Accurate time domain simulation of frequency dependent transmission line models for large time steps

H.M.J.S.P. De Silva, K.K.M.A. Kariyawasam, A. M. Gole and J.E. Nordstrom

***Abstract*-- A method to accurately simulate frequency dependent transmission line models for large simulation time steps is introduced in this paper. This method is described with a phase domain model called Universal Line Model; however in general the method can be applied to any frequency dependent transmission line model in emtp-type programs. The proposed model is validated by comparing the time domain simulations with an analytical solution obtained using the inverse Laplace transform method for simple linear terminations.**

***Keywords*: large time step, Transmission line models, recursive convolution.**

I. INTRODUCTION

Time domain simulations involving frequency dependent transmission line models are widely used in power system transient studies. One limitation in traditional travelling wave based transmission line models is that the time step of the simulation should be less than the travel time of the lines. In a typical electromagnetic transient (emt) study using emtp-type software, the minimum simulation time step is limited by the shortest transmission line (or more precisely the transmission line having shortest travel time).

Reference paper [1] discusses a new transmission line model to simulate transients with a large time step. The model was developed using a linear interpolation method on Bergeron type model. One limitation of such model is that it does not consider the frequency dependent behaviour of the transmission lines (accurate only at the specified frequency). In contrast, frequency dependent transmission line models accurately consider both the frequency dependency of the line parameters due to skin effect as well as distributed nature of the line and hence these models are widely used in many electromagnetic transient studies [7], [8].

In transmission line modelling, the transmission line equations in terms of propagation function and characteristic admittance are first formulated in frequency domain. Next using techniques such as vector-fitting, above functions are approximated (curve-fitted) using rational functions [2],[6]. Finally, the recursive convolution algorithm is used as a means to convert frequency domain equations with curve-fitted functions into a time domain equivalent circuit, which can be readily implemented in emtp-type algorithms [3]. The recursive convolution algorithm associated with the propagation function contains a travel time term. However, this approach requires that time step is smaller than the travel time of the transmission line. The typical time steps can be $1/10^{th}$ or $1/5^{th}$ of the travel time.

This paper proposes and discusses a new method in which the recursive convolution algorithm associated with the propagation function is modified so that the convolution gives accurate results even when the time step is larger than the propagation delay. The method is based on modifying the coefficients of convolution algorithm to approximate the effect of the small time step and also by applying linear interpolation technique. The validity of the proposed method is demonstrated using an example involving a short single-conductor transmission line model. This method gives fast, accurate and numerically efficient time domain simulations for electrical networks having very short transmission lines modelled with frequency dependent characteristics.

II. LARGE TIME STEP MODELLING

The convolution associated with the propagation function and arbitrary function $q(t)$ can be written in discrete time as [3],

$$s(t) = k_1 q(t-\tau) + k_2 q(t-\Delta t-\tau) + k_3 s(t-\Delta t) \qquad (1)$$

where, $s(t)$ is the value of the convolution at time t and k_1, k_2 and k_3 are constants. τ is the travel time (see Appendix B for further details).

Equation (1) is evaluated by selecting a small time step compared to travel time (Δt, $\Delta t < \tau$). Also the ($N+1$, $N = \tau/\Delta t$) history values of $q(t)$ are stored in memory to calculate the convolution at current time step.

For a large time step ($\tau < dt$), equation (1) can be modified based on linear interpolation as shown below. The linear variation of the function $q(t)$ is assumed between time steps as

H.M.J.S.P. De Silva (jeewantha@hvdc.ca) and John Nordstrom (johnn@hvdc.ca) are with the Manitoba HVDC Research Centre, a division of Manitoba Hydro International Ltd. Canada.

K. K. M. A. Kariyawasam (umkapuge@cc.umanitoba.ca) and A. M. Gole (ani_gole@umanitoba.ca) are with the Dept. of Electrical and Computer Engineering, University of Manitoba, Canada.

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shown in figure 1.

