Custom-Coded Models in the State Space Nodal Solver of ARTEMiS

Christian Dufour Hani Saad Jean Mahseredjian Jean Bélanger

***Abstract*-- This paper explains the users' custom model feature of the State-Space-Nodal (SSN) solver of ARTEMiS. Using this feature, users can interface directly with the SSN nodal solver their own discretized models. This can lead to great improvements in real-time performance with some models that exhibit many different operation modes and/or models having a high degree of internal redundancy. Direct discretized solver coding also enables the user to write custom stabilization code or to use different solvers coupled in the same nodal admittance matrix. On-Load Tap Changer (OLTC) transformer, Modular Multilevel Converter (MMC) and frequency dependent line models (modal and phase domain types) are given as examples.**

***Keywords*: Real-time simulation, nodal admittance method, SSN, MMC, line models.**

I. Introduction

THE solvers used today for real-time simulation are mainly derived from either the nodal admittance method with trapezoidal discretization of systems branches, like in Hypersim or RTDS real-time simulators or full state-space system description, as in SimPowerSystems (SPS) and the eMEGAsim real-time simulator from Opal-RT.

Experience has shown that both approaches have their advantages. One of the main advantages of nodal-based solvers in real-time application is their ability to quickly recompute their equations in systems with many topologically connected switches or piece-wise non-linear models. State-space modeling has the advantage of enabling the use of high precision matrix exponential approximants of higher degree than the trapezoidal rule. Within the Simulink environment, the state-space formulation can be also easily interfaced with Simulink advanced control system simulation toolboxes.

It is also worth noting that some 'specialty' models, like the frequency dependent line models are much better suited for simulation inside a nodal approach than a full state-space approach. In the latter, these frequency dependent line models would simply make the number of states explode, making real-time simulation very difficult.

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Recently, a new real-time simulation solver called State-Space-Nodal (SSN)[1][2] was developed to try to take advantage or both nodal and state-space approaches. SSN computes the state-space equations and nodal equivalent of any group of RLC elements, including sources, switches and non-linear elements (piece-wise or injection types). It does so using a unique mathematical foundation for all element types. Within SimPowerSystems (SPS) for example, SSN uses the state-space routines already coded in SPS to find nodal equivalents for all types of elements and groups of elements. These SSN 'state-space groups' (SSG) can be discretized using a variety of discretization rules. After discretization, the implicit terms of each SSG are combined into a nodal admittance matrix to find a common and simultaneous solution to all SSG.

This paper will explain the methodology for connecting user's custom models (also called SSN external models) to the main ARTEMiS-SSN solver and within the Simulink environment. On-Load Tap Changer (OTLC) transformer, Modular Multilevel Converter (MMC) HVDC systems and frequency dependent line examples will serve to demonstrate this feature of SSN.

II. SSN Mathematical Framework

The SSN method is a generalisation of the EMTP nodal admittance method. SSN uses arbitrary sized clusters (or partitions) of electrical elements, each describable by state-space equations, and solve them simultaneously using nodal admittance method[1].

A. SSN State-space modeled groups

For a group of resistance, inductance, capacitance, transformers, voltage/current sources and other electrical elements connected to a terminal of unknown voltage and current (a 'nodal connection point'), the state-space equation exists:

$$\begin{aligned} x' &= A_k x + B_k u \\ y &= C_k x + D_k u \end{aligned} \tag{1}$$

where

x: states of the system

u: inputs of the system

y: output of the system

A_k , B_k , C_k , D_k : state space matrices corresponding to the k-th permutation of switches and other piece-wise linear element segments.

When discretized, these equations result into

$$x_{n+1} = A_d x_n + B_{d1} u_n + \begin{bmatrix} B_{d2(in)} & B_{d2(no)} \end{bmatrix} \begin{bmatrix} u_{n+1(in)} \\ u_{n+1(no)} \end{bmatrix} \tag{2}$$

$$\begin{bmatrix} y_{n+1(in)} \\ y_{n+1(no)} \end{bmatrix} = \begin{bmatrix} C_{(in)} \\ C_{(no)} \end{bmatrix} x_{n+1} + \begin{bmatrix} D_{(in,in)} & D_{(in,no)} \\ D_{(no,in)} & D_{(no,no)} \end{bmatrix} \begin{bmatrix} u_{n+1(in)} \\ u_{n+1(no)} \end{bmatrix}$$

where:

A_d, B_{d1}, B_{d2}, C_d, D_d are the discrete state-space matrices for the given pattern of binary switches modeled inside the group (the *k* subscript of switch permutation is not written in Eq. 2 but still present). The trapezoidal rule of integration will produce B_{d1}=B_{d2}. Subscript *n* and *n+1* indicate the time instants: *n* is the last time step known solution values and *n+1* is the current time step.

