Effects of Subharmonic Frequencies in Phasor Estimation Algorithms for Distance Protection of Series Compensated Transmission Lines

D. S. Moura, F. A. Moreira, K. M. Silva

Abstract-- **The main purpose of this work is the presentation of the behavior of the distance protection in transmission lines with series compensation, evaluating some phasor estimation algorithms under the influence of subharmonic frequencies. The series compensation introduces several problems for the distance protection of transmission lines, such as voltage and current inversions and the presence of subharmonic frequencies Simulations are performed in the ATP program for different compensation degrees and fault scenarios (type and location) and the results obtained are then used as input data for the phasor estimation algorithms that are implemented in Matlab. It is possible then to evaluate the performance of the distance protection when subjected to subharmonic frequencies.**

Keywords: Distance protection, Transmission lines, Series compensation, Phasor estimation, Subharmonic frequency.

I. Introduction

THE distance protection of transmission lines operates according to the measurement of the positive sequence impedance of the transmission line from the point where the relay is installed up to the point where the fault occurs [1].

In transmission lines, the resistance and inductive reactance are responsible for the voltage drops. Also, the inductive reactance is responsible for the power angle of the transmission line, and consequently has influence on the steady-state and transient stability of the system. One alternative to reduce the effect of the inductive reactance in transmission lines is the introduction of series compensation, through the installation of series capacitors. The main advantages of the series compensation in transmission lines are [2]:

- Increase in the active power transmission capacity;
- Improvement in the steady-state and transient stability of the power system.

On the other hand the installation of series capacitors becomes a problem for the protection system, especially for the distance protection. The main problems introduced by the series compensation are [3]:

- Current inversion;
- Voltage inversion;
- Over and underreaching;
- Subharmonic frequencies.

The main purpose of this paper is to show in detail the problems related to the subharmonic frequencies that may occur with some of the phasor estimation algorithms used in distance protection.

II. Theoretical Background

A. Subharmonic Frequency

The installation of series compensation introduces a capacitive reactance in series with the inductive reactance of the transmission line, forming a resonant series circuit, where the resonant frequency f_r is given by

$$f_r = \frac{1}{2\pi\sqrt{LC}} = f\sqrt{\frac{X_C}{X_S+X_{LT}+X_R}} \tag{1}$$

where f is the fundamental frequency, X_C is the capacitive reactance, X_S and X_R are the inductive reactances of the sources in the sending and receiving ends of the line, respectively, and X_{LT} is the inductive reactance of the transmission line.

Since the number in the square root is always smaller than 1, f_r will be a subharmonic frequency of the system. The subharmonic transients may last for several cycles until the desenergization of the transmission line [4].

The subharmonic frequencies, also called subsynchronous frequencies are always below the fundamental frequency and usually vary from 25 to 30 Hz [3]. These frequencies are not eliminated in the process of phasor estimation, since they are not precisely known due to the variation of the compensation degree of the lines. Also, the algorithms traditionally used in the phasor estimation process are FIR (finite impulse response) filters, whose frequency responses are tuned to provide zero gain only for harmonic frequencies and therefore present lateral lobes for subharmonic and interharmonic frequencies.

B. Frequency Response

Fig. 1 illustrates the frequency responses of the $\boldsymbol{h_C}$ (cosine)

