

Synchronous Generator Withstand against Transformer Energization

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Abstract--In oil and gas plants large power distribution transformers are often directly connected at the same voltage level to turbo-generators having almost the same rating as the transformer size: hence the generator can be subjected several times to the transformer energization, both during plant black start and during normal operating conditions.

In this paper, a typical electrical distribution from an industrial plant is taken into consideration: the phenomenon of transformer energization under one gas turbine synchronous generator is studied and particular emphasis is put on the transient behavior of the following generator magnitudes: minimum and maximum stator voltage, field current/voltage, stator winding current, electromagnetic torque, turbine speed/generator frequency. A simple analytical method is also provided to evaluate, starting from the calculating transformer magnetization inrush current waveform, the stator winding equivalent thermal stress to be compared to the generator capability against unbalanced and non-linear current waveform ($I_2^2 \cdot t$ withstand) as foreseen by IEC and ANSI standards.

Simulations are carried out by means of EMTP-ATP program: main equipment (synchronous machine, generator automatic voltage regulator and exciter, transformer) is modeled using parameters from relevant manufacturers.

Design data from generator manufacturer regarding withstand against the transformer energization are finally compared to the results obtained with the EMTP-ATP program, and useful conclusions are derived in terms of choice of the best settings for frequency and voltage protection relays in order to avoid undue generator trip during energization.

Keywords: magnetizing inrush current, transformer energization, AVR, excitation system, synchronous generator, negative sequence current, voltage sags / swells.

I. INTRODUCTION

THE use of distributed generation (synchronous turbo-generators with typical rating between 5 MW and 40 MW) within oil & gas plants has become quite widespread in latest years [3], [5]. More often each one of these generators is coupled directly to the plant distribution network, without a dedicated step-up transformer like in classical power station schemes. Hence it is not more possible to energize each plant distribution transformer in a gradual and soft way during the start-up of the generator and the gradual build-up of its excitation voltage versus its speed increasing, but the

transformer energization takes place only after the generator black-start through an emergency diesel generator or from an external grid supply.

However, a synchronous generator can be very sensitive to the energization of a transformer having approximately the same kVA generator rating [3], [4], in terms of stator winding thermal stress, over-excitation, excessive electro-magnetic torque oscillations, and abnormal stator voltage and frequency behavior.

The phenomenon of transformer energization is well known in technical literature since many years [2], [10], and a lot of work has been done with regard to the transformer modeling for the aim of its magnetizing inrush current calculation [7], [8], [9]. Several recent works have shown clearly that transformer energization transients could affect the successful completion of a generator black-start procedure [11] or of a plant emergency restoration plan [4], as well as they could impact on the malfunction of generator differential protection relay [6]: following these investigations, here particular attention is focused on the generator electro-mechanical stress, in spite of a very detailed transformer modeling, and a term of comparison is looked for between calculated results and some prescriptions foreseen by IEC standards for rotating machines [15] and ANSI standards for synchronous generators [14].

The aim of this work is to analyze the possibility of one synchronous generator to energize successfully one distribution transformer without exceeding the electro-mechanical withstand limits declared by the generator manufacturer and preventing undue trips of voltage and frequency generator protection relays.

II. SYSTEM DATA AND MODELING

A. System Data

The electrical distribution scheme of a typical industrial plant, in which a synchronous generator, being driven by a gas turbine, is operated in parallel to an external supply grid during plant normal operating conditions, is shown in Fig. 1.

Main electro-mechanical parameters, for each network component, are reported in the Appendix.

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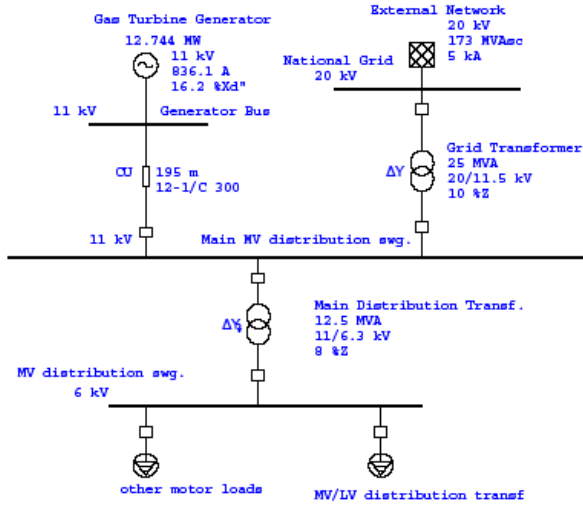


Fig. 1. Simplified single-line diagram of the industrial electrical system

B. Modeling

The electrical network is simplified and modeled in ATP (alternative transient program) [13] as shown in Fig. 2, following the general guidelines presented in [12].

