



The current can also be taken from supplying transformer, if only one centralized type digital fault recorder is installed at the substation. The fault location technique is based on the fundamental frequency component of voltages and currents measured at the line terminal. The assumption of the algorithm is that the network impedance is obtained for particular symmetrical components. The investigation on the accuracy of the fault location algorithms in case non-symmetrical structure of the line conductors is beyond the scope of this paper.

In the following sections, short description of the proposed algorithm as well as distance to fault estimation for different fault types are briefly discussed. The impact of Distributed Generation (DG) on the distance to fault location on Distribution Network is mentioned. The simulation for a particular network is described and showed that the proposed algorithm has sufficient accuracy and robustness.

## II. THE PROPOSED ALGORITHM

The presented method overcomes the above mentioned difficulties by utilizing the following two-step procedure for fault location in distribution networks (Fig. 2).

- First, the equivalent positive- ( $\underline{Z}_{1k}^f$ ) and zero-sequence ( $\underline{Z}_{0k}^f$ ) impedance of the network is computed in pre-fault steady-state for all  $k = 1, \dots, M$  nodes of the network based on existing topology, loads and feeder parameters. These values represent positive- and zero-sequence impedance as seen from the substation up to a given  $k$  node of the network.
- Then after the fault the specific fault-loop parameters are calculated depending on the fault type (phase-to-phase or phase-to-ground) and the place of measurements (at the supplying transformer or at the faulty feeder).

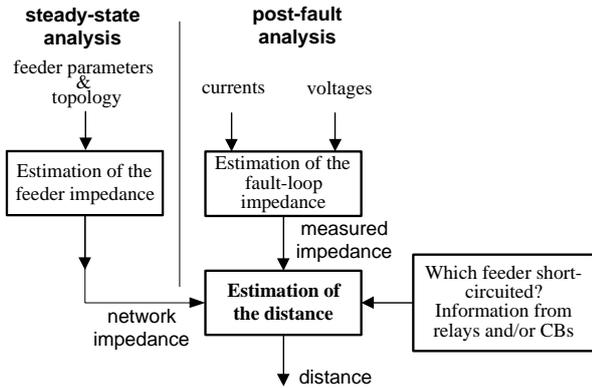


Fig. 2. Basic block diagram of the proposed fault location algorithm

The fault place is determined as a result of checking the following set of conditions for consecutive nodes of the network:

$$\text{Im}(\underline{Z}_{ek}^f) \geq 0, k = 1, 2, \dots, M \quad (1)$$

where:

$$\underline{Z}_{ek}^f = \begin{cases} \underline{Z}_{1k}^f - \underline{Z}_{1f} & \text{– for ph - to - ph fault} \\ \underline{Z}_{1k}^f + k_I \underline{Z}_{0k}^f - \underline{Z}_{1N} & \text{– for ph - to - grnd fault} \end{cases} \quad (2)$$

$$k_I = \frac{\underline{I}_{pN}}{3\underline{I}_{-p} - \underline{I}_{pN}}, \quad \underline{Z}_{1N} = \frac{3\underline{V}_{-ph}}{3\underline{I}_{-p} - \underline{I}_{pN}} \quad (3)$$

and:  $\underline{V}_{ph}$  - voltage at the faulty phase,  $\underline{Z}_{1f}$  - positive-sequence fault-loop impedance obtained from measurements,  $\underline{I}_p, \underline{I}_{pN}$  - adequately: fault-loop and residual currents obtained from measurements.

The final distance to fault will be chosen when the condition as in (1) is fulfilled. The method of calculation of the parameters ( $\underline{Z}_{1f}, \underline{I}_p, \underline{I}_{pN}$ ) depends on the place of measurement (at the substation or at the feeder).

### A. Measurements at the faulty feeder

As far as only one-end supplied radial networks are considered, the positive sequence fault-loop impedance is calculated according to well-known equations depending on the type of fault (Fig. 3).