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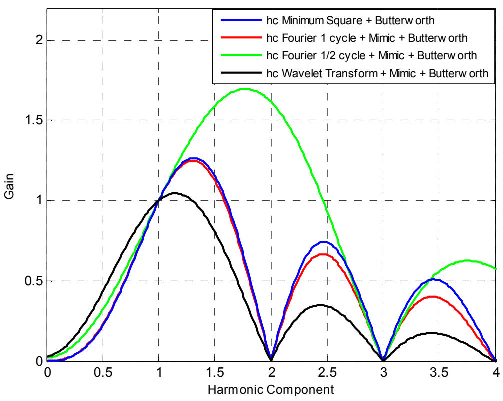
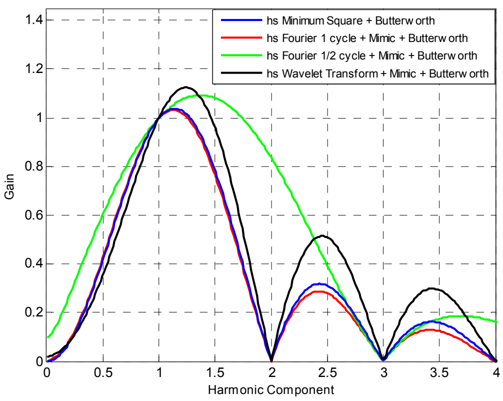
and $\boldsymbol{h_S}$ (sine) filters for each of the algorithms analyzed [5]. It is possible to observe that all algorithms are subjected to the subharmonic frequencies. The function of the mimic filter [6] is the reduction of the DC component and the Butterworth filter is the anti-aliasing filter. The one cycle Fourier and the minimum squares algorithms are the most immune to the subharmonic frequencies, following similar characteristics until the first harmonic component in the $\boldsymbol{h_C}$ filters and with a good performance in the $\boldsymbol{h_S}$ filters. The $\boldsymbol{h_C}$ half-cycle Fourier filter has a frequency response similar to the $\boldsymbol{h_C}$ filter based on the Wavelet transform [7] up to the fundamental frequency. On the other hand, concerning the $\boldsymbol{h_S}$ filters, the half-cycle Fourier filter is the most affected by the subharmonic frequencies. Therefore, the half-cycle Fourier filter is the most affected by the subharmonic frequencies. This characteristic is intrinsically connected to its smaller data window when compared to the other algorithms.



(a)



(b)

Fig. 1. Frequency response of the filters: (a) $\boldsymbol{h_C}$ and (b) $\boldsymbol{h_S}$ for the one and half-cycle Fourier, least squares, and the one based on the Wavelet transform.

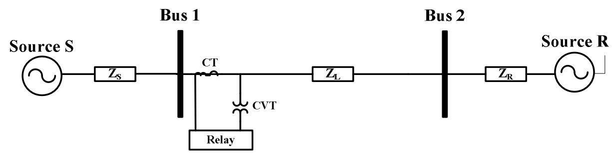
The immunity of the filters to the subharmonic frequencies is decreased with the shift of these frequencies in direction to the fundamental frequency. This leads to the conclusion that the larger the subharmonic frequencies, the bigger the effects caused by them.

III. Simulations with the ATP (Alternative Transients Program)

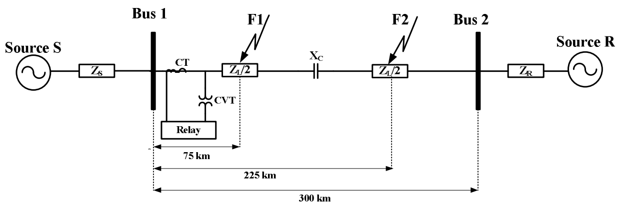
Regarding the simulations, all the databank was created in the ATP. A simplified power system was simulated for different fault conditions and different compensation degrees. A time step of 10.416 µs, corresponding to a sampling rate of 1600 samples per cycle, was used. The signals of interest then pass through an anti-aliasing filter composed by an analogue low pass third order Butterworth filter with a cut-off frequency of 187.9 Hz. Through a process of sample discard, the signals are then converted to a sample rate of 16 samples per cycle, quantity normally used in distance relays. The conversion of the sampling rate is performed with the subroutine MODELS that allow the execution of the program with a sampling rate different from the one used in the ATP. The phasor estimation algorithms were implemented in MATLAB where their performances were also verified.