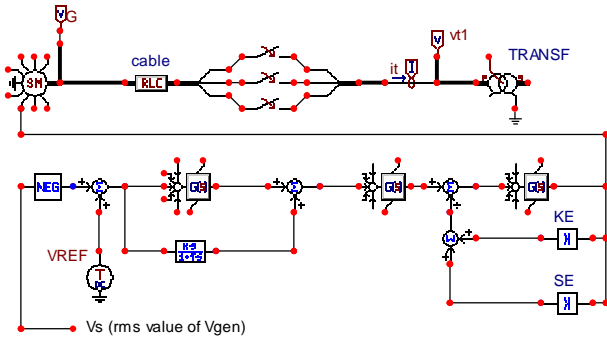


Fig. 2. ATP model of the electrical system

Medium voltage (11 kV) cables are modeled as constant impedances, since they are electrically short lines and: due to their non significant extension (less than 1 km), also the cable shunt capacitance can be neglected.

The turbine-generator is modeled by means of ATP SM59 model for synchronous machines [13], based on d-q reactance Park's theory [1] and with transient control system (TACS) and magnetic saturation being taken into account.

The automatic voltage regulator (AVR) of the generator excitation system plays an important role during the energization since it can respond quickly soon after the stator winding voltage undergoes dips caused by the transformer inrush current, and stator voltage swells and overshoots can follow the initial dips due to the interaction between AVR and inrush currents [3], [5]: the AVR and the generator exciter are modeled with equivalent transfer functions in Laplace s-domain.

In contrast to the necessity of modeling the AVR, the turbine governor can be neglected [4]: in fact, it does not impact on the accuracy of the overall analysis since its time constants are longer than the duration of the transients being

studied. Moreover, the transformer energization is a phenomenon essentially of reactive type: the levels of active power consumed during this transient are very low and negligible.

The transformer is modeled by means of ATP saturable transformer component (STC), based on the nonlinear version of the Steinmetz model [13]: the non-linear saturation curve and air-core inductance are provided by the manufacturer on the basis of no-load test [10], while the winding resistance and leakage inductance from manufacturer short circuit tests are equally split on a per unit base between primary and secondary winding. This simple model cannot represent accurately the inrush current decay, due to lack of detailed representation of core hysteresis and iron losses [7], but this is deemed more conservative for the evaluation of the generator winding stress during transformer energization. The residual flux is not taken into account due to the lack of a topology-correct transformer model able of accounting of magnetic fluxes outside the core and windings [8]: however, this does not affect the analysis, since the first energization during black-start or during restoring after a long maintenance period is studied here, instead of a de-energization and a fast subsequent re-energization transients.

The circuit breaker of the transformer feeder is modeled as a time controlled switch, i.e. a switch that closes at a pre-determined time.

III. PRE-ANALYSIS AND STUDY CASE

A. Validation of Transformer Model

The transformer model, being assumed for the simulations, is first tested in case of an ideal supply network (stiff supply provided with infinite short circuit power), as shown in Fig. 3.

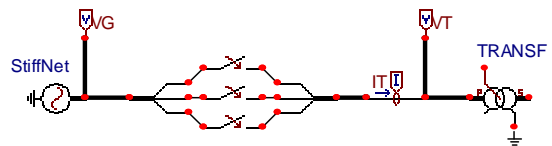


Fig. 3. ATP model of the electrical system: ideal network supply

The circuit breaker poles are closed at the instant when the voltage on phase A passes through the zero value, to get the maximum current peak value on the same phase [2]. The inrush current waveform shown in Fig. 4 results.

The peak value on phase A is around 7050 A (corresponding approximately to 10.7 p.u. of transformer rated current and to 8.4 p.u. of generator rated current) and its relevant r.m.s. value (4985 A) is quite in line with the symmetrical r.m.s. value provided by the manufacturer (5000 A).

