Improvement of Transformer Saturation Modeling for Electromagnetic Transient Programs

M. Salimi, A. M. Gole, R. P. Jayasinghe

***Abstract*—This paper compares the errors made when the saturation is modeled as a current source placed at the terminals of the transformer, rather than in between the primary and secondary leakage inductances. Such an approach is often used in nodal analysis based electromagnetic transient programs in order to increase the computational efficiency of transformer model. The results are satisfactory for most studies. However, there are some inaccuracies which can be observed in some situations.**

The paper also introduces a slight modification, where the saturation current injections can still be located at the terminals, but the injections can be calculated exactly as though the magnetizing branch is located in between the leakages. This approach can accurately represent magnetizing current in transformers without sacrificing the computational efficiency of the previous model. This new model is compared with the conventional model by conducting several simulations using PSCAD/EMTDC.

***Keywords*: Transformer saturation, Simulation, Modeling, Current injection.**

I. Introduction

THE transformer is a widely used element in power systems. It is also one of the most common nonlinear elements in power systems since the ferromagnetic materials used in its core have nonlinear magnetization characteristics. In most applications, the magnetizing current of the power transformer is negligible under steady state operation [1]; therefore it can be modeled as a linear device. However, when such transformer works under abnormal operating conditions, it may experience saturation.

Many electromagnetic transient studies require an accurate modeling of the transformer saturation phenomena. Examples of such studies are inrush currents from transformer energizing and overvoltage caused by ferroresonance or faults [2]-[5]. Therefore modeling this nonlinear phenomenon is necessary for electromagnetic transient (EMT) type simulation tools.

In nodal analysis based programs, transformer saturation is modeled by an extra nonlinear inductance or injecting current at the transformer terminal [6]-[9]. The benefit of this method is that internal nodes of the transformer are not added to the total number of nodes in the system and it makes the model computationally efficient. This model has decent accuracy for most power system studies including faults and other types of transients.

Although placing the saturation model at the transformer terminal reduces the resources required for simulation, it also causes inaccuracies. This is because the saturation current is computed by assuming that the magnetizing branch is directly across one of the transformer ports, rather than in between the two leakage impedances (for a two winding transformer).

This paper proposes an approach based on the mathematical equivalent circuit of transformer for modeling transformer saturation. Using this approach, the saturation current with the short circuit impedance effect can be modeled accurately while the computational efficiency is retained in EMTP-type simulation tools. The new model also shows that modeling the saturation in this way results in more accurate results as compared with the previous approach.

II. Transformer Mathematical Model

There are several methods to approximate the equivalent circuit of transformer depending on the winding structure. One of the most commonly used equivalent circuit of transformer known as T model emphasizes the unity of the magnetizing current and resolves the leakage inductance into primary and secondary components [1], [10]. This electric model is derived from the equivalent magnetic circuit of transformers as shown in Fig. 1.

This model consists of series resistor-inductor branches for the primary (R_1, L_{l1}) and secondary (R_2, L_{l2}) windings, representing the copper losses and leakage flux. A shunt resistance (R_{sh}) is used to model no-load losses, and a shunt, non-linear inductance (L_{mag}) is used to model magnetization. The nonlinear nature of this inductance is shown in Fig. 2, where flux is plotted as a function of current.

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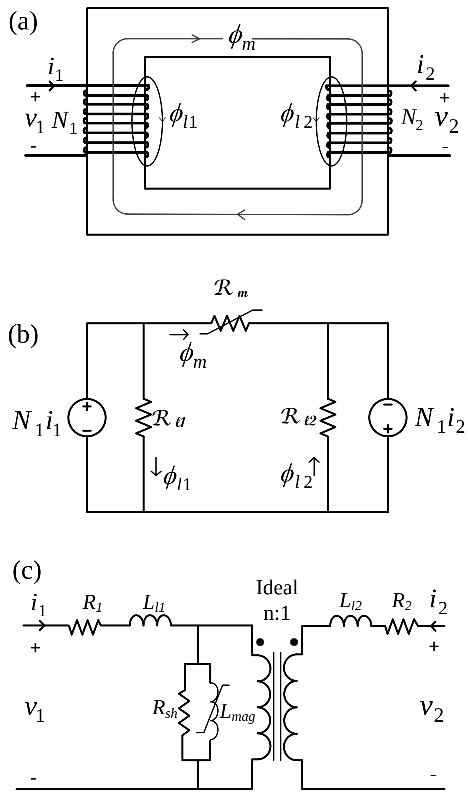


Fig. 1. (a) Transformer structure, (b) Magnetic circuit model of transformer, (c) Electric circuit model of transformer.