A. Base Case

The simulations were performed for a base system of 230 kV with a line length of 300 km. The faults were basically applied in two different locations: at a distance of 75 km and 225 km from bus 1, with distinct compensation degrees and with series capacitors installed in the middle and half at each line end. Different types of faults were also considered. Fig. 2 (a), (b), and (c), show the simulated systems: without series compensation, with series compensation in the middle of the line and with series compensation located half at each line end, respectively.



(a)



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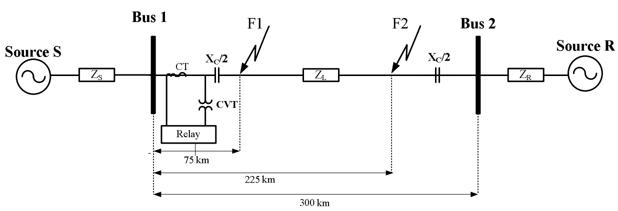

(c)

Fig. 2. Base model of the electric system used in the simulation: (a) no series compensation, (b) compensation with series capacitors installed in the middle of the line (c) compensation with series capacitors installed in the sending and receiving ends of the line.

The input signals for the model of the distance relay are the voltages and currents measured by the CVT and the CT installed at bus 1, respectively. The values of the parameters shown in Fig. 1 are depicted in Table I.

TABLE I
PARAMETERS OF THE SIMULATED ELECTRICAL SYSTEM

Line	Source S	Source R
300 km	$\hat{V}_S = 1{,}02\angle 0°$ pu	$\hat{V}_R = 0{,}98\angle -10°$ pu
$Z_{L,0} = (0{,}532 + j1{,}541)\ \Omega/km$	$Z_{S,1} = (1{,}014 + j18{,}754)\ \Omega$	$Z_{R,1} = (1{,}127 + j20{,}838)\ \Omega$
$Y_{L,0} = j2{,}293\ S/km$	$Z_{S,0} = (0{,}871 + j25{,}661)\ \Omega$	$Z_{R,0} = (0{,}968 + j28{,}513)\ \Omega$
$Z_{L,1} = (0{,}098 + j0{,}510)\ \Omega/km$		
$Y_{L,1} = j3{,}252\ S/km$		

B. Transmission Line without Series Compensation

Fig. 3 illustrates the trajectory in the R-X plane of the apparent impedance of the Z_{AG} unit of the distance relay for a single-phase to ground fault in phase A when there is no series compensation in the line. The fault occurs at a distance of 75 km from bus 1. It is possible to observe that all algorithms correctly identify the fault in their first zone and the convergence is smooth. This case will serve as comparison to the cases when series compensation is considered.

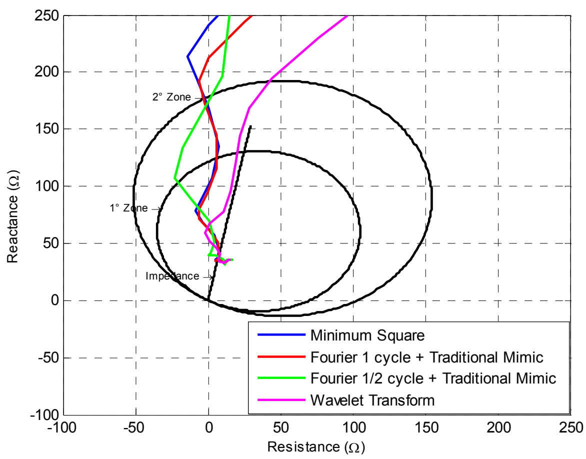

Fig.3. Trajectory in the R-X plane of the apparent impedance of the Z_{AG} unit of the distance relay for a single-phase to ground fault in phase A located at 75 km from bus 1 for the line without series compensation.

C. Series Compensation with Capacitors Installed in the Middle of the Line

When the series capacitors are installed in the middle of the line, the effects of the subharmonic frequencies are more pronounced when the faults occur in the second half of the line. Fig. 4 shows the frequency spectrum for the current signal in phase A for a single-phase to ground fault in phase A. The fault is applied at distances of 75 km (first half) and 225 km (second half) from bus 1. The compensation degree considered was 40%.