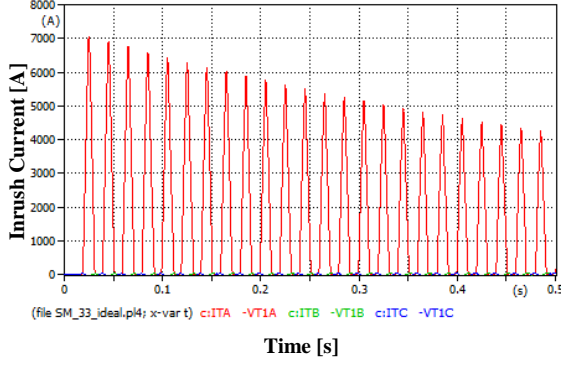


Fig. 4. Inrush current (phase A) with transformer fed by an ideal supply

It is clearly visible the typical waveform being completely asymmetrical for the phase A and made of one peak followed by a hole in each 20 ms cycle (corresponding to the fundamental frequency of 50 Hz).

B. Approximate evaluation of equivalent energy content for the inrush current

Rigorously speaking, since the transformer inductance is non-linear and the transformer resistance reflects the core losses as well as the winding resistance, a time constant for the transformer inductive-resistive equivalent circuit cannot be exactly defined [4]: however, the envelope of the transient waveform in Fig. 4 resembles an exponential decay behavior, which can be simply represented as one time constant decay as follows:

$$I = I_G \cdot e^{-\frac{t}{T}} \quad (1)$$

where

I is the instantaneous envelope inrush current (p.u. of generator rated current)

I_G is the peak value of the inrush current (p.u. of generator rated current)

e is the exponential function (neperian number)

t is the time variable (s)

T is the equivalent time constant of the envelope decay (s).

The energetic content (I^2t) of the inrush current is calculated through the integration of the function expressed by “(1)”, which gives the area under the current waveform exponential envelope, considering that half of this shape is null. The following formula results:

$$(I^2t) = \frac{1}{2} \int_0^{\infty} I^2 dt = \frac{1}{2} \int_0^{\infty} \left(I_G \cdot e^{-\frac{t}{T}} \right)^2 dt = \frac{T}{4} I_G^2 \quad (2)$$

where

(I^2t) is the let-through energy of the inrush current (s)

I is the instantaneous envelope inrush current (p.u. of generator rated current)

$\int dt$ is the integral operator

I_G is the peak value of the inrush current (p.u. of generator

rated current)

e is the exponential function (neperian number)

t is the time variable (s)

T is the equivalent time constant of the envelope decay (s).

IEC standard [15], for air indirect cooled generators, gives a thermal withstand capability (I_2^2t) against unbalanced negative sequence currents (at fundamental frequency) equal to 20 s, while ANSI standards [14] prescribe, for the same type of machines, a value of 30 s. ANSI standard [14] states also that in principle it is possible to define a more general withstand capability (I_2^2t), by taking into account also unbalanced harmonic components with order higher than the fundamental: anyway, these harmonics are defined only during steady state operation and not during transients like the energization.

The transformer energization current waveform has a harmonic spectrum whose most significant components are usually the 2nd and the 5th orders [6].

In general, not all the harmonic components of the inrush current are of negative sequence type: however, since the prevailing 2nd and 5th harmonics are negative sequence components [16], from an engineering point of view it is deemed a conservative approach to compare at a first glance the let-through energy (I^2t), calculated by “(2)”, to the withstand capability (I_2^2t) defined by the standards in steady state conditions, i.e. to verify the following relationship:

$$I^2t \leq I_2^2t \quad (3)$$

where

I^2t is the let-through energy calculated by “(2)”

I_2^2t is the unbalance current withstand defined by [14], [15].

Indeed, since “(2)” takes into account, although in a simplified way, the whole energetic content developed by the transient energization current, if “(3)” holds true one can be quite confident that a good safe margin does exist on the actual thermal stress of the generator stator winding, before performing any more detailed harmonic analysis which, however, would require more construction data from the generator manufacturer and in situ measurement tests.

