2(t_{m+p-1})) = 0 \tag{20}$$

C. PETSc: Portable Extensible Toolkit for Scientific computation

Our parallel implementation is based on the mathematical and computing framework of the high-performance library PETSc [8]. PETSc is an open source package (BSD-style license) for the numerical solution of large-scale applications and provides the building blocks for the implementation of large-scale application codes on parallel (and serial) computers. The wide range of sequential and parallel linear solvers, preconditioners, reordering strategies, flexible run-time options, ease of code implementation, debugging options, and a comprehensive source code profiler make PETSc an attractive experimentation platform for developing our parallel dynamics simulator. A review of PETSc and its use for developing scalable power system simulations can be found in [4]. The PETSc package consists of a set of libraries for creating parallel vectors, matrices, and distributed arrays, scalable linear, nonlinear, and time-stepping solvers. A few of the parallel components in PETSc are shown in Figure 5.

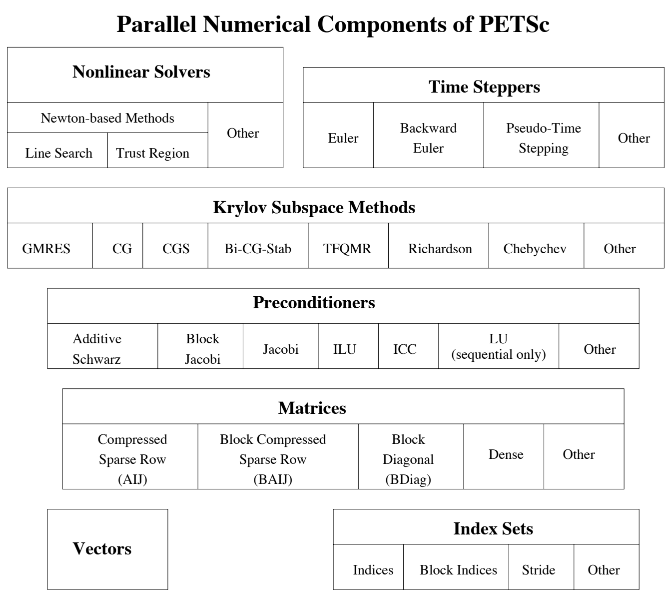

Fig. 5. Numerical libraries of PETSc [7]

V. Parallel TSEMT performance results

We present the results on four test systems: a IEEE 118-bus system and three systems formed by replicating the external system 6×, 10×, and 16×. The detailed system for all the test cases consists of the radial system formed by buses 20-23 with three transmission lines and four loads as shown in 6. The disturbance considered was a temporary three-phase balanced fault inside the EMT region applied for 6 cycles from 0.1 seconds to 0.2 seconds. The TS time step is 0.01667 seconds while the EMT time step is 0.0001667 seconds; that is, $\Delta t_{TS}/\Delta t_{EMT}$ = 100. The simulation time length was set to 1 second. The parallel performance runs were done on a shared-memory machine with four 2.2 GHz AMD Opteron 6274 processors. Each processor has 16 cores, giving a total of 64 cores. The TSEMT code is written in C by using the PETSc library framework and compiled with GNU's gcc compiler with -O3 optimization.

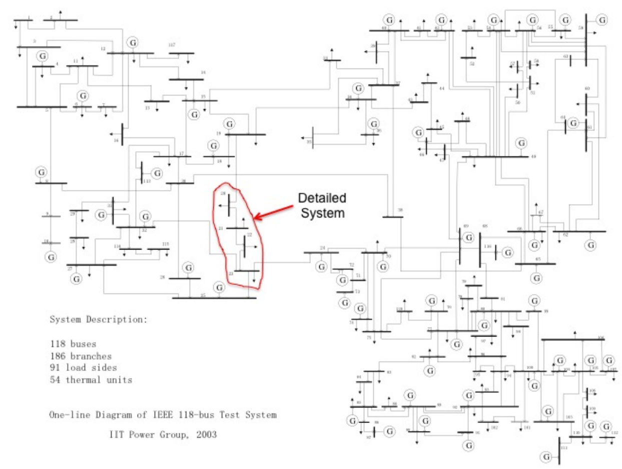

Fig. 6. Detailed and External system division for the IEEE 118 bus system

The scalability of the proposed parallel-in-space-and-time approach was tested for two linear solution schemes:

- **Direct parallel LU factorization using MUMPS:** MUMPS is a parallel sparse direct linear solver that uses a multifrontal approach [5] for the parallel solution of $Ax = b$.
- **GMRES with block-Jacobi preconditioner:** Krylov subspace based iterative solver GMRES [15] is a linear solver along with a block-Jacobi preconditioner. The block-Jacobi preconditioner uses the diagonal blocks of the Jacobian matrix on each processor. On each block we do a LU factorization with a quotient minimum degree (QMD) reordering scheme to form the block-Jacobi preconditioner.