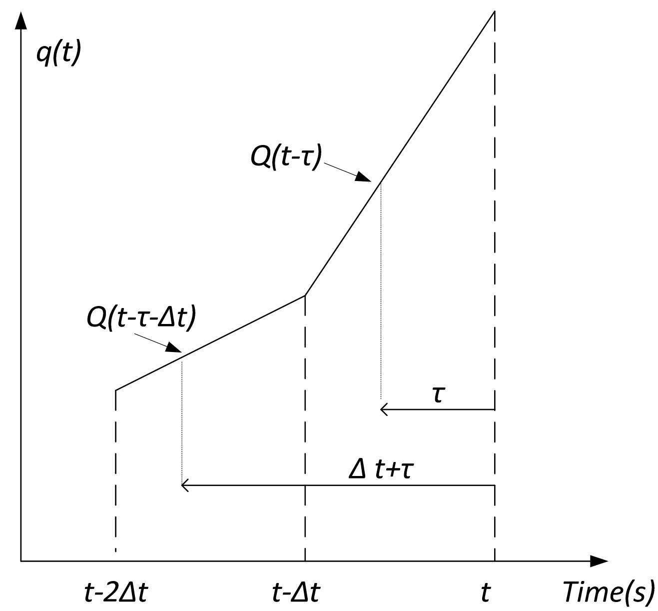

Fig. 1. Linear interpolation between time steps (*t*), *(t- Δt)* and *(t-2Δt)*

Using linear interpolation, the function *q(t)* can be written as,

$$q(t-\tau)=\alpha_1 q(t)+\alpha_2 q(t-\Delta t) \qquad (2)$$
$$q(t-\tau-\Delta t)=\alpha_1 q(t-\Delta t)+\alpha_2 q(t-2\Delta t)$$

By substituting in (1), the convolution becomes

$$s(t)=\beta_0 q(t)+\beta_1 q(t-\Delta t)+\beta_2 q(t-2\Delta t)+\beta_3 s(t-\Delta t) \qquad (3)$$

where,

$$\alpha_1=\frac{\Delta t-\tau}{\Delta t}$$
$$\alpha_2=\frac{\tau}{\Delta t}$$
$$\beta_0=k_1\alpha_1$$
$$\beta_1=k_1\alpha_2+k_2\alpha_1$$
$$\beta_2=k_2\alpha_2$$
$$\beta_3=k_3\alpha_1$$

In contrast to (1), it can be seen that only two history values of function (*q(t)*) are required to be stored in memory to use at the current time (t); hence, the computer memory requirement is less compared to simulating a short transmission line with small time step (*Δt, Δt< τ).* Another difference is that the modified convolution is not merely a history term. It uses the current value of *q(t)*.

Figure 2 represents a short single-phase transmission line where *v* and *i* are time domain voltages and currents at the sending and receiving-end terminals respectively. Subscripts '*k*' and '*m*' respectively denote the sending and receiving-ends of the line.

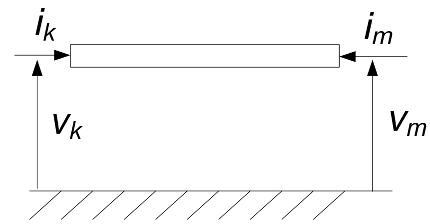

Fig. 2. A short transmission line showing receiving-end and sending-end voltages and currents.

By applying the proposed modified convolution technique, the transmission line equations in time domain becomes (see Appendix A for more details).

$$\begin{bmatrix} i_k(t) \\ i_m(t) \end{bmatrix}=\begin{bmatrix} \gamma_1 & \gamma_2 \\ \gamma_2 & \gamma_1 \end{bmatrix}\begin{bmatrix} v_k(t) \\ v_m(t) \end{bmatrix}+\begin{bmatrix} i_{hist1} \\ i_{hist2} \end{bmatrix} \qquad (4)$$

where, γ_1 and γ_2 are constants and i_{hist1} and i_{hist2} are derived based on history voltages and currents only for last two time steps (at $t = t\text{-}\Delta t$ and $t = t\text{-}2\Delta t$). In the traditional line model, i_k (t) is only a function of v_k *(t)* and v_m(t) affects only through the history terms. However, once the interpolation is carried out, even the present value of v_m(t) affects i_k(t) i.e., γ_2 is non-zero. Hence it can be seen that the voltages and current at either end of the line are coupled in this formulation.

III. Simulation Examples

Time domain simulations involving short transmission lines with large time step were conducted to verify the validity of the proposed method.

A. Short circuit test

A short circuit test is performed in order to validate the modified transmission line model for large time steps. The simulation is compared with the results from Numerical Inverse Laplace Transform method (NILT). The 3 km cable was energized with a step voltage (1 kV) and the receiving-end is connected to ground through a very small resistance. The cable data is shown in the table 1.