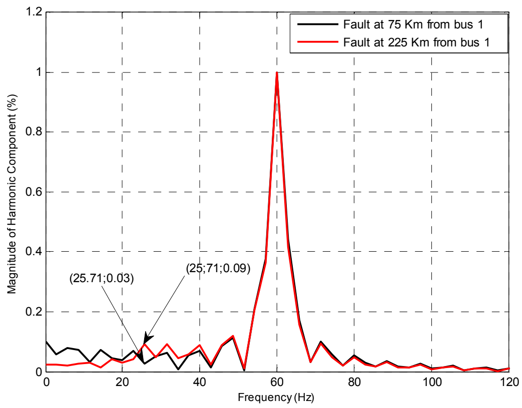

Fig. 4. Frequency spectrum of the current in phase A for a single-phase to ground fault in phase A at 75/225 km from bus 1 with series capacitors in the middle of the line and a compensation degree of 40%.

It is possible to observe that for a frequency of 25.71 Hz, the magnitude of the current signal corresponds to 9% of the magnitude at the fundamental frequency in the case of the fault applied at a distance of 225 km from bus 1. This is the maximum magnitude in the range of frequency from 25 to 30 Hz, which as previously mentioned, is usually a critical range for the presence of high subharmonic frequency components [3]. On the other hand, in case the fault is applied at a distance of 75 km from bus 1, the magnitude of the current signal decreases to 3% of the magnitude of the fundamental frequency.

Fig. 5 presents the trajectory in the R-X plane of the apparent impedance for the situation of the single-phase to ground fault in phase A applied at a distance of 225 km from bus 1. In this situation the algorithms present a swirling trajectory which is due to the subharmonic frequencies.

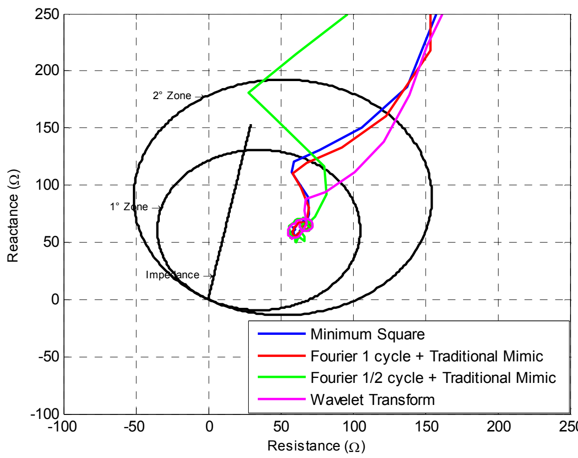

Fig. 5. Trajectory in the R-X plane of the apparent impedance of the Z_{AG} unit of the distance relay for a single-phase to ground fault in phase A located at 225 km from bus 1 with series compensation in the middle of the line and a compensation degree of 40%.

When faults are located in the first half of the line and the series capacitors are installed in the middle of the line, there is no significant influence of subharmonic frequencies since the series capacitors are beyond the location of the fault.

The increase in the compensation degree shifts the largest subharmonic frequency magnitudes towards the fundamental frequency, making the phasor estimation algorithms even more sensitive to the subharmonic frequencies as previously observed in the frequency response of the filters. Fig. 6 shows this situation.

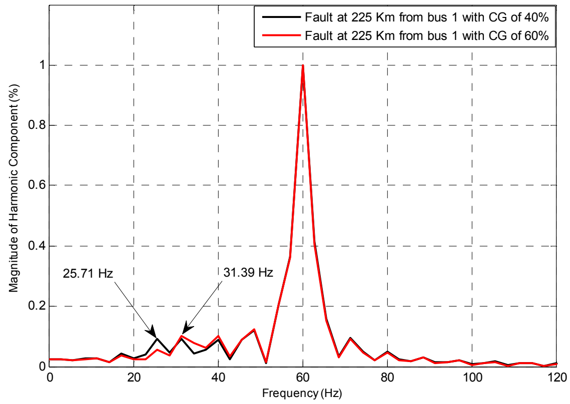

Fig. 6. Frequency spectrum of the current in phase A for a single-phase to ground fault in phase A at 225 km from bus 1 with series capacitors in the middle of the line and compensation degrees of 40% and 60%.