$u_{(in)}$: internal sources of the state-space model. Like in standard state-space modeling, these include known forced sources and sources from non-linear current injections. *(in)* means internal.

$u_{(no)}$: unknown sources of the state-space model at the present time n+1. This represents the nodal voltage or the current injection that can only be resolved by simultaneous solution of all groups connected to the nodes of the network. *(no)* means nodal.

$y_{(in)}$: internal output of the model. These are current and voltage measurements to be taken inside the group.

$y_{(no)}$: nodal output of the state-space model. This is the voltage output or current output of the group that needs to be solved simultaneously along with all groups connected to the system nodes.

The following relationship can now be derived:

$$\begin{aligned} y_{n+1(no)} &= C_{(no)} \{A_d x_n + B_{d1} u_n + B_{d2(in)} u_{n+1(in)} + \ldots \\ & B_{d2(no)} u_{n+1(no)}\} + D_{(no,in)} u_{n+1(in)} + D_{(no,no)} u_{n+1(no)} \end{aligned} \tag{3}$$

Equation 3 has two types of terms, one known from past history with *(n)* and one which is unknown *(n+1)*. It represents a history term

$$\begin{aligned} y_{hist} &= C_{(no)} \{A_d x_n + B_{d1} u_n + \ldots \\ & B_{d2} u_{n+1(in)}\} + D_{(no,in)} u_{n+1(in)} \end{aligned} \tag{4}$$

in parallel/series with a discrete admittance/resistance, i.e. discrete ratio of voltage/current input-output values,

$$W = C_{(no)} B_{d2(no)} + D_{(no,no)} \tag{5}$$

This can be interpreted in two ways:

1) Case *y(no)* is a current and *u(no)* is a voltage: *W* is an admittance and y_{hist} is a history current source. Equation 3 describes a Norton equivalent. In the SPS implementation of SSN, we call this a 'V-type SSG' because the external inputs of the groups are voltage sources.

2) Case *y(no)* is a voltage and *u(no)* is a current: *W* is an impedance and y_{hist} is a history voltage source. Equation 3 then describes a Thevenin equivalent. In the SPS implementation of SSN, we call this an 'I-type SSG' because the external inputs of the groups are current sources.

It is important to understand that the circuit topology will determine if a V-type or I-type group must be used in practice. For example, an SSG of inductive type (for example, a series RLC branch) will require a V-type SSN description because one cannot obtain a state-space description in the form of Eq. 1 by connecting a current input source in series with the inductance of the SSG. Mathematical derivation is also possible when the nodal inputs of the SSG are a mix of voltage and current sources [2].

This approach can also be coupled with the standard nodal admittance method in which the discrete equations are derived from the reduction of the individual elements discrete equations. This is what is done in many of the SSN external models presented in the next section.

B. Usage of Higher-Order L-Stable Matrix Exponentials

It is well-known that the exact solution to Eq. 1 is equal to:

$$x_{n+1} = e^{Ah} x_n + \int_t^{t+h} e^{A(t-\tau)} Bu(\tau) d\tau \tag{6}$$

where *h* is the discretization time step. It should be recognized that 2 distinct approximations are necessary to obtain a numerically computable expression:

1- The approximation to the matrix exponential e^{Ah}
2- The way the input *u* is approximated during integration

The traditional EMTP approach uses the trapezoidal approximation (Padé 1,1) of the matrix exponential, equal to:

$$e^{Ah} \cong \frac{I + hA/2}{I - hA/2} \tag{7}$$

combined with a linear interpolation of the input during the integration step. The trapezoidal rule, however, can exhibit numerical oscillations during discontinuities; therefore the Backward Euler method is used during discontinuities in EMTP-RV.