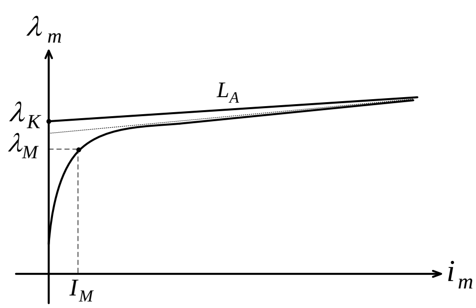


Fig. 2. Core magnetization characteristic of the classical transformer.

In Fig. 2, the air core inductance L_A is represented by the straight-line characteristic, which bisects the flux axis at λ_K. The actual magnetizing characteristic is asymptotic to both the vertical flux axis and the air core inductance characteristic. The sharpness of the knee point is defined by λ_M and I_M, which can represent the peak magnetizing flux and current at rated voltage. It is possible to define an asymptotic equation for current in the non-linear magnetizing inductance if L_A, λ_K, λ_M and I_M are known [6]. Magnetizing current i_m can be defined as

$$i_m = \frac{\sqrt{(\lambda_m - \lambda_k)^2 + 4 \cdot D \cdot L_A} + \lambda_m - \lambda_K}{2 \cdot L_A} - \frac{D}{\lambda_K} \tag{1}$$

In this equation, constants λ_M, λ_K, A, B, C, D are defined as follows.

$$\begin{aligned}
\lambda_M &= \frac{V_M}{2\pi f} \\
\lambda_K &= K\lambda_M \\
A &= \frac{L_A}{\lambda_K^2} \\
B &= \frac{L_A I_M - \lambda_M}{\lambda_K} \\
C &= I_M\left(L_A I_M - \lambda_M + \lambda_K\right) \\
D &= \frac{-B - \sqrt{B^2 - 4AC}}{2A}
\end{aligned} \tag{2}$$

III. Saturation Model in EMT-Type Programs

The underlying solution approach of Electromagnetic Transient programs is based on discretizing the differential equations for each circuit component using a trapezoidal rule for integration [6], [7]. The network nodal equation has the following general form.

$$GV_n = I_h \tag{3}$$

where G is the network nodal construction matrix, the vector V_n contains the node voltages and the vector I_h includes the history current sources and independent sources injected to the nodes.

Saturation can be represented in one of these two ways: first, with a varying inductance as the magnetizing branch second, with a compensating current source along with the linear magnetizing branch. Current source representation is preferred in nodal analysis based solutions (such as PSCAD/EMTDC) since it does not involve change to the subsystem matrix during saturation and reduces the computational effort comparing to modeling with a nonlinear inductance. Note that using a varying inductor requires calculating a new G matrix every time the inductor value changes. This can add a significant amount of load to the processor and increases the simulation time.

The representation of the transformer without saturation (i.e. the coupled circuits) can be in the form of resistance and inductance branches. Alternatively, in order to reduce the number of nodes, this representation can be collapsed to include only the terminal nodes. To do so, the no load losses are moved to the transformer terminals and the linear transformer model is represented by a branch resistance and inductance matrix which is obtained by voltage-current relationship based on the mutual coupling theory as stated in (4). For ease of explanation, we use a 2 winding transformer, but extension to more windings is obvious.

$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} R_1 & 0 \\ 0 & R_2 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} + \begin{bmatrix} L_m + L_{l1} & L_m / n \\ L_m / n & L_m / n^2 + L_{l2} \end{bmatrix} \frac{d}{dt} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} \tag{4}$$

In (4) n is transformer turns ratio and L_m is the linear magnetizing inductance.

In this approach, there is no access to the middle node of the T-model to inject the saturation current. Therefore the saturation current injection is placed at the transformer terminal, i.e. across the primary or secondary winding (usually winding wound closest to the core). In each case the corresponding voltage is used to calculate the magnetizing flux. Different saturation models for a two-winding single-phase transformer saturation are shown in Fig. 3.