TABLE I
CABLE DATA

Parameter	Units	Value
Inner radius of the conductor	m	0.022
Resistivity of the conductor	Ωm	1.68e-8
Relative permittivity of insulation		4.1
Outer radius of the insulation	m	0.0395
Sheath outer radius	m	0.044
Resistivity of the sheath	Ωm	2.2e-7
Outer radius of outer insulator	m	0.0475
Relative permittivity of outer insulator		2.3

Figure 3 shows the sending-end current for different simulation time steps ($\Delta t = \tau$, 5τ, 10τ and 20τ) and also for the reference solution (NILT) for the time frame [0 to 2s]. The travel time (τ) for the cable is 20.258μs. The simulation results for large time step are in a close agreement with the reference. Figure 4 plots the error as compared to the reference solution. Figures 5 and 6 show the same simulation results for the time frame [0 to 0.01s]. As the simulation time step increases, the error increases due to approximations in the linear interpolation algorithm.

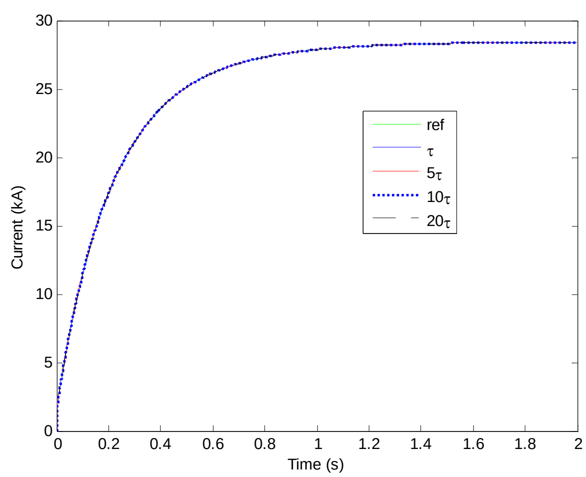

Fig. 3. Sending-end current vs. time for t = [0 to 2s]

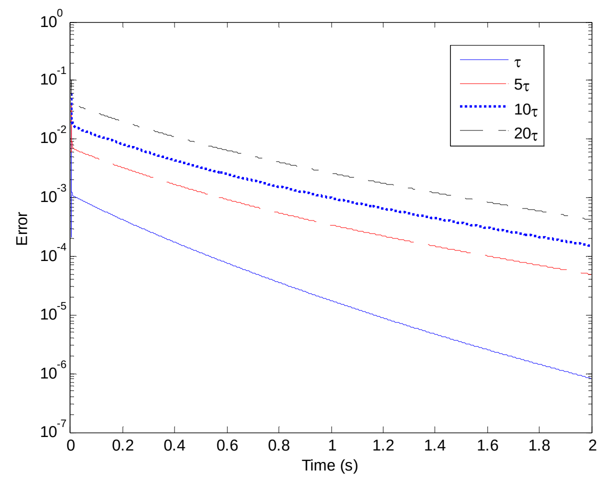

Fig. 4. Log of absolute error vs. time for t = [0 to 2s]

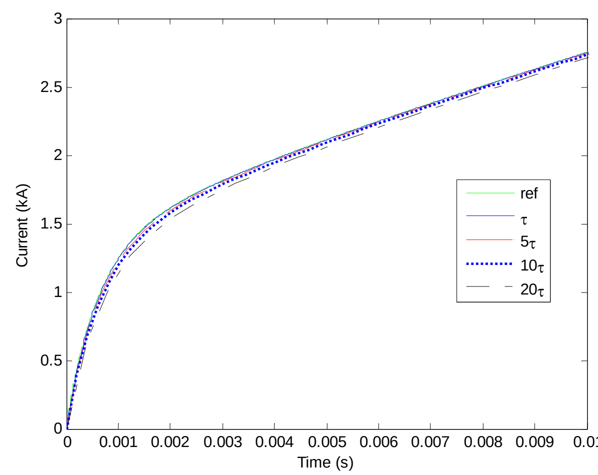

Fig. 5. Sending-end current vs. time for t = [0 to 0.01s]

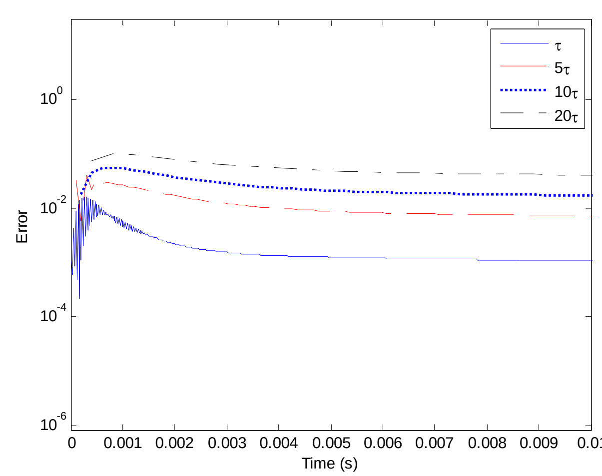

Fig. 6. Log of absolute error vs. time for t = [0 to 0.01s]