- Phase-to-phase fault loop (phase-to-phase, phase-to-phase-to-ground or three phase fault):

$$\underline{Z}_k = \frac{\underline{V}_{pp}}{\underline{I}_{kpp}} \quad (4)$$

where:

$\underline{V}_{pp}$  - phase-to-phase fault loop voltage, for example:

$$\underline{V}_{pp} = \underline{V}_A - \underline{V}_B,$$

$\underline{I}_{kpp}$  - phase-to-phase fault loop current, for example:

$$\underline{I}_{kpp} = \underline{I}_{kA} - \underline{I}_{kB}.$$

- Phase-to-ground fault loop (a phase-to-ground fault):

$$\underline{Z}_k = \frac{\underline{V}_{ph}}{\underline{I}_{kph} + k_{kN} \underline{I}_{kN}} \quad (5)$$

where:

$\underline{V}_{ph}, \underline{I}_{kph}$  - voltage and current from a faulty phase,

$$k_{kN} = \frac{\underline{Z}'_0 - \underline{Z}'_1}{3\underline{Z}'_1} \quad (6)$$

$\underline{Z}'_0, \underline{Z}'_1$  - zero and positive sequence impedance per length of the faulted feeder,

$$\underline{I}_{kN} = \underline{I}_{kA} + \underline{I}_{kB} + \underline{I}_{kC} \quad (7)$$

### B. Measurements at the substation level

In this case we assume that the faulty line is identified. Moreover, some of the described below pre-fault parameters of the network are also known or can be estimated from the SCADA information.

Let a faulty feeder (say feeder  $k$ ) from the considered radial network has the pre-fault equivalent impedance  $\underline{Z}_{Lk}$  (Fig. 3). The remaining parallel-connected feeders are represented with the equivalent branch of the impedance  $\underline{Z}_L$  (i.e.  $1/\underline{Z}_L = 1/\underline{Z}_1 + 1/\underline{Z}_2 + \dots + 1/\underline{Z}_m$ ). Both  $\underline{Z}_{Lk}$  and  $\underline{Z}_L$  are assumed to be the positive sequence impedance.

The aim of the analysis is to determine the post-fault positive sequence impedance  $\underline{Z}_k$  under assumption that the

equivalent impedance  $\underline{Z}_L$  remains unchanged during a fault. The following equation is valid for the pre-fault state (Fig. 4):

$$\underline{Z}_{pre} = \frac{V_{pre}}{I_{pre}} = \frac{\underline{Z}_L \underline{Z}_{Lk}}{\underline{Z}_L + \underline{Z}_{Lk}} \quad (8)$$

where:  $V_{pre}, I_{pre}$  - are phase-to-phase or phase-to-ground (for symmetrical condition) variables.

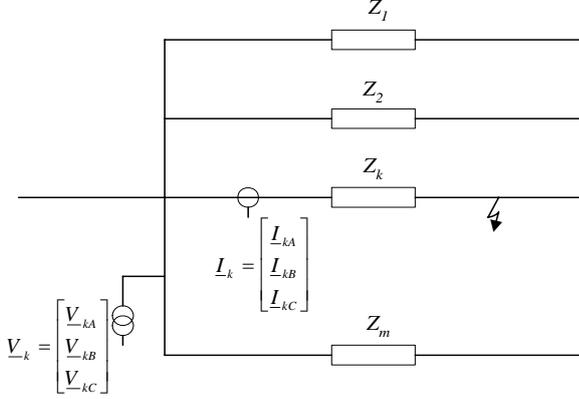


Fig. 3. Diagram of the network: measurements are taken in the faulty feeder

Two post-fault cases should be considered:

- Phase-to-phase fault loop (phase-to-phase, phase-to-phase-to-ground or three phase fault).

The positive sequence impedance seen from the substation is obtained from the equation:

$$\underline{Z} = \frac{V_{pp}}{I_{pp}} = \frac{\underline{Z}_L \underline{Z}_k}{\underline{Z}_L + \underline{Z}_k} \quad (9)$$

where:

$V_{pp}$  - phase-to-phase fault loop voltage, for example:

$$\underline{V}_{pp} = \underline{V}_A - \underline{V}_B,$$

$I_{pp}$  - phase-to-phase fault loop current taken at the substation,

$$\text{for example: } \underline{I}_{pp} = \underline{I}_A - \underline{I}_B,$$

Combining (8) and (9) yields:

$$\underline{Z}_k = \frac{\underline{Z} \underline{Z}_{pre}}{\underline{Z}_{pre} - \underline{Z}(1 - k_{zk})} \quad (10)$$

where:

$$k_{zk} = \frac{\underline{Z}_{pre}}{\underline{Z}_{Lk}} = \frac{S_{Lk}}{S_{\Sigma}} \quad (11)$$

$S_{Lk}$  - power in the faulty line in the pre-fault conditions,

$S_{\Sigma}$  - power in all the lines in the pre-fault conditions.