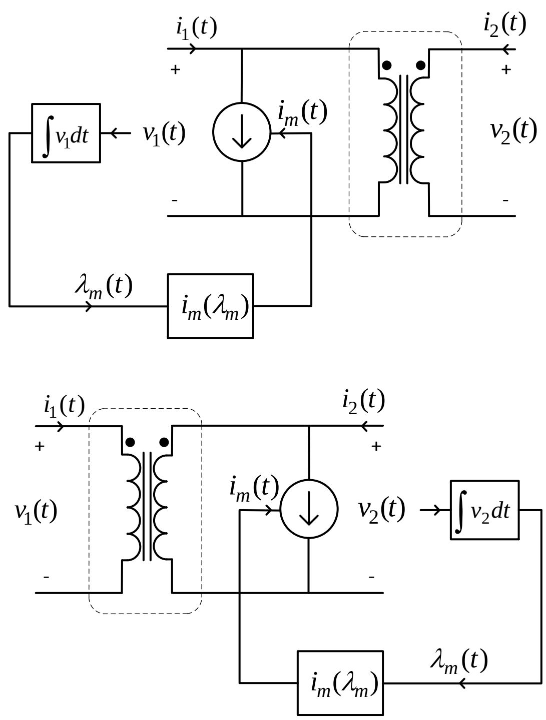


Fig. 3. Formulation of nonlinear magnetization current in classical transformers (a) primary winding current injection, (b) secondary winding current injection.

The approach of injecting the current source at the transformer terminal as discussed above usually provides acceptable simulation results for many large power system studies. However, the impact of voltage drops across the leakage impedance is significant during the saturation of transformer, since the magnetizing inductance will drop dramatically during the saturation. This phenomenon is not properly included in these models because the saturation current source location has been moved from its true location across the magnetizing impedance to the new location across transformer terminal. To demonstrate the inaccurate results, two simulation cases that show the transformer inrush current in no load condition have been carried out. The transformer parameters are listed in Table I. In the first simulation case the current source is placed at the primary winding terminal and in the second one it is at the secondary winding terminal. The simulation results of transformer magnetizing current and secondary voltage for both scenarios are shown in Fig. 4 and 5 respectively.

TABLE I
TRANSFORMER PARAMETERS

Transformer MVA	100 [MVA]
Base frequency	60 [Hz]
Primary winding voltage	230 [kV]
Secondary winding voltage	115 [kV]
Leakage Reactance	0.1 [pu]
Copper losses	0.01[pu]
No load losses	0.01[pu]
Air core reactance	0.2 [pu]
Knee voltage	1.17[pu]
Magnetizing Current	0.4 [%]

It can be observed that when the saturation current is injected to the terminal connected to a strong ac source (low thevenin impedance), the secondary voltage waveform is not distorted. This is because the distorted magnetizing current is injected into the strong source and has negligible effect on distorting this strong voltage. On the other hand, injecting the saturation current to the secondary winding increases the distortion in its voltage waveform. Since part of the magnetizing current also flows through the leakage impedances and causes non-linear voltage drop.

In addition, in the first case the calculated magnetizing flux is more than the actual value since the leakage flux is not subtracted from it. This will result in significant increase in magnetizing current during the saturation comparing to second case as it is show in Fig. 5.