B. Comparison with alternate approaches (π- model)

For the time domain simulation studies, involving very short and relatively long transmission lines, the short transmission line is sometimes modelled using a π-model. This allows time-steps larger than the travel time of the small line to be used. This section compares the performance of the proposed short transmission line model with that of π-model. Note that the π-model is accurate only at a specified frequency.

Figure 7 shows the test circuit used to compare the two transmission line models (proposed method and π-model). Two transmission lines (a short cable and a relatively long overhead line) are connected in series and energized with a dc voltage (1 kV) with an initial 0.2 ms ramp-up time. The cable data is the same as that of the cable used in section (III.A). The lengths of the cable and the overhead transmission line are 1 km and 30 km respectively with corresponding travel times 6.737 μs and 100.226 μs.

The line is first modeled using the conventional approach on the PSCAD/EMTDC using a very small time step ($\triangle t = \tau/10$, where τ = 6.737 μs). These results are considered as the accurate template for comparing the various higher-timestep models (reference simulation).

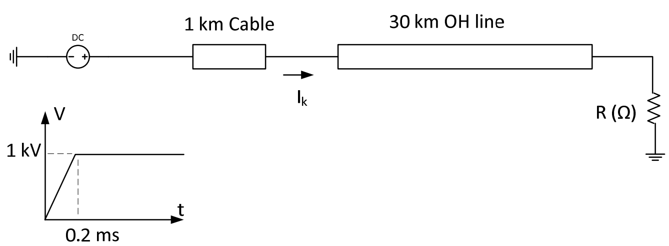

Fig.7. example circuit

In the first test, the receiving-end of the 30 km line is kept open. Three different sets of simulation are conducted:

i. using π-model for the short line, and the conventional approach ($\triangle t < \tau$, where $\triangle t$ = 0.6737 μs) for the long line
ii. using proposed line model with time step ($\triangle t$) = τ, where tau is travel time of the short line
iii. using proposed line model with time step ($\triangle t = 5\tau$).

Figure 8 shows the current (I_k) measured between two transmission lines for the time period [0, 5ms]. In this case, the equivalent π-model is created for the specified frequency of 0.01 Hz (as the excitation is almost dc) so that the steady state response is accurate. It is clearly seen that the proposed short line model is in close agreement with the reference for time steps $\triangle t = \tau$ and even for $\triangle t = 5\tau$. However a noticeable difference can be seen when using the π-model due to the fact that the pi-model is accurate only at very low frequency and gives some error at higher frequencies.

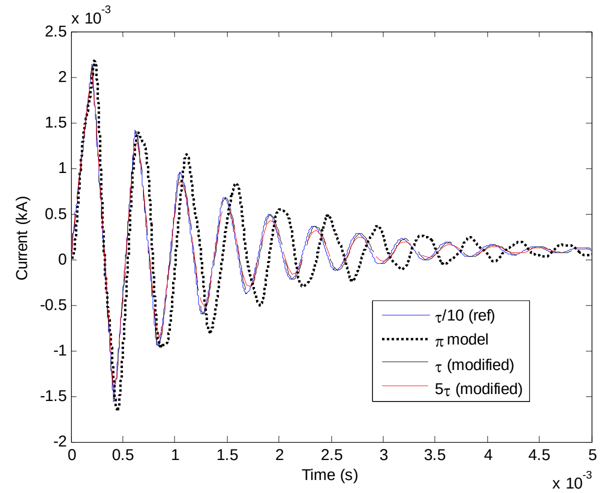

Fig.8. Sending-end current vs. time for open circuit study

In the second test, the equivalent π-model is derived for a higher frequency (2.3 kHz – of the order of the observed frequency in the reference simulation). Figure 9 shows the corresponding results for the open circuit test. The figure shows the reference simulation, the pi model (2.3 kHz fit), and the results of the proposed model with the two different time steps. In this case, the π-model is also in a close agreement with the reference.