Using a higher order in Equation 6 can lead to interesting results especially with regards to stability issues. The ARTEMiS 'Art5' solver, based on the (2,3)-Padé order 5 approximation of the matrix exponential of Eq. 8,

$$e^{Ah} \cong \frac{I + 2hA/5 + (hA)^2/20}{I - 3hA/5 + 3(hA)^2/20 - (hA)^3/60} \tag{8}$$

has a property called L-stability, an extension of A-Stability, which makes it immune to the kind of numerical instability of the trapezoidal rule. It should be noted that the Backward Euler rule is also an L-stable Padé approximation.

Some interesting possibilities can be foreseen from Eq. 6. For example, one is not forced to use a linear interpolation of the input of each SSG. Indeed, one of the available solvers of the ARTEMiS-SSN uses a 'Backward-Euler' input type for the input terms. It means that the input u is considered constant and equal to the solution at time *(n+1),* making this term fully implicit. This solver is called 'Art5 with Backward Euler nodal interface' within ARTEMiS-SSN. Experiments with this latter solver showed that is can improve the numerical stability in some difficult cases, like the simulation of MMCs [8].

C. Nodal admittance solution and switch management.

In the ARTEMiS-SSN, the user manually defines the SSG by selecting the nodal matrix nodes location in the SPS diagram. This in return automatically determines the SSG limits. Once the SSG limits are defined, the SSN approach will automatically derive all SSN equations, that is, the SSG update equations and nodal admittance matrix parts.

When a switch is present *inside an SSG*, SSN will make the complete precalculation of all possible sets of ABCD equations for this SSG. For its part, *the nodal admittance matrix is retriangularized continuously during the simulation*, dynamically taking into account the changes that can occur within all the SSGs. *This guarantees that circuits with an arbitrary large number of switches can be simulated without memory issues, provided that the user limits the number of switches within each SSG.*

One interesting aspect of SSN is that the user can modulate the size of the nodal matrix by changing the size and grouping of the SSGs. In a classic nodal method, a fixed set of basic elements (such as R, series RL, series RLC, transformer, switches) are discretized individually. Their discretized equivalents are solved simultaneously with the use of a nodal admittance matrix. Because the basic elements can be small and numerous, the resulting nodal matrix can in return be huge and therefore pose a challenge during real-time simulation. By comparison, the SSN solver can make larger groups, or partitions, of the network to be simulated and obtain a *reduced size nodal admittance matrix*, thus facilitating real-time simulation.

D. External user-coded SSN Groups

The SSN mathematical framework allows user to directly interface custom-coded nodal groups with the main nodal admittance matrix solver. For this, the user has to derive the discretized equations of a device and interface directly with the nodal solver of ARTEMiS-SSN. There are several cases in which such an approach can be advantageous:

1- Models that are naturally better solved with a nodal approach.

FD-lines, modal or phase domain types, are much more efficiently computed in a nodal approach than in state-space mainly because of the large number of states within these models.

2- Models with high-switch number and switching dependencies.

MMC devices are made of several thousands of switches that can instantaneously switch dependencies (i.e. some IGBT turn-off can lead to instantaneous diode turn-on). The dependencies make it difficult to put the different switches into different SSG as SSN cannot detect these dependencies across SSGs. Since MMC consists of many identical cells in series, it is better to code the MMC arms as a whole in a 'C' code function than to consider all IGBT/Diode blocks individually.