Fig. 7 presents the trajectory of the apparent impedance of the Z_{AG} unit of the distance relay again for a single-phase to ground fault in phase A, located at a distance of 225 km from bus 1, but now for a compensation degree of 60%. Comparing with Fig. 5, it is possible to observe that the radius of the swirling trajectory increased. This behavior is due to the shifting of the subharmonic frequencies towards the fundamental frequency.

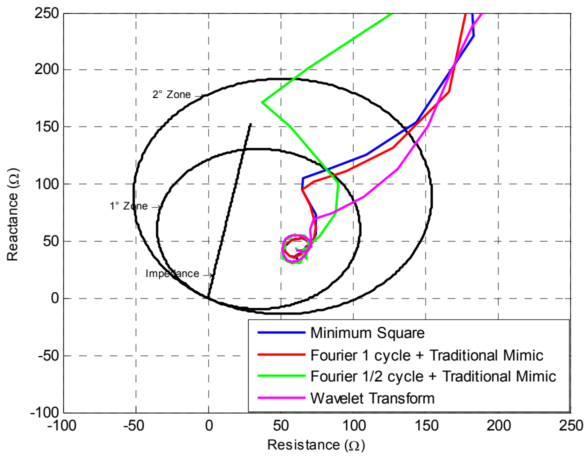

Fig. 7. Trajectory of the apparent impedance of the Z_{AG} unit of the distance relay for a single-phase to ground fault in phase A at 225 km from bus 1 with capacitors in the middle of the line and a compensation degree of 60%.

D. Series Compensation with Capacitors Installed in the Sending and Receiving Ends of the Line

When the series compensation is performed through the installation of capacitors in both ends of the transmission line, the effects on the phasor estimation algorithms are more severe than in the case when the capacitors are located in the middle of the line, independently of the fault location.

Fig. 8 shows that the maximum magnitude in the frequency spectrum of the subharmonic frequencies reaches 36% of the value at the fundamental frequency, a much larger value than when the compensation is performed in the middle of the line. A double-phase to ground fault, involving phases A and B is considered in this case. The reason why the type of fault has been changed from a single-phase to ground to a double-phase to ground fault is just to show that the effects of the subharmonic frequencies are not restricted to a particular type of fault. The fault is applied at a distance of 75 km from bus 1. In this figure it is also possible to verify that the increase in the compensation degree shifts the subharmonic frequency with maximum current magnitude towards the fundamental frequency, as already observed in Fig. 6.

Fig. 9 presents the trajectory in the R-X plane of the apparent impedance of the Z_{BG} unit of the distance relay for a double-phase to ground fault involving phases A and B at a distance of 75 from bus 1 and a compensation degree of 60%. In this case the radius of the swirling trajectory increased significantly and it takes a very long time for the stabilization of the estimated impedance seen by the relay.

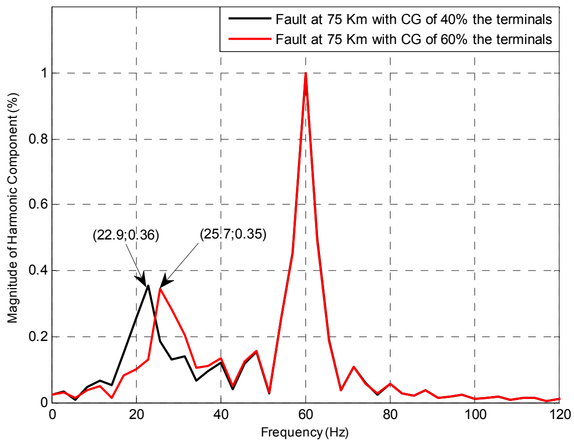

Fig. 8. Frequency spectrum of the current in phase A for a double-phase to ground fault in phases A and B, located at 75 km from bus 1 with series capacitors installed at both terminals of the line and compensation degrees of 40% and 60%.