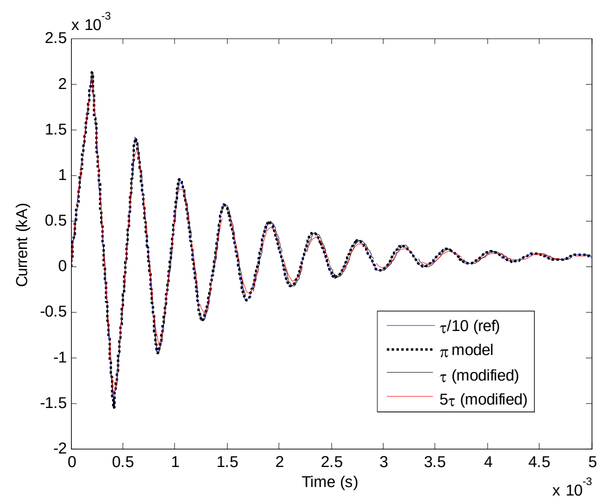

Fig.9. Sending-end current vs. time with different π-model for open circuit study

However, when a remote line to ground fault is applied at the overhead line's receiving end, there can be errors. Figure 10 shows the current (I_k) for the time period from 0 to 0.8s. The current in case of equivalent π-model does reach the correct steady state value. This is not surprising because evaluating the π-section for 2.3 kHz results in an inaccurate model at dc frequency. Nevertheless, the proposed model with large time-steps as large as 5τ accurately reproduces the reference waveform. Note however, that the $\triangle t$ must be small enough (i.e., smaller than that dictated by Nyquist–Shannon's sampling theorem [9]) to handle the maximum frequency that is applied to the circuit.

As a summary, the proposed model gives accurate results for a wide frequency range. However the equivalent π-model is only accurate at specified frequency. For an example, if the specified frequency of π-model is selected to obtain a correct steady state values, then transient response may not be accurate and vice versa.

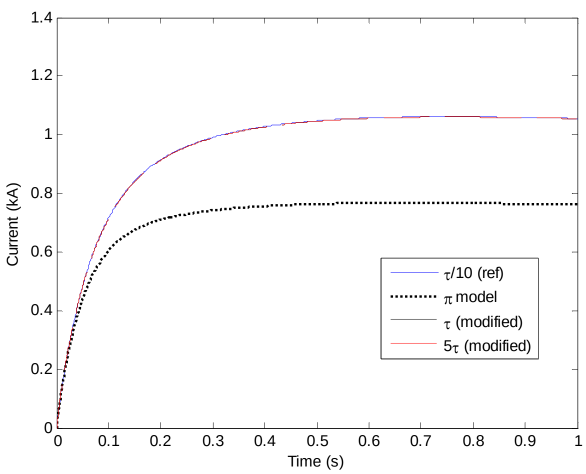

Fig.10. Sending-end current vs. time for short circuit study

IV. CONCLUSIONS

The proposed transmission line model can be used to approximately simulate a short transmission line using a time-step much larger than the travel time. The time step however, should still be small enough so that a sufficient number of samples of the applied excitation waveforms are obtained to provide an accurate representation of the signal (Nyquist–Shannon's Theorem). The traditional model requires that the time simulation step is less than the travel time of the transmission line, a limiting factor in a typical emt study with short transmission lines.

Compared to the traditional frequency dependent transmission line model with small time step, the proposed model uses significantly larger time-steps and fewer steps in the convolution (only information from the last two (large) time-steps is required).

It should be noted that simulating transmission lines with very small simulation time step (*Δt<< τ)* always gives accurate results, but results in significant computational effort. However, the proposed model gives sufficiently accurate results with significantly less computation effort. It is recommended for simulating an electrical network having a mixture of long transmission lines and as well as very short transmission. The proposed line model includes frequency dependent effects and so has a wide frequency range of accuracy. Depending on the study, the emt user may carefully decide which short transmission lines are to be represented by the proposed model, resulting in minimum impact on the simulation result.

Application of this method to more general multi-conductor frequency dependent transmission line models is currently under investigation and will be presented in a future publication.

V. APPENDIX A

A. Transmission line equations

This section briefly describes the modeling of frequency dependent transmission line modelling for a single conductor lines. The frequency domain solution of the traveling wave equation can be expressed as [4],

$$I_k = Y_c V_k - A(Y_c V_m + I_m)$$
$$I_m = Y_c V_m - A(Y_c V_k + I_k) \qquad \text{(A.1)}$$

In the above equations, V and I voltage and currents and subscripts 'k' and 'm' denote sending-end and receiving-end of the line. Y_c and A are the characteristic admittance and the propagation functions respectively. In order to implement the model in the time domain, Y_c and A are approximated with rational functions of suitable orders M and N [5] in the form shown below in (A.2) and (A.3).

$$A(s) = \sum_{p=1}^{N} \frac{c_p e^{-s\tau}}{s - a_p} \qquad \text{(A.2)}$$

$$Yc(s) = \sum_{q=1}^{N} \frac{c_q}{s - a_q} + d \qquad \text{(A.3)}$$

The unknown coefficients, c_p, a_p a_q, c_q and d are calculated using a technique called Vector Fitting [2]. Note that the time delay (τ) in equation (A.2) is estimated before the fitting procedure.