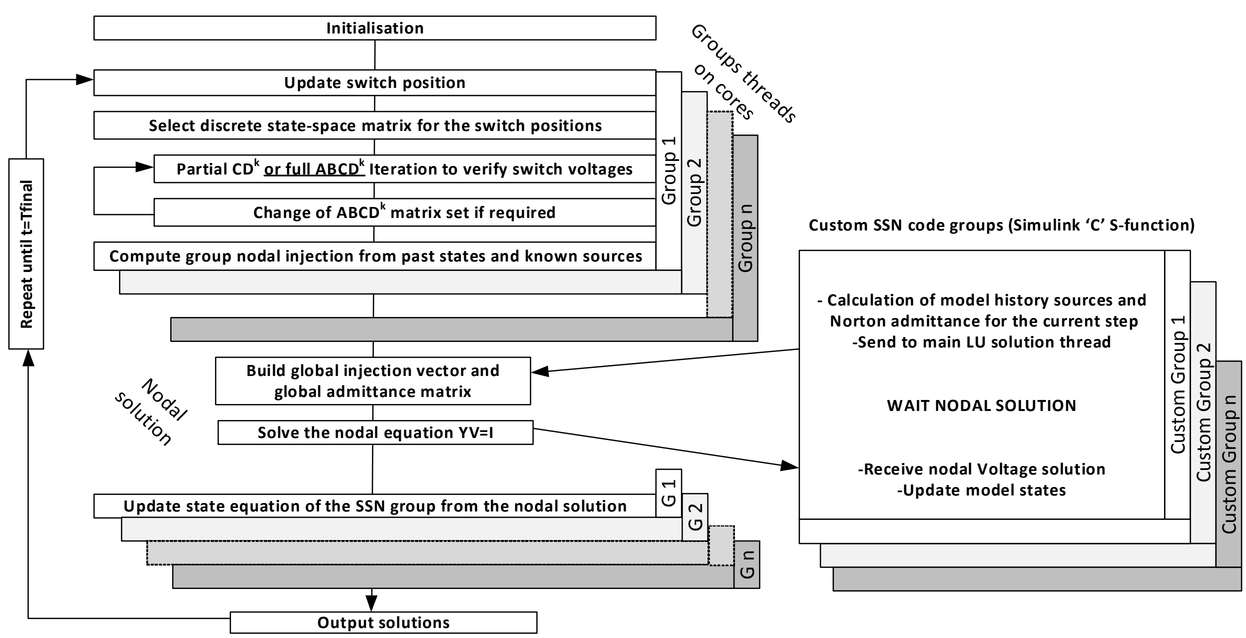

Fig. 1. Simulink implementation of the SSN algorithm with external models

3- SSN group code optimization

OLTC consists of multi-tap transformer with a large number of switches to change the taps. This can lead to a complex implementation if real switches used in the model. Alternatively, on OLTC can be viewed as a transformer with varying turn ratio and leakage inductance. This latter interpretation is easy to code into a nodal approach.

E. Preferred coding method using Simulink 'S-function builder' blocks

The way to connect external user-coded SSN groups is rather simple when you consider that SSN uses a classic nodal admittance method at the core of the solver. Then, these external SSN groups a) read the last voltage, b) build the new history current source and nodal matrix solution, according to the last systems inputs, like source or switch position, c) send this history vector and matrix contribution to the main SSN nodal admittance solution solver. This is depicted in Fig. 1

In the ARTEMiS implementation of the SSN solver, within Simulink, this is done normally with Simulink 'S-function Builder' blocks. It can also be done directly into a 'C' code S-function. In all cases, the main difficulty is to synchronize the different algorithmic steps of SSN within Simulink's simulation that is typically made of 2 different steps: update of each block outputs followed by the update of each block internal states.

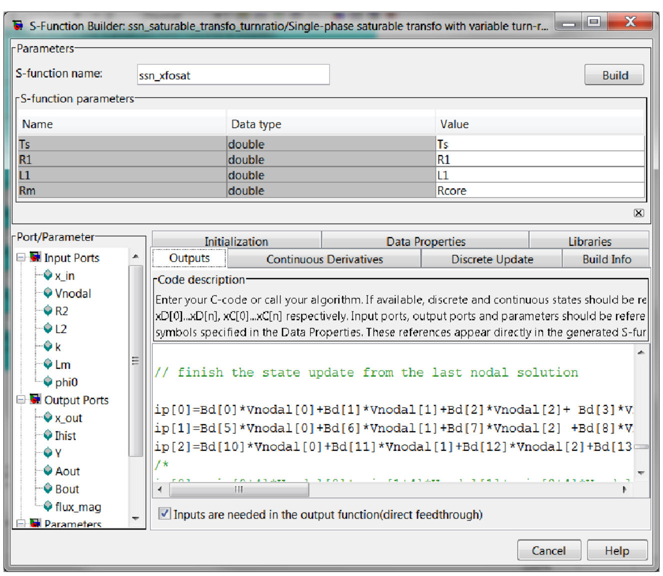

Fig. 2. Simulink S-function builder interface

The easiest way to code Custom SSN blocks is certainly to use Simulink 'S-function builder' blocks. With this block, the user can write its own SSN blocks directly in 'C' code, validate its behavior offline in Simulink and directly compile it for real-time simulation in the RT-LAB real-time environment. The user interface of the 'Simulink S-function builder' block is easy to use and is depicted in Fig. 2.