Figures 7-10 show the execution times, while Figures 11-14 show the speedup for the 118, 6×118, 10×118, and 16×118 bus test systems respectively. The speedup is compared with a direct LU factorization using the MUMPS package on 1 processor. As seen, GMRES with the block-Jacobi preconditioner was found to be more scalable than a parallel direct solution using MUMPS. For the largest test case, GMRES with the block-Jacobi preconditioner achieved a speedup of 11.54 on 16 cores. Note that a superlinear speedup observed using GMRES with block-Jacobi preconditioner for 2,4, and 8 core runs for some cases can be attributed to cache effect.

VI. Conclusions

This paper presented a parallel implementation of the implicitly-coupled electromechanical and electromagnetic transients simulation using a space-and-time partitioning strategy. Test results presented on several test systems show the scalability of the proposed parallel implementation using two linear solution schemes. For all the test systems the iterative solver GMRES using a block-Jacobi preconditioned was found to be more scalable than a parallel direct solution using the MUMPS package.

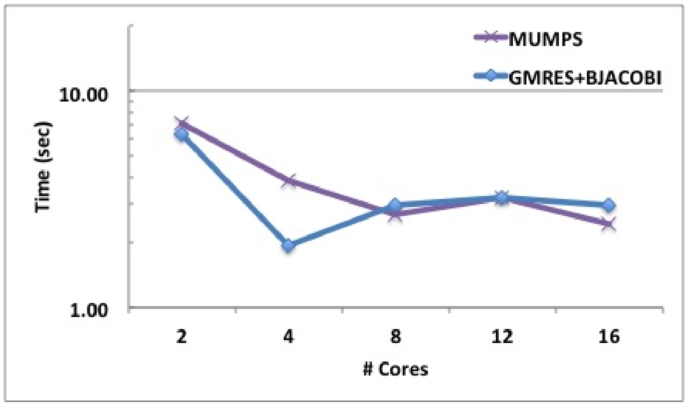

Fig. 7. Comparison of TSEMT runtimes for the 118 bus system

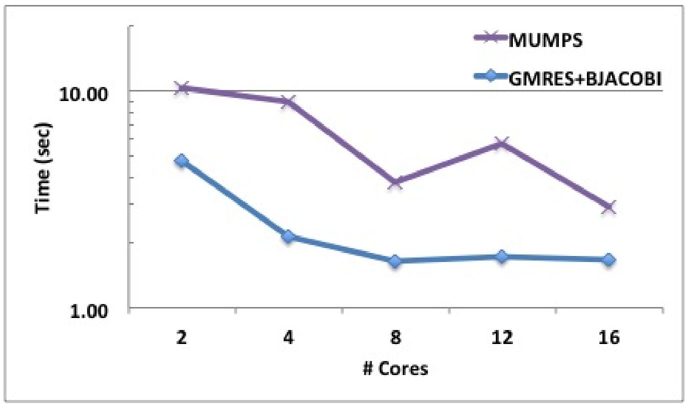

Fig. 8. Comparison of TSEMT runtimes for the 708 bus system

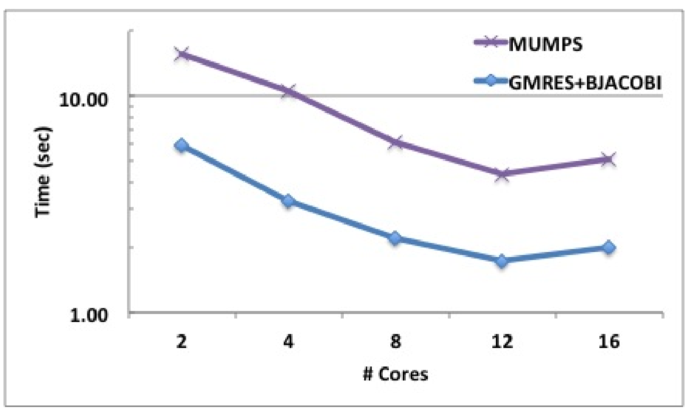

Fig. 9. Comparison of TSEMT runtimes for the 1180 bus system

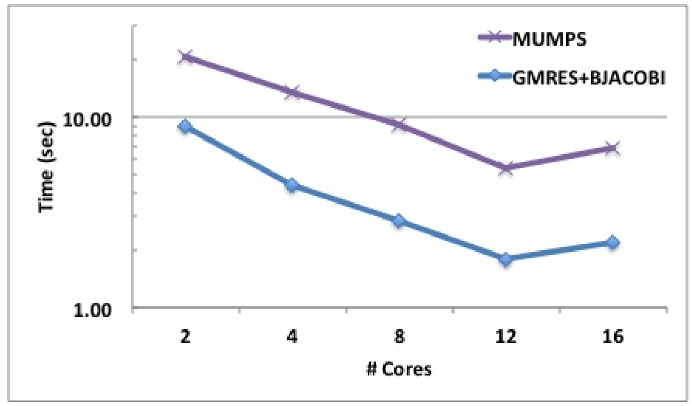

Fig. 10. Comparison of TSEMT runtimes for the 1888 bus system

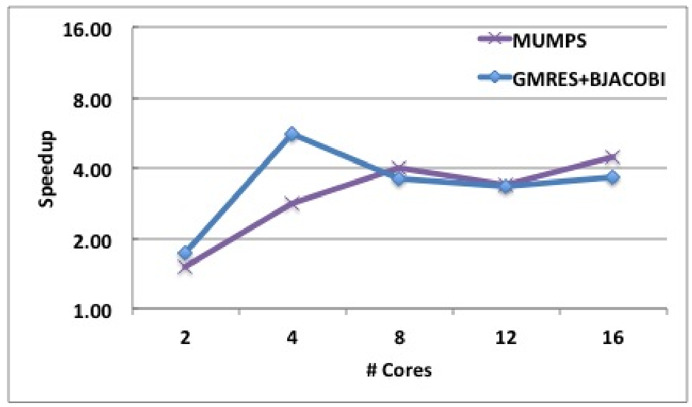

Fig. 11. Comparison of TSEMT speedup for the 118 bus system

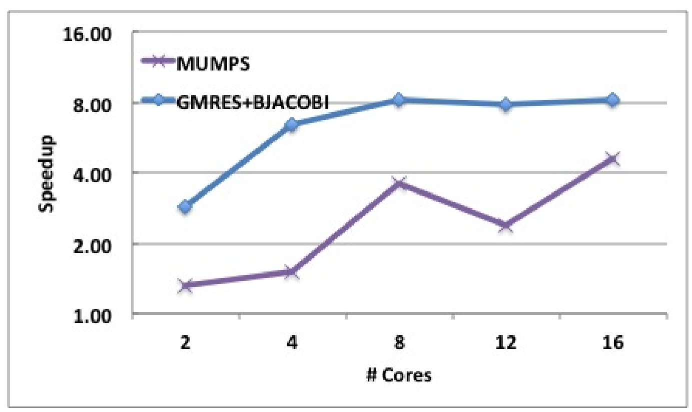

Fig. 12. Comparison of TSEMT speedup for the 708 bus system

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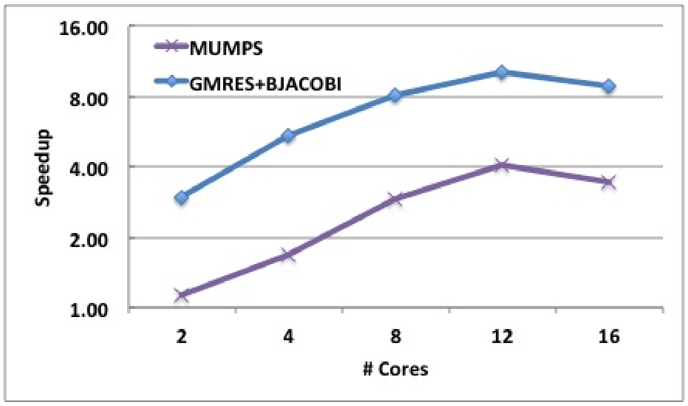

Fig. 13. Comparison of TSEMT speedup for the 1180 bus system

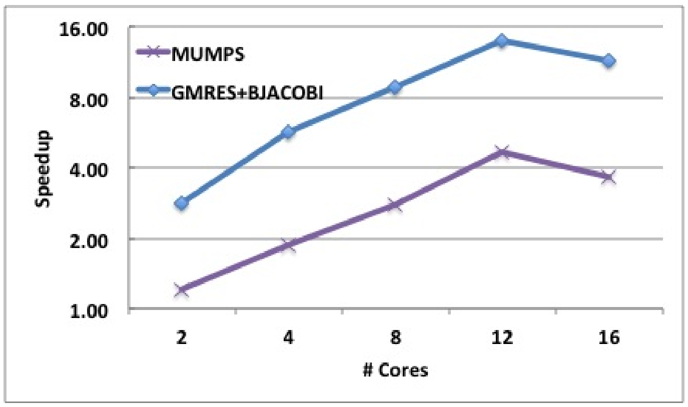

Fig. 14. Comparison of TSEMT speedup for the 1888 bus system

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