C. Study Case

The transformer energization takes place only after the gas turbine generator has been started and is operating under no-load conditions. Hence it is assumed that, during all the plant black-start sequence, the low voltage auxiliary loads relevant to gas turbine and plant oil & gas process are fed by an emergency diesel generator or by an external supply start-up supply grid.

IV. RESULTS

The results of numerical simulations are shown graphically in the following figures. A comparison between results and generator manufacturer data is then performed.

A. Electromechanical Magnitudes

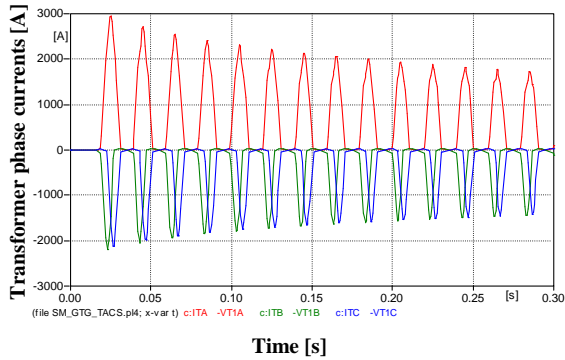


Fig. 5. Transformer inrush currents – zoom view on first waveform cycles

The maximum peak value on phase A is around 2950 A and it corresponds to approximately 3.5 p.u. of the generator rated current. From the Fig. 6, it is visible that the transient inrush currents damp out in about 10 s to reach the steady state values: an equivalent time constant equal to 2 s can be assumed.

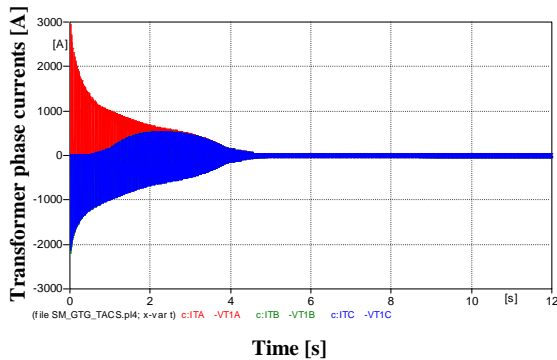


Fig. 6. Transformer inrush currents – view on complete waveform

The generator stator voltage is shown in Fig. 7.

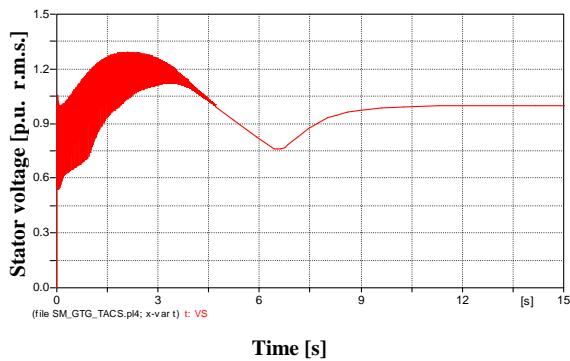


Fig. 7. Generator stator winding voltage as a function of time

It is clearly visible that the voltage sag as well as the voltage swell at generator terminals are neither excessive nor detrimental for the generator, and are compatible with under-voltage and over-voltage generator protection relay settings

provided in the following. Due to the initial small generator voltage (less than 0.8 p.u. for about 0.8 s), the low voltage black-start direct loads, made of direct on line motors and variable speed drive motors, cannot be fed by the gas turbine generator but shall be supplied by the emergency diesel generator or the external supply grid for all the duration of the black-start procedure and of the transformer energization. For the stator voltage the steady state value is reached after approximately 10 s.

From the angular speed deviation, shown in Fig. 8, the corresponding generator frequency can be computed by the following formula:

$$f = f_n \left(1 + \frac{D}{2\pi} \right) \quad (4)$$

where

f is the generator frequency (Hz)

f_n is the rated generator frequency (equal to 50 Hz)

D is the angular speed deviation (centi rad/s)

2π is the circle center angle (rad).