E. Comparison of the Lines with Compensation in the Middle and at the Terminals of the Lines

Comparing the effects of a double-phase to ground fault in a transmission line with series capacitors installed in the middle and at both terminals of the line for a fault located at 75 km from bus 1 and a compensation degree of 40%, it may be verified that the convergence of the estimated current phasor is much faster when the series compensation is located in the middle of the line. This behavior is illustrated in Fig. 10

In Fig. 10 (b), it may be observed that the estimated current phasor magnitude oscillates for a long time when capacitors are installed in both ends of the line. The amplitudes of these oscillations are higher for the half-cycle Fourier algorithm and for the algorithm based on the Wavelet transform. The reason for these oscillations is the presence of the subharmonic frequencies that provokes the swirling trajectory as seen in

Fig. 9. When the fault is in the first half of the line and the series capacitors are located in the middle of the line, then the convergence of the phasor estimation algorithms is much faster since the capacitors are beyond the fault location.

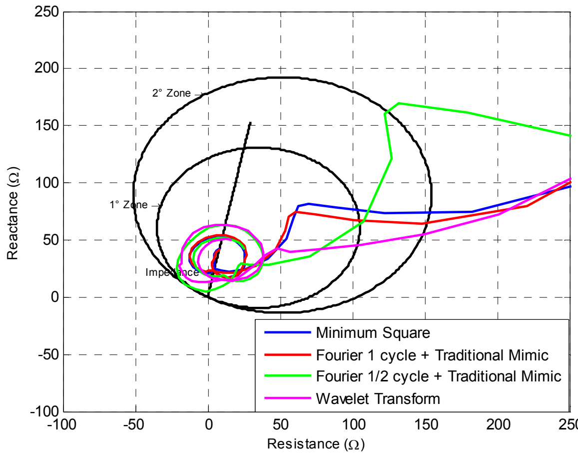

Fig. 9. Trajectory of the apparent impedance of the Z_{BG} unit of the distance relay for a double-phase to ground fault in phases A and B at 75 km from bus 1 with series capacitors installed in both terminals of the line and a compensation degree of 60%.

The reason why in this section the phase B current and the apparent impedance of the Z_{BG} unit have been shown, whereas, in the previous section, the phase A current has been shown together with the apparent impedance of the Z_{AG} unit is because any of the ground units of the phases involved in the fault should detect a double-phase to ground fault.

In transmission lines with series compensation in both terminals, the DC component in the current estimation is considerably reduced when compared to the situation when the series compensation is installed in the middle of the line for a fault in the first half of the line. However, the effect of subharmonic frequencies is much more pronounced. Fig. 11 shows this situation by comparing the frequency spectrum of the current in phase B when the series capacitors are installed in the middle and at both ends of the line. The fault considered in this situation is a double-phase to ground fault involving phases A and B at a distance of 75 km from bus 1 (first half of the line) and a compensation degree of 40%.

Even though the DC component is high for the situation when the series capacitors are installed in the middle of the line and the fault occurs at the first half of the line, this is not a problem, since the DC component is virtually eliminated by the mimic filter implemented in the phasor estimation algorithms. However, the subharmonic frequencies are not eliminated and its effect becomes very relevant for the phasor estimation of transmission lines with series compensation.

Fig. 12 shows a comparison between the frequency spectrum of the current in phase B when the series compensation is installed in the middle and at the terminals of the line. The fault considered is again a double-phase to ground fault involving phases A and B at 225 km from bus 1 (second half of the line) and a compensation degree of 40%.