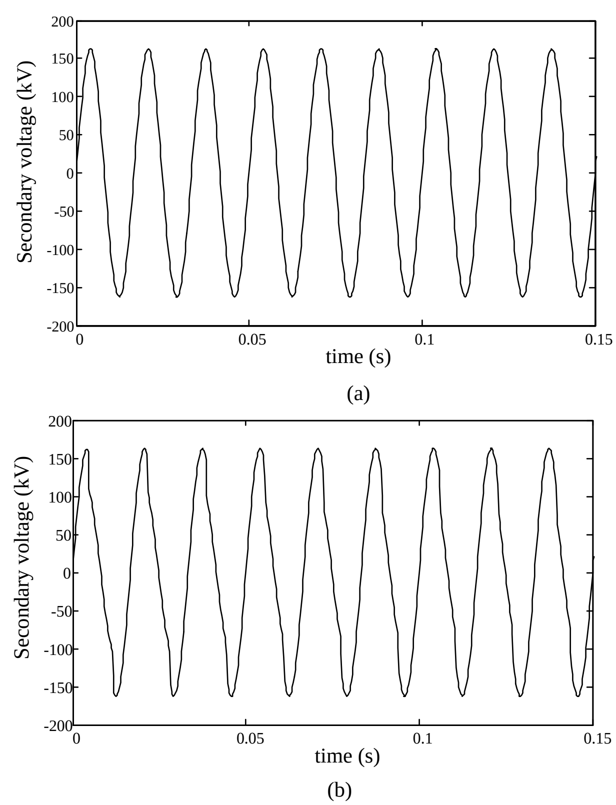
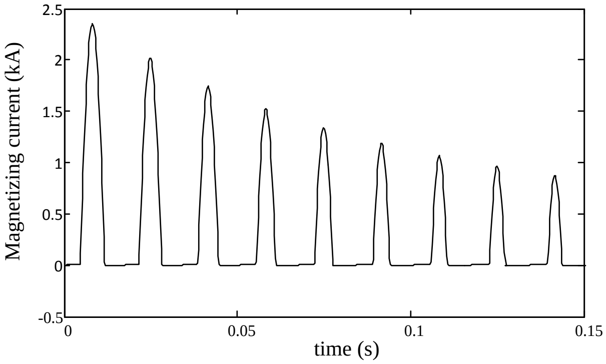
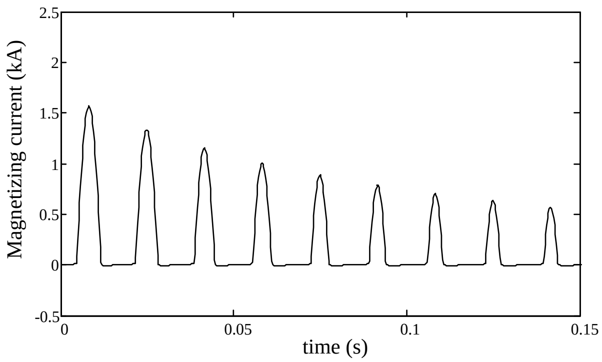


Fig. 4. Transformer secondary voltage where magnetizing current injection is (a) at primary terminal, (b) at secondary terminal.



(a)



(b)

Fig. 5. Transformer primary current where magnetizing current injection is (a) at primary terminal, (b) at secondary terminal.

These issues can be solved using the proposed method for modeling saturation in transformers, explained in the next section.

IV. Modified Representation of Saturation Current

Fig. 6 shows the true location of magnetizing branch in the transformer T model. The magnetizing current can be divided into two components; the linear component which is represented as a linear inductance L_m defined as

$$L_m = \frac{\lambda_M}{I_M} \tag{5}$$

where λ_M and I_M are the peak magnetizing flux and current at rated voltage as shown in Fig. 2., and the nonlinear component that is the difference between the total magnetizing current and the linear current as illustrated in Fig. 7. The nonlinear component or saturation current can be calculated as follows.

$$i_s(\lambda_m) = i_m(\lambda_m) - \frac{\lambda_m}{L_m} \tag{6}$$

In (6) i_m is the total magnetizing current and λ_m is the magnetizing flux calculated by (7).

$$\lambda_m = \int (v_1 - R_1 i_1 - L_{l1}\frac{di_1}{dt})dt = \int (v_1 - R_1 i_1)dt - L_{l1} i_1 \tag{7}$$

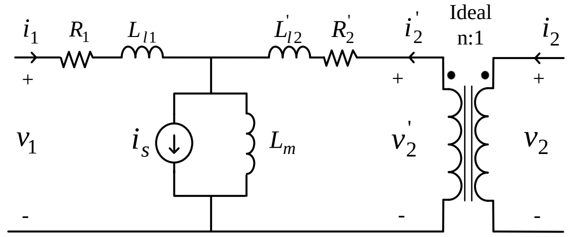


Fig. 6. Transformer T model including saturation current at the middle.

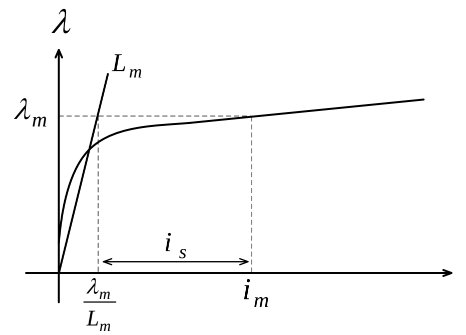


Fig. 7. Magnetizing current consists of linear current and saturation current.