Combining (8) and (11) one also obtains

$$k_{zk} = \frac{\underline{Z}_L}{\underline{Z}_L + \underline{Z}_{Lk}} \quad (12)$$

The coefficient  $k_{zk}$  for each line is estimated on the basis of the pre-fault steady-state conditions. In a substation with a large number of feeders these coefficients are close to zero and change only a little, e.g. for two identical lines:  $k_{zk} = 0.5$  (if

only line reactance is taking into account) while for twenty lines:  $k_{zk} = 0.05$ . One should observe that, in general,  $k_{zk}$  is a complex number.

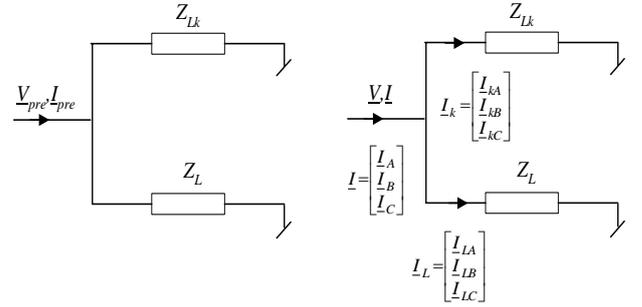


Fig. 4. Equivalent circuits of the distribution network

From equation (10) one can calculate the fault loop impedance using the measurements from the substation. Dividing numerator and denominator of (10) by  $\underline{Z}_{pre}$  and substituting (9) for  $\underline{Z}$ , equation (10) can be rewritten in a more convenient form:

$$\underline{Z}_k = \frac{V_{pp}}{\underline{I}_{pp} - (1 - k_{zk}) \frac{V_{pp}}{\underline{Z}_{pre}}} \quad (13)$$

- Phase-to-ground fault loop (phase-to-ground fault).

In the case of a phase-to-ground fault, the positive sequence fault loop impedance is calculated according to the second equation in (2). One can observe that as only a single phase-to-ground fault is considered (say, in feeder  $k$ ) the zero sequence current measured in the substation contains the faulty feeder current  $\underline{I}_{kN}$  and the zero-sequence current flows through capacitances of the healthy feeders  $\underline{I}_{CL}$ . Knowing voltage and current measurements at the substation and network parameters the fault loop impedance can be established in the similar way as for measurements from the feeder [6].

Summarizing the above derivations we can represent currents  $\underline{I}_p, \underline{I}_{pN}$  in (3) as follows:

- for measurements in the feeder:

$$\underline{I}_p = \underline{I}_{ph}, \quad (14a)$$

$$\underline{I}_{pN} = \underline{I}_N = \underline{I}_A + \underline{I}_B + \underline{I}_C \quad (14b)$$

- for measurements at the substation:

$$\underline{I}_p = \underline{I}_{ph} - (1 - k_{zk}) \frac{V_{ph} - V_0}{\underline{Z}_{pre}} \quad (15a)$$

$$\underline{I}_{pN} = \underline{I}_N - \frac{(1 - k_{zk0}) V_0}{-jX_{C0}} \quad (15b)$$

where:

$$k_{zk0} = \frac{X_{C0k}}{X_{C0}} = \frac{C_{0k}}{C_{C0}}$$

- $C_{0k}$  - zero-sequence capacitance of the faulty feeder,
- $C_{C0}$  - zero-sequence capacitance of all MV network, as in (11),

$Z_{pre} = \frac{V_{pre}}{I_{pre}}$  - pre-fault positive-sequence impedance at the

supplying transformer,  
index  $ph$  stands for the faulty phase.

Moreover, the positive sequence fault loop impedance  $Z_{1f}$  seen from the substation can be obtained from division of adequate voltage drop by difference of currents:

$$Z_{1f} = \frac{V_{pp}}{I_{pp}} \quad (16)$$

where:  $V_{pp}$  - phase-phase voltage,  $I_{pp}$  - phase-phase current, e.g. for  $A-B$  fault:  $V_{pp} = V_A - V_B$ ,  $I_{pp} = I_A - I_B$ .

Having the network impedance  $Z_{1k}^f$  and  $Z_{0k}^f$  for steady-state condition, and fault loop parameters:  $Z_{1f}$ ,  $k$ ,  $Z_{1N}$  given from measurements according to (2, 3) with respect to (4-6) it is possible to utilize the criterion (1) for distance to fault calculation.