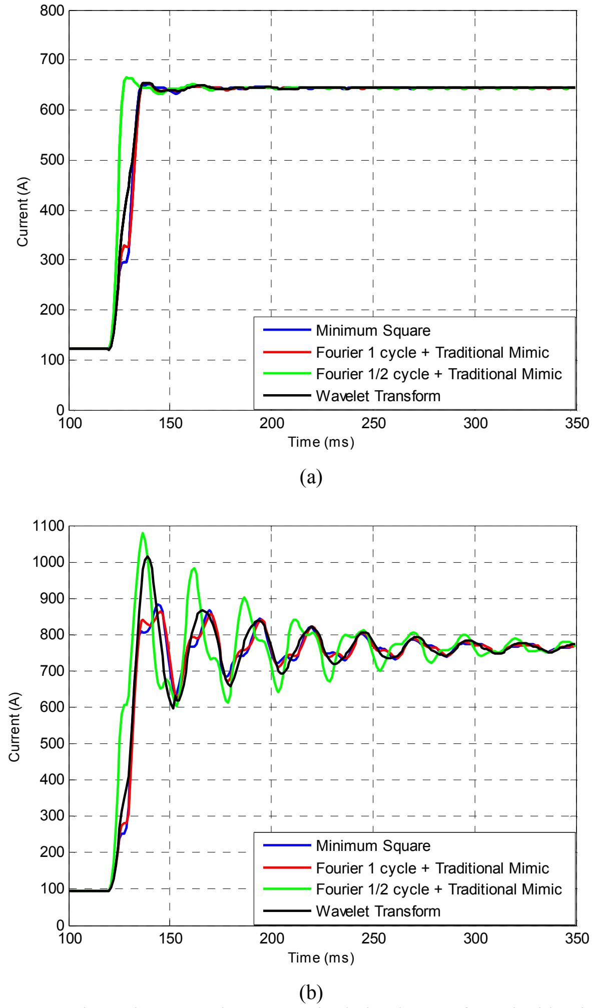

Fig. 10. Estimated current phasor magnitude in phase B for a double-phase to ground fault, occurring at t = 118ms, in phases A and B at 75 km from bus 1 and a compensation degree of 40%. (a) Capacitors installed in the middle of the line. (b) Capacitors installed at both terminals of the line.

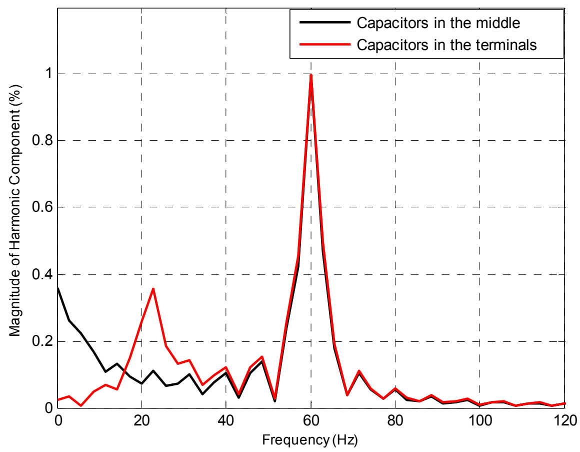

Fig. 11. Frequency spectrum of the current in phase B for a double-phase to ground fault in phases A and B at 75 km from bus 1 with capacitors installed in the middle and at both terminals of the line and a compensation degree of 40%.

For faults occuring in the second half of the line there will always be series capacitors between the relay and the fault location, independently of where the capacitors are installed. This reduces the DC component but increases the amplitude of the subharmonic frequency components, as can be observed in Fig. 12. It can also be verified that the amplitude of the subharmonic frequency components are higher in the situation when the series compensation is installed in the middle of the line, since in this case the whole compensation is located between the relay and the fault location while when the series compensation is split between both terminal of the line, only half of the total series compensation is located between the relay and the fault location. This can also be observed in Fig. 13 that shows the trajectory of the apparent impedance in this situation. The radius of the swirling trajectory is larger in Fig. 13 (a) (capacitors in the middle of the line) than in Fig. 13 (b) (capacitors in both terminals of the line).