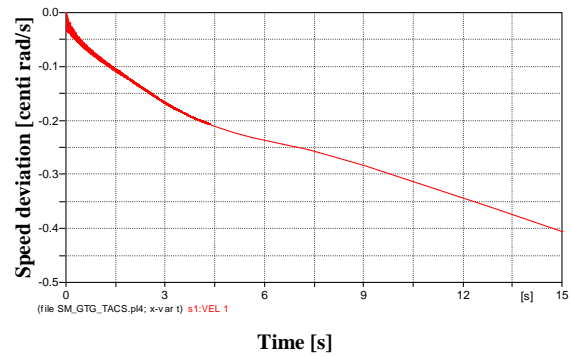


Fig. 8. Generator angular speed deviation as a function of time

Hence by means of “(4)”, the negative speed deviation of - 0.40 rad/s, reached at the end of the energization transient, corresponds to the minimum frequency of 49.97 Hz.

The generator frequency pattern is shown in Fig. 9. It is visible that the frequency is practically not affected by the energization transient.

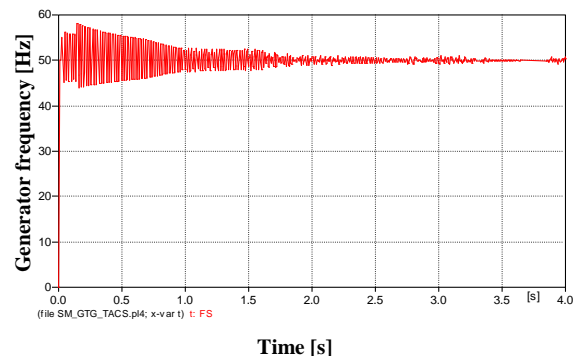


Fig. 9. Generator frequency as a function of time

The zoom views on the frequency, reported in Fig. 10 and Fig. 11, show that the transient oscillations are compatible with under-frequency and over-frequency generator protection relay settings provided in the following.

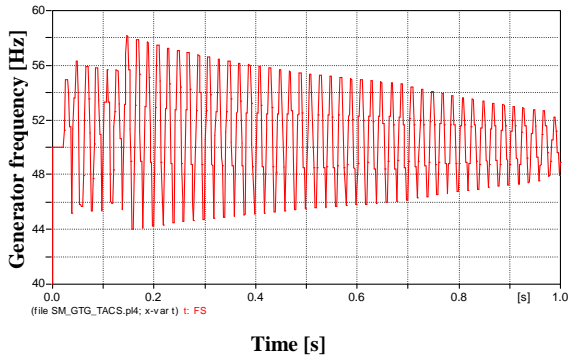


Fig. 10. Generator frequency – zoom view on first waveform cycles

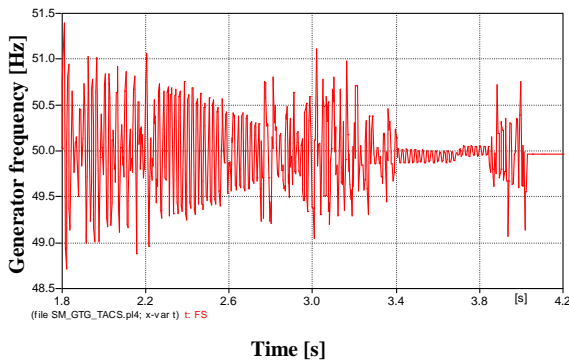


Fig. 11. Generator frequency – zoom view on subsequent waveform cycles

From Fig. 12, it is visible that the field current trend is compatible with the transient ceiling current withstand capability provided by the generator manufacturer and being equal to 194% of the rated field base current for a duration of 10 s. The base field current is equal to 253.1 A and the relevant ceiling value is equal to 491 A.

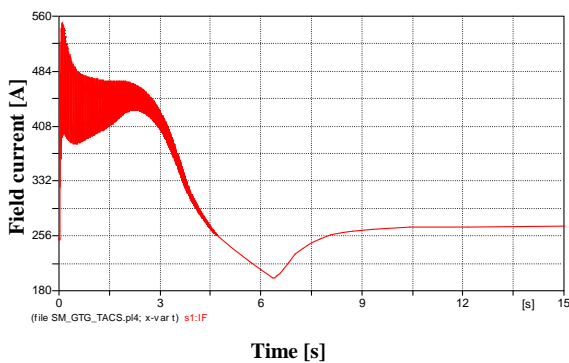


Fig. 12. Generator field current as a function of time

The transient field voltage, shown in Fig. 13, is compatible with the maximum design limit of 15.31 p.u., corresponding to

the value of 356 V with a base field voltage equal to 23.25 V.