III. EXAMPLE CASES

In this section, we will show and explain several cases of SSN user coded models. The examples are: modal-domain frequency dependent lines, phase-domain frequency dependent line models, MMC based HVDC transmission system and OLTC) transformer.

A. Modal Domain FD-line

The FD-line model of [3] was the first line model in EMTP to take into account the variation of wire and ground return path inductance and resistance variation with regards to frequency. The model equations are solved in the modal domain and this results in a model that is faster than phase-domain FD-line models. The main approximation of the FD line model is that it uses frequency independent modal transformation matrices. This approximation is exact for continuously transposed line, but can become less accurate for unbalanced lines and cannot be used for cables.

Within SSN, the FD-line model is computed in the exact same way as in EMTP. It is using direct reduction methods of the discretized equations of the FD-line model. In this model (as well as for the WideBand line of the next section), each end of the line contributes matrix-wise to the nodal admittance matrix and global history vector as seen in Fig. 3.

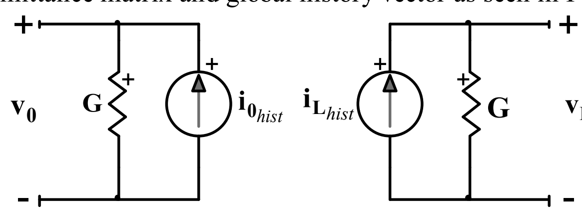

Fig. 3. Reduced discrete time domain circuit of a multi–conductor line

B. Phase Domain FD-line or WideBand (WB-line)

The Phase Domain FD-line model, also called Universal Line model or WideBand (WB) line model [4][5], is another line model that takes into account the variation of wire and ground return path inductance and resistance variation with regards to frequency. However, this frequency dependence is computed directly into the phase domain, without modal transformations. As a result, the WB-line model is more accurate for all arbitrary line and cable configurations but its complexity results into a larger model, slower to compute than the FD-line counterpart. In [5] notably, the WB model has been optimized for use in real-time applications.

Fig. 5 shows the simple energizing of a DC-link with a WB-line model and compares it with a Constant Parameter line model (also called Distributed Parameter Line (DPL) model or Bergeron with losses line model), with the cable configuration of Fig. 4. One can observe that WB-line has much higher damping of high frequency components than the Constant Parameter line (DPL in the figure). This is to be expected because of the increase of ground return impedance when the frequency increases.

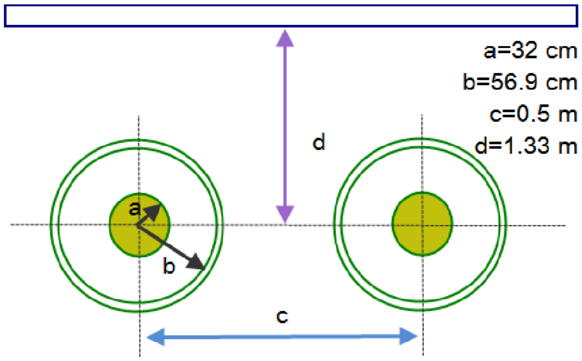

Fig. 4. Cable configuration

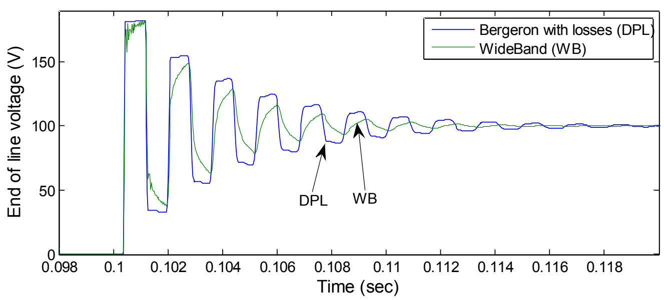

Fig. 5. Open-ended energization test of cable for DPL and WB line models.

C. Multi-level converter (MMC) applications

The Modular Multilevel Converter (MMC) topology is now a common solution for HVDC and FACTS systems. MMC structures are composed from several hundreds to thousands of half-bridge converters. Such power electronic topologies pose an important challenge in electromagnetic transient (EMT) type simulation programs and especially real-time simulation software.