B. Modified equations for large time steps

The product of two frequency domain functions in (A.1) can be written as a convolution in time domain.

$$i_k(t) = y_c(t) \times v_k(t) - a(t) \times f_m(t) \qquad \text{(A.4)}$$

where,

$$f_m(t) = y_c(t) \times v_m(t) + i_m(t)$$

The lower case letters represent the corresponding time domain form of the upper case variables. Using recursive convolution technique [3], $y_c(t) \times v_m(t)$ and $y_c(t) \times v_k(t)$ can be written as

$$y_c(t) \times v_m(t) = y_{eq} v_m(t) + i_{hm} \qquad \text{(A.5)}$$
$$y_c(t) \times v_k(t) = y_{eq} v_k(t) + i_{hk}$$

then,

$$f_m(t) = y_{eq} v_m(t) + i_m(t) + i_{hm}$$

The y_{eq} is a constant and i_{hm} and i_{hk} are history current terms. The convolution $a(t)\times f_m(t)$ in (A.4) depends on the travel time (τ). Based on the modified interpolation technique discussed in section II for the large time step, $a(t)\times f_m(t)$ can be written as,

$$s(t)=a(t)\times f(t)=\lambda_0 v_m(t)+\lambda_1 i_m(t)+i_{ha} \qquad \text{(A.6)}$$

The λ_0 and λ_1 are constants and i_{ha} is a history current term. By substituting in equation (A.4), the sending end current becomes,

$$i_k(t)=y_{eq}v_k(t)-\lambda_0 v_m(t)-\lambda_1 i_m(t)-i_{ha}+i_{hk} \qquad \text{(A.7)}$$

Similar equation can be derived for the receiving-end and the formulas can be rearranged in the following matrix form shown below.

$$\begin{bmatrix} i_k \\ i_m \end{bmatrix}=\begin{bmatrix} \gamma_1 & \gamma_2 \\ \gamma_2 & \gamma_1 \end{bmatrix}\begin{bmatrix} v_k \\ v_m \end{bmatrix}+\begin{bmatrix} i_{hist1} \\ i_{hist2} \end{bmatrix} \qquad \text{(A.8)}$$

where, γ_1 and γ_2 are constants and i_{hist1} and i_{hist2} are derived based on history voltages and currents only for last two time steps (at $t = t\text{-}\Delta t$ and $t = t\text{-}2\Delta t$).

VI. APPENDIX B

A. Recursive convolution

Frequency dependent transmission line modeling involves the evaluation of the convolution in discrete time. The mathematical technique called recursive convolution is a numerically efficient technique to perform the evalution [3]. Consider the convolution of two frequency domain functions p and q.

$$s(t)=p(t)\times q(t) \qquad \text{(B.1)}$$

For the recursive convolution, both of the two functions in (B.1) should be expressed as a sum of exponentials as shown in (B.2).

$$p(t)=\sum_{i=1}^{n} c_i e^{a_i(t-\tau)} \qquad \text{(B.2)}$$

The terms c_i, a_i are pre-determined using the curve-fitting technique and the time delay (τ) is the travel time of the transmission line (see Appendix A for further details). Then discrete form of $s(t)$ is shown in (B.3) in terms of history values of $s(t)$ and $q(t)$.

$$s(t)=k_1 q(t-\tau)+k_2 q(t-\Delta t-\tau)+k_3 s(t-\Delta t) \qquad \text{(B.3)}$$

The constants k_1, k_2, k_3 in (3) are derived based on the assumption that $q(t)$ has a linear variation between $t\text{-}\tau\text{-}\Delta t$ and $t\text{-}\tau$ and the constants are,

$$k_1=-\frac{c}{a}\left(1+\frac{1-e^{a\Delta t}}{a\Delta t}\right) \qquad \text{(B.4)}$$

$$k_2=\frac{c}{a}\left(e^{a\Delta t}+\frac{1-e^{a\Delta t}}{a\Delta t}\right)$$

$$k_3=e^{a\Delta t}$$

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