In this case, the voltage equation of the transformer model in terms of terminal currents and saturation current is expressed in (8). In this paper, all the primed variables are reflected values from secondary to primary side of the transformer.

$$\begin{aligned} &V = RI + L\frac{d}{dt}I + L_m\frac{d}{dt}I_s \\ &V = \begin{bmatrix} v_1 \\ v_2' \end{bmatrix}, \quad I = \begin{bmatrix} i_1 \\ i_2' \end{bmatrix}, \quad I_s = \begin{bmatrix} 1 \\ 1 \end{bmatrix} i_s \\ &R = \begin{bmatrix} R_1 & 0 \\ 0 & R_2' \end{bmatrix}, \quad L = \begin{bmatrix} L_m + L_{l1} & L_m \\ L_m & L_m + L_{l2}' \end{bmatrix} \end{aligned} \tag{8}$$

To calculate the value of terminal currents at every time step, the trapezoidal rule [7] is applied to (8) as stated in (9).

$$\frac{V + V_{old}}{2} = R\frac{I + I_{old}}{2} + L\frac{I - I_{old}}{\Delta t} + L_m\frac{I_s - I_{sold}}{\Delta t} \tag{9}$$

In (9) Δt is the simulation time step and V_{old}, I_{old} and I_{sold} are the electrical signals from previous time step. Therefore the terminal currents of the transformer at every time step using T model can be calculated using (10)

$$I = GV + GV_{old} - G(R - \frac{2L}{\Delta t})I_{old} - \frac{2L_m}{\Delta t}G(I_s - I_{sold}) \tag{10}$$

where $G = (R + \frac{2L}{\Delta t})^{-1}$.

In the proposed method of saturation modeling, the location of the saturation branch should be maintained at the terminals of the transformer in order to retain the computational efficiency. Therefore the true saturation current is partitioned between the terminal current sources by appropriate ratios as shown in Fig. 8. To do so, the injected currents at both terminals should be accurately computed to be mathematically equivalent to injection at the magnetizing branch so that the new model behaves exactly the same as the transformer T model.

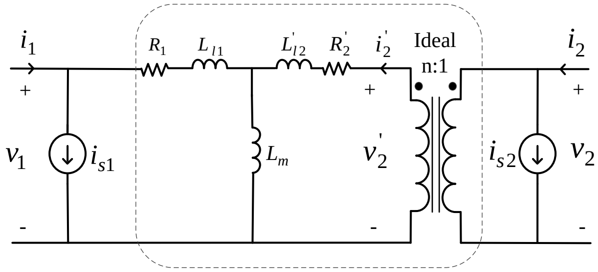


Fig. 8. mathematically equivalent model of transformer electric circuit.

The voltage equation of the new model shown in Fig. 8 is expressed in (11).

$$V = RI + L\frac{d}{dt}I + RI_{s12} + L_m\frac{d}{dt}I_{s12}$$
$$V = \begin{bmatrix} v_1 \\ v_2^{'} \end{bmatrix}, \quad I = \begin{bmatrix} i_1 \\ i_2^{'} \end{bmatrix}, \quad I_{s12} = \begin{bmatrix} i_{s1} \\ i_{s2}^{'} \end{bmatrix} \tag{11}$$
$$R = \begin{bmatrix} R_1 & 0 \\ 0 & R_2^{'} \end{bmatrix}, \qquad L = \begin{bmatrix} L_m + L_{l1} & L_m \\ L_m & L_m + L_{l2}^{'} \end{bmatrix}$$

The terminal currents of this model at every time step are also calculated using trapezoidal rule as follows.

$$I = GV + GV_{old} - G(R - \frac{2L}{\Delta t})I_{old}$$
$$- I_{s12} - G(R - \frac{2L}{\Delta t})I_{s12old} \tag{12}$$

The values I_{s12} can be derived using (10) and (12) as expressed in (13). Here the value I_s is calculated based on magnetizing flux stated in (6).

$$I_{s12} = \frac{2L_m}{\Delta t}GI_s - \frac{2L_m}{\Delta t}GI_{sold} - G(R - \frac{2L}{\Delta t})I_{s12old} \tag{13}$$

The simulation results of primary current and secondary voltage of the same case mentioned in section II, using the new model, along with the previous simulations for the placement of the saturation at either ends is presented in Fig. 9. Comparing with old model simulation results, the new model magnetizing current is lower than the primary winding current injection and higher than the secondary winding current injection. Moreover the voltage distortion of secondary winding is higher than the first case and lower than the second case. This is due to the fact that the effect of leakage flux is substantial during saturation since the magnetizing inductance is significantly low. Therefore injecting the saturation current to the terminals results in significant changes in simulation results, and the terminal voltages cannot be used to calculate the magnetizing flux.