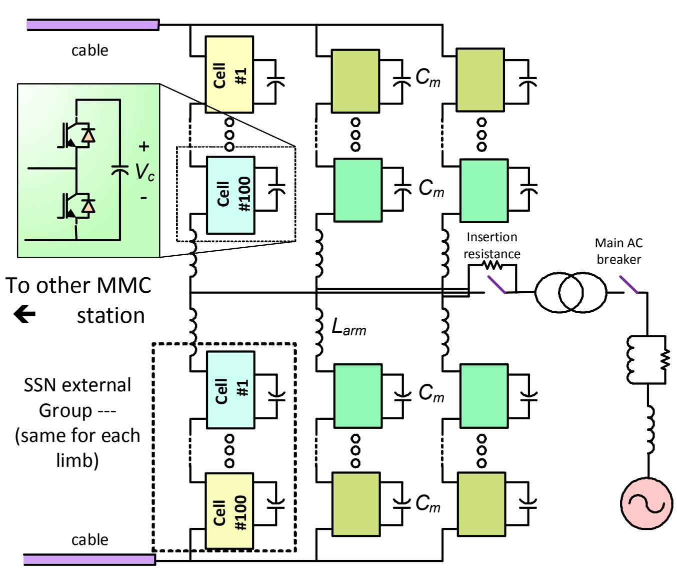

Fig. 6. One MMC-HVDC terminal model

In [8], SSN is used to make the real-time simulation of a 101-level MMC-based HVDC system, with all controllers connected, in normal and faulty working modes.

The use of an external SSN model to simulate the MMC within SSN, a nodal admittance method, enables the simulation of difficult cases such as when all IGBT gates are turned off and the MMC arms go into a high impedance state. This type of simulation case is typically hard to make using switching-function methods. In this paper, we show such a test using the circuit of Fig. 6 connected to a resistive load of 1 pu at the other end of the transmission line. In the test, the MMC control system regulates the DC voltage, that is before t=1sec.in Fig. 7. At this time, the IGBT pulses are shut off and the MMC enters in natural rectification mode. Thus, DC voltage drops to the natural rectification value:

$$V_{dc\_natural\_rectification} = \frac{3\sqrt{2}}{\pi} V_{LL} \tag{9}$$

which is lower than the desired DC voltage that is set at $V_{dc\_reference} = 2V_{LL}$

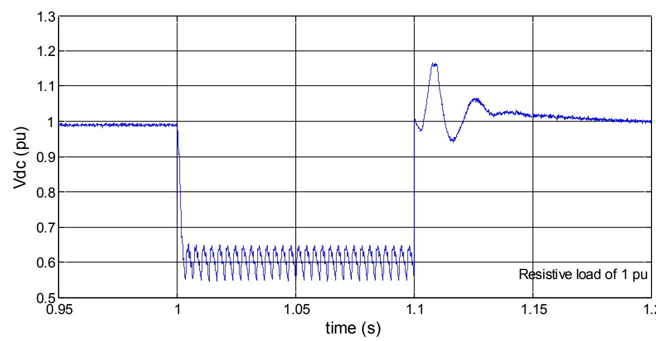

Fig. 7. Blocked state applied in Station#1 for all phase.

The complete MMC-HVDC system (101-levels, 2 terminals connected with Bergeron with losses lines) can be simulated in real-time in RT-LAB at a time step of 25µs.

D. On-Load Tap Changer transformer applications

The 'good and hard way' to simulate an OLTC transformer is to include the tap switches in the model. The resulting model could, however, be difficult to simulate in real-time because of the large number of switches and nodes. Alternatively, one can model the OLTC like a simple transformer with a continuously varying turn-ratio and leakage inductance. In terms of simulation code, this is only a matter of re-discretizing the transformer equation when the turn ratio and leakage inductance change, without involving any notion of switches.

This is what is done in the following example case of an OLTC transformer feeding a distribution network. The network is composed of many inductive loads and capacitive compensation banks. The lines have 5 km each and are modeled with the pi-line model. Parameters of the case are given in TABLE 1.