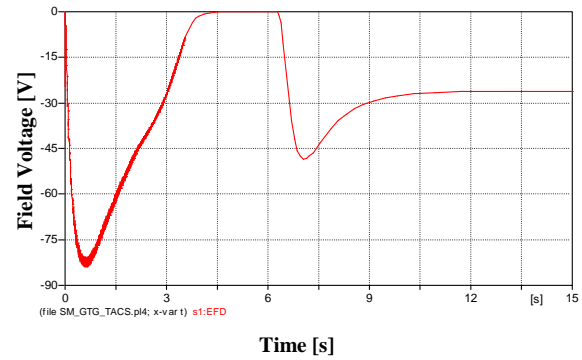


Fig. 13. Generator field voltage as a function of time

Both field current and field voltage behaviors show that the relevant steady state values are reached after 10 s.

Finally, the electromagnetic torque developed during the energization transient results well within the limit that the turbine-generator can withstand during 3-phase and 2-phase short circuit events: the maximum reached value is around 50 kNm and is also lower than the rated torque value equal to 82.78 kNm.

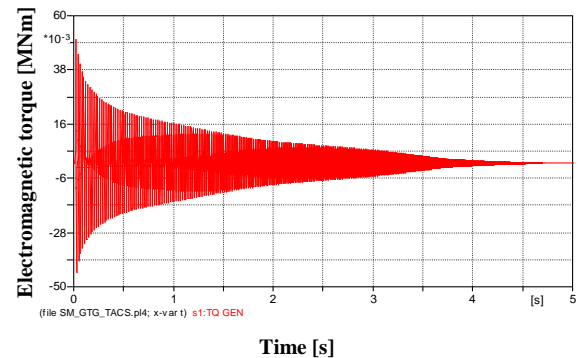


Fig. 14. Generator electromagnetic torque as a function of time

B. I_2^2t unbalanced current withstand

By applying “(2)”, with $T = 2$ s and I_G equal to 3.5 p.u., a let-through energy (I^2t) equal to 6.12 s results. Since the generator has a withstand capability against unbalanced currents (I_2^2t) equal to 20 s, the relationship in “(3)” is quite well satisfied and therefore no problem does exist for the thermal stress of the generator stator winding.

C. Voltage and Frequency protection settings

On the basis of the calculated generator stator voltage during transformer energization, the following under-voltage (ANSI code 27) and over-voltage (ANSI code 59) settings can be used for generator protection relays:

$$\begin{aligned}
 U < (27 \text{ 1}^{\text{st}} \text{ threshold}) &= 90\% \text{ of } 11 \text{ kV} && \text{for } 10 \text{ s} \\
 U << (27 \text{ 2}^{\text{nd}} \text{ threshold}) &= 80\% \text{ of } 11 \text{ kV} && \text{for } 3 \text{ s}
 \end{aligned}$$

U> (59 1st threshold) = 110% of 11 kV for 10 s
 U>> (59 2nd threshold) = 120% of 11 kV for 3 s.

On the basis of the calculated generator frequency during transformer energization, the following under-frequency (ANSI code 81U) and over-frequency (ANSI code 81O) settings can be used for generator protection relays:

f< (81U) = 48.5 Hz = 97% of 50 Hz for 5 s
 f<< (81U) = 47.5 Hz = 95% of 50 Hz for 1 s
 f> (81O) = 51.5 Hz = 103% of 50 Hz for 10 s
 f>> (81O) = 53 Hz = 106% of 50 Hz for 1 s.

The above protection relay settings do not hinder the energization of the distribution transformer and their validity has been confirmed by the turbine-generator manufacturer.

V. CONCLUSIONS

In this paper a typical industrial system, provided with distributed generation, is taken into consideration and the analysis is focused on the effects of transformer energization on one turbo-generator.

For plant configurations in which the generator is directly connected to a distribution switchgear, it results not more possible to energize gradually through a soft voltage versus frequency ramp each distribution transformer during the start-up of the turbine-generator and the build-up of generator excitation, like it usually happens for the generator step-up transformer of a power plant, but the transformer energization takes place once the generator is already on-line and operating in no-load conditions. As a rule of thumb for the likelihood and necessary condition for transformer energization, it is always advisable that the largest transformer to be fed has a rated power not greater than the rated power of the generator.