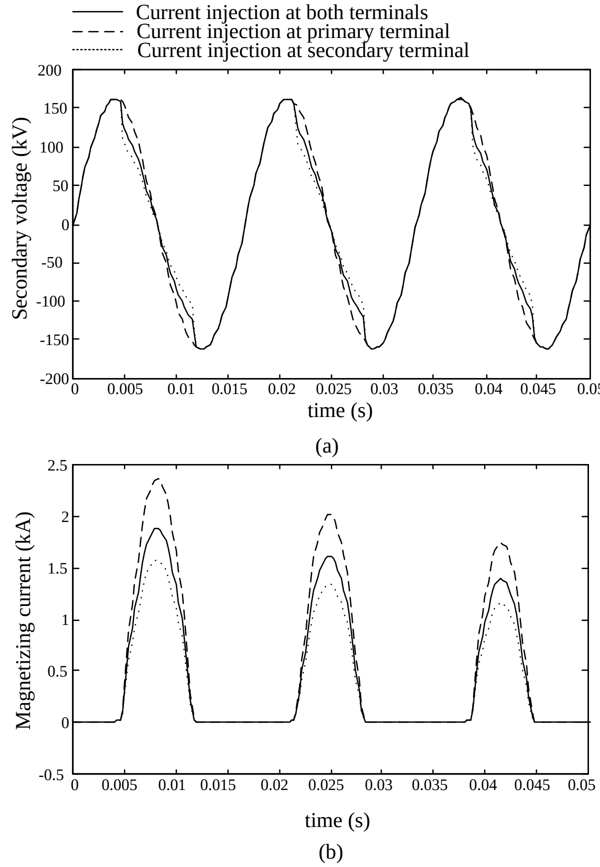


Fig. 9. The simulation results of old and new modeling approaches, (a) secondary voltage, (b) primary current of transformer.

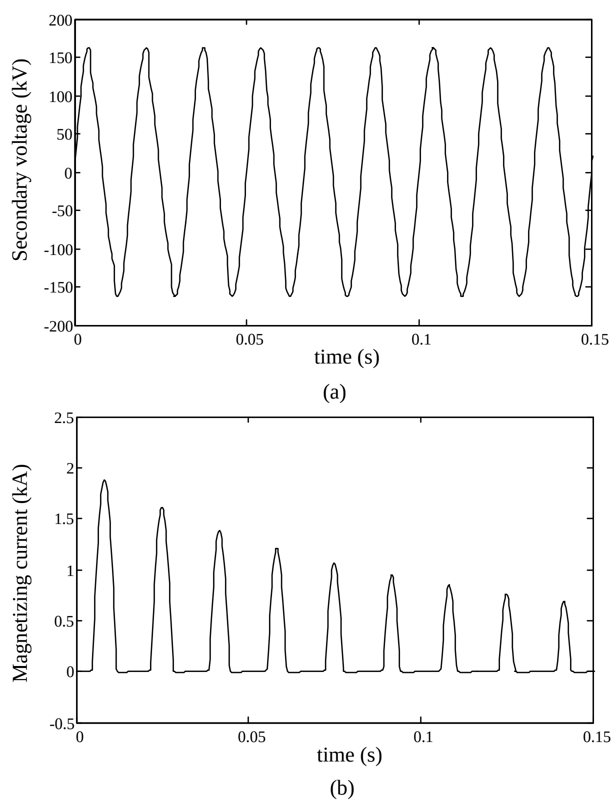


Fig. 10 New model simulation results compared with explicit simulation, (a) secondary voltage, (b) primary current of transformer.

In order to validate this transformer model, the equivalent circuit of Fig. 6 was explicitly simulated using individual lumped elements. The results of both cases are shown in Fig.10 and are exactly the same.

V. Conclusions

This paper proposed an approach based on mathematical equivalent circuits to model the saturation current for transformer models in EMT-type programs. The new model is capable of modeling saturation current precisely considering the effect of leakage flux while the computational efficiency is retained. The old and new modelling approaches were compared by conducting several simulations using PSCAD/EMTDC. Simulation results show that the proposed method leads to higher accuracy.

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VII. References

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