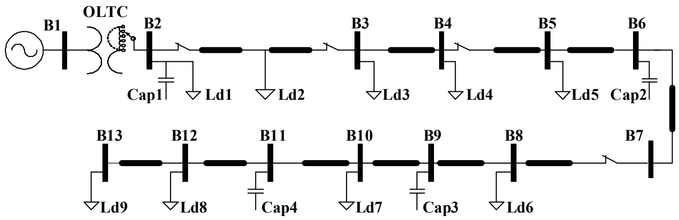

Fig. 8. OLTC-fed distribution network

TABLE 1 OLTC-FED MAIN DISTRIBUTION NETWORK PARAMETERS

Elements	Value
Loads Ld1…Ld9	P=100kW Q=800kVar
Capacitor banks Cap1…Cap4	Q= -3000 kVar
π-lines (all)	R1, R0= 0.06, 0.18 Ohms/km L1, L0=1.2675e-3 3.8025e-3 H/km C1, C0= 1e-9 1e-9 F/km Length=5 km
Fault resistances	0.001 Ohms
Transformer	R=0.4% X=8% Turn ratio :1 Config : Yg-Yg, no saturation
AC source	25kV, 60 Hz, X=0.01%, 1 GVA

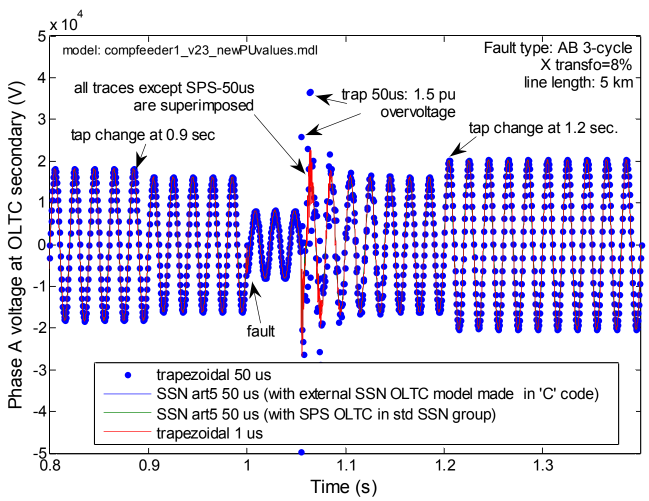

Fig. 9. Comparison between SSN, SSN external model and SPS for two tap changes and phase-phase fault.

In this test, we compared: the standard SPS multi-tap transformer with switches for taps inside 3 SSN groups with art5 solver; the external SSN model of OLTC, that is a transformer model that is continuously discretized for turn-ratio and leakage inductance change; and SPS simulation using the Trapezoidal rule of integration at 50 µs and 1 µs.

The test consisted on making a tap change, followed by a 3-cycle AB fault and followed by another tap change. The results for the OLTC secondary voltage, depicted in Fig. 9, show that the external model approach is as accurate as the traditional way using 'switched' taps in SSN using the art5 solver. The trapezoidal rule of SPS was however inaccurate to simulate the fault clearing at 50 µs due to the small fixed-step induced current chopping at breaker opening and the presence of very high frequency modes induced by the short line sections ('*' without lines in the figure).

Another test consisting on the transformer and grid energization (Fig. 10) from null initial conditions showed a good match between all solvers and all time steps.

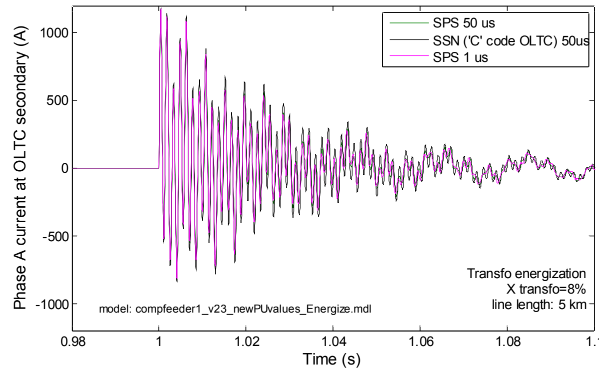

Fig. 10. Distribution grid energization currents (phase A)

IV. Conclusion

In this paper, we have presented and explained how users can integrate their own nodal codes inside the ARTEMiS-SSN solver. Examples were given to explain this feature such as frequency dependent line models, MMC-HVDC model and OLTC transformer modeling. Such a feature is often very important to optimize complex models and enable their real-time simulation.

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