During the energization transient, the stator voltage at generator terminals undergoes initially voltage sags due to the consumed reactive power, followed by voltage swells due to the interaction between the fast response of the automatic voltage regulator (AVR) and the transformer inrush magnetizing current: the AVR shall then always be modeled for this type of studies. The initial stator voltage dips caused by transformer energization forbid the supply of direct on line motors and variable speed drive motors by the generator during the energization: in fact, low voltage contactors within direct on line motor starters are prone to drop-out for minimum voltage, as well as variable speed drives are very susceptible to excessive under-voltages with even few hundred milliseconds duration.

To evaluate the impact of transformer energization on the generator, it is necessary to analyze not only the stress on stator winding, mainly in terms of withstand capability (I_2^2t) against unbalanced currents, but also the transient behavior of rotor field current and field voltage and the transient electromagnetic torque oscillations developed on turbine-

generator shaft: it is therefore always important to get from the turbine-generator manufacturer all the essential data, as shown in the Appendix, to carry out a careful analysis.

Under/over voltage and under/over frequency generator protection relays are usually set according to short circuit and transient stability studies: in this context their settings shall also take care of the transformer energization transient, without causing undue trips which could compromise the commissioning of the turbo-generator unit and the plant black-start operation. Since the energization is mainly a reactive power phenomenon, between voltage and frequency magnitudes the stator voltage results the one being most affected, and particularly attention must be paid to check that the relevant temporary overshoot during energization does not exceed the design limit foreseen by the generator manufacturer.

VI. APPENDIX

A. Turbo-Generator Data

TABLE I
GENERATOR

4-pole synchronous generator driven by a gas turbine	
all reactance and resistance p.u. (per unit) values are referred to the base power $S_b = 15930$ kVA	
parameter	value
rated power	15930 kVA
rated power factor	0.8
rated voltage (r.m.s.)	11000 V (line to line)
rated stator current	836 A
rated angular speed	157.1 rad/s
rated no-load field current	274 A
armature resistance	$R_a = 0.0027$ p.u.
d-axis synchronous reactance	$X_d = 1.53$ p.u.
q-axis synchronous reactance	$X_q = 0.67$ p.u.
d-axis transient reactance (un-saturated value)	$X'_d = 0.267$ p.u.
q-axis transient reactance (un-saturated value)	$X'_q = 0.267$ p.u.
d-axis sub-transient reactance (saturated value)	$X''_d = 0.162$ p.u.
q-axis sub-transient reactance (saturated value)	$X''_q = 0.243$ p.u.
leakage reactance	$X_l = 0.118$ p.u.
zero sequence reactance	$X_0 = 0.06$ p.u.
negative sequence reactance	$X_2 = 0.211$ p.u.
transient d-axis open circuit time constant	$T'_{do} = 6.759$ s
sub-transient d-axis open circuit time constant	$T''_{do} = 0.039$ s
sub-transient q-axis open circuit time constant	$T''_{qo} = 0.102$ s
moment of inertia of generator + turbine + gear	$J = 4653$ kg m ²
total inertia time constant	$H = 3.6$ s
I _{FIELD} CEILING	194% for 10 s
I _{FIELD} BASE	253 A
rated torque	$T_N = 82.78$ kNm
3-ph short circuit torque	$T_{3\text{-phase short circuit}} = 8.35 * T_N$

2-ph short circuit torque	$T_{2\text{-phase short circuit}} = 9.79 * T_N$
unbalanced current withstand capability	$I_2^2 t = 20 \text{ s}$
$S_{100} = I_{f100} / I_f$ d-axis saturation	1.1
$S_{120} = I_{f120} / (1.2 * I_f)$ d-axis saturation	1.67
I_f = field current at 100% terminal voltage on air gap line I_{f100} = field current at 100% terminal voltage on open-circuit saturation curve I_{f120} = field current at 120% terminal voltage on open-circuit saturation curve	

TABLE II

GENERATOR OVERVOLTAGE WITHSTAND CHARACTERISTIC

voltage (%)	time (s)
120	3
125	1
130	0.4
140	0.3
145	0.2
155	0.1

TABLE III
GENERATOR AVR

parameter	value
K_P AVR proportional gain	40
K_I AVR integral gain	30
K_D AVR derivative gain	6.7
K_A AVR actuator gain	0.99
T_D derivative time constant	0.01 s
V_{RMAX} saturation max. limit	10.5 p.u.
V_{RMIN} saturation min. limit	0.0 p.u.
V_{RBASE}	15.90 V
T_A AVR actuator time constant	0.0001 s