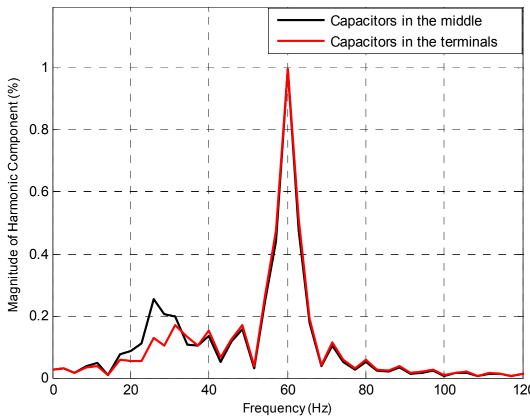

Fig. 12. Frequency spectrum of the current in phase B for a double-phase to ground fault in phases A and B at 225 km from bus 1 with series capacitors in the middle and at both ends of the line and a compensation degree of 40%.

IV. Conclusions

It has been verified that the effect of the subharmonic frequencies is more critical when the series compensation is performed at the transmission line terminals, since in this situation the relay will always see a series capacitance between its terminal and the fault location. The trajectory in the R-X plane will present a swirling characteristic and depending on the radius of this trajectory, the relay may even overreach its protection zone.

Another important observation is that the increase in the magnitude of the subharmonic frequency components is directly related to the increase of the compensation degree and also with the distance of the fault location to the distance relay. The increase in the compensation degree also shifts the subharmonic frequencies with maximum amplitude in the frequency spectrum towards the fundamental frequency.

The frequency response analysis shows that all algorithms analyzed are subject to the effects provoked by the subharmonic frequencies since they are adjusted to have zero gain only for harmonic frequencies.

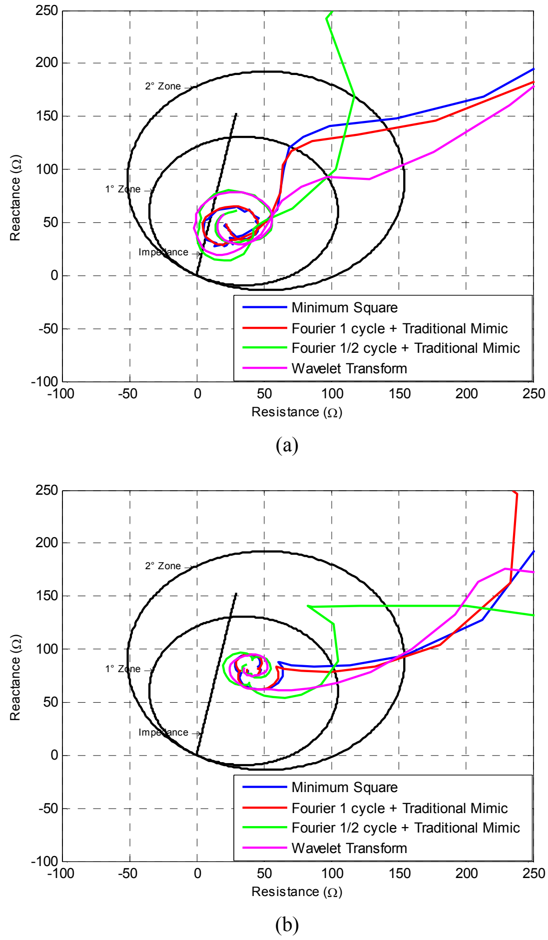

Fig. 13. Trajectory of the apparent impedance of the Z_{AB} unit of the distance relay for a double-phase to ground fault in phases A and B at 225 km from bus 1 and a compensation degree of 40%. (a) Capacitors installed in the middle of the line. (b) Capacitors installed in both ends of the line.

V. Acknowledgement

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VI. References

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