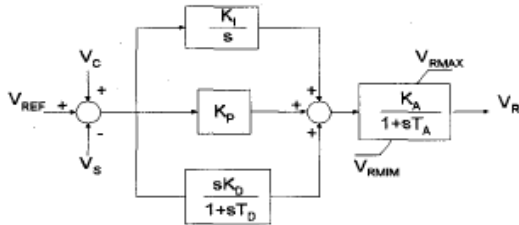


Fig. 15. AVR transfer function typical representation

TABLE IV
GENERATOR EXCITER

IEEE 421.5 AC brushless type model parameter	value
K_E	1.00
T_E	0.229 s
S_E ($E_{FD} = 8.8 \text{ p.u.}$)	0.02
S_E ($E_{FD} = 4.6 \text{ p.u.}$)	0.02
E_{FDMIN}	0 p.u.
E_{FDMAX}	15.31 p.u.
E_{FDBASE}	23.25 V
I_{FDBASE}	253.08 A

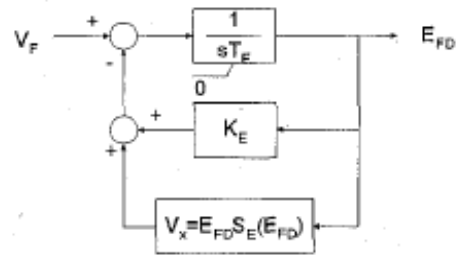


Fig. 16. Exciter transfer function typical representation (simplified version of IEEE Std. 421.5 AC8B excitation system model)

B. Distribution Transformer and Cable Data

TABLE V
MAIN TRANSFORMER

parameter	value
S_r rated power	12500 kVA (base power)
Z short circuit impedance	6% (referred to S_r)
V_{1n}/V_{2n} rated voltage ratio	11 kV / 6.3 kV
I_1 primary winding current	656 A (base current)
I_o no-load current at rated primary voltage	0.03 p.u. (20 A at 11 kV side)
P_o no-load losses at rated primary voltage	0.0008 p.u. (10 kW)
P_{cc} short circuit losses	0.00656 p.u. (82 kW)
I_m inrush current (from ideal supply network)	5000 A r.m.s. (at 11 kV)
L_o air-core inductance	1.027 H

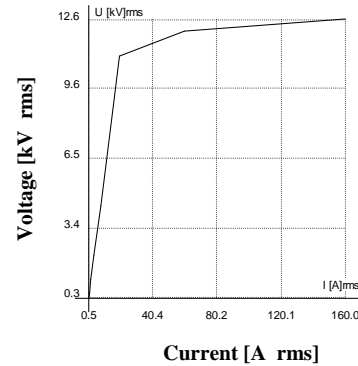


Fig. 17. Main transformer magnetization curve

TABLE VI
MAIN TRANSFORMER EQUIVALENT ELECTRICAL CIRCUIT

parameter	value
L_1 winding leakage inductance (at primary voltage side)	0.9188 mH
R_1 winding resistance (at primary voltage side)	0.03176 ohm
L_2 winding leakage inductance (at secondary voltage side)	0.3014 mH
R_2 winding resistance (at secondary voltage side)	0.010418 ohm
R_o no-load losses resistance (at primary voltage side)	12100 ohm

The parameters of the transformer equivalent electrical

circuit in Table VI are derived from manufacturer no-load and short circuit tests data reported in Table V, following the general procedure reported in [1].

TABLE VII
CABLE

equipment	parameters
cable feeder from generator to main distribution transformer	300 m length
	240 mm ² cross section
	3-core copper conductors 3 parallel runs
	R _c = 0.0072 ohm resistance / phase
	X _c = 0.01 ohm reactance / phase

- [14] IEEE Standard for Cylindrical-Rotor 50 Hz and 60 Hz Synchronous Generators Rated 10 MVA and Above, IEEE Std. C50.13, Dec. 2005.
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VII. ACKNOWLEDGMENT

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