### III. DISTANCE TO FAULT ESTIMATION

Distance to fault can be determined on the basis of criterion (1). In the searching algorithm two impedances: first one calculated for steady state while the second - obtained from measurements should be compared against matching the criterion. Two different algorithms are used depending of the fault loop type: phase-to-phase or phase-to-ground fault loops.

#### A. Phase-to-phase fault

In this case the measured impedance in (2) is represented by the positive-sequence impedance  $Z_{1f}$  which is compared with the impedance  $Z_{1k}^f$  for successive  $k = 1, \dots, M$  network nodes. Let us consider the phase-to-phase fault at node  $k$  of the network as in Fig. 5. It is assumed that the impedance  $Z_{1k}^f$  (positive-sequence network impedance as seen from the substation under assumption that the fault with no resistance occurs at the node  $k$ ) is known from steady-state calculations and  $Z_{1f}$  is obtained from measurements according to (6).

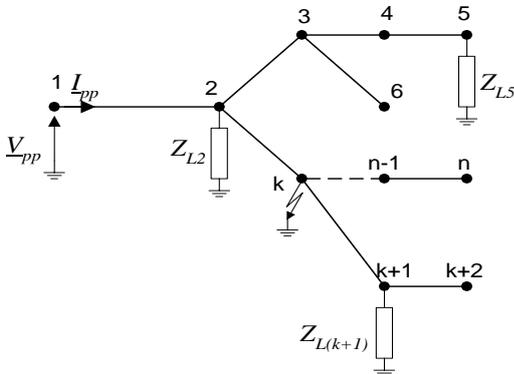


Fig. 5. Scheme of the network for phase-to-phase fault at node  $k$

For further analysis the fault loop seen from the substation is represented with the equivalent scheme as in Fig. 7. The following condition is fulfilled for this scheme:

$$Z_{1k}^f = (1-m)Z_{1k}^{f1} + \frac{mZ_{1k}^{f1}Z_{1k}^{f2}}{mZ_{1k}^{f1} + Z_{1k}^{f2}} \quad (17)$$

The separate impedance in (7) can be easily determined from the known impedance  $Z_{1k}^f$  by choosing the parameter  $m$  ( $0 < m \leq 1$ ).

Representation of the impedance  $Z_{1k}^f$  in a form as in Fig. 6 gives possibility to include the fault resistance into considered fault loop what is depicted in Fig. 6a. The residual impedance  $\Delta Z_f$  represents the equivalent impedance involved in the fault loop due to the fault resistance  $R_f$ , if the fault occurs at node  $k$  or behind them. The equivalent scheme for representing the impedance  $\Delta Z_f$  is shown in Fig. 6b. Here:

$Z_{1k}$  - equivalent shunt impedance at node  $k$ ,  
 $Z_L$  - impedance of the cable section between nodes  $k, k+1$ ,  
 $Z_{1(k+1)}^u$  - equivalent impedance of the network seen from the node  $k+1$  up to the end of the feeder.

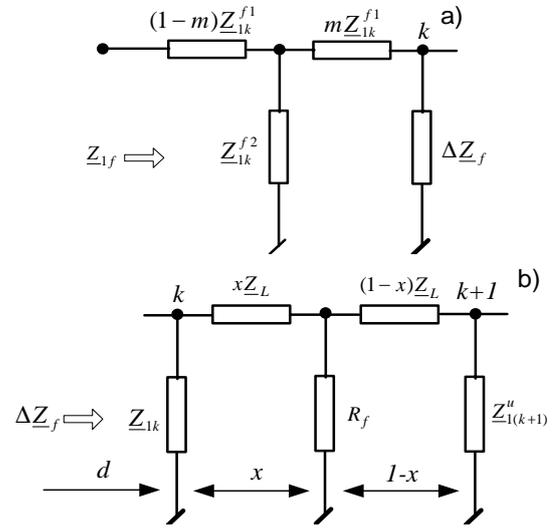


Fig. 6. Equivalent scheme of the fault loop: a - from the substation up to the fault point, b - beyond the fault point

The impedance  $Z_{1(k+1)}^u$  should be also calculated for steady-state conditions for all network nodes and stored in the database.

The distance to fault  $d_f$  (m) is determined as a sum of a distance  $d$  (m) from substation up to node  $k$  (Fig. 6b) and a distance  $xl_k$  (m) inside a given section:

$$d_f = d + xl_k \quad (18)$$

where  $l_k$  is a length of the section.

The algorithm for the distance  $x$  calculation is derived as follows:

1. The fault-loop impedance  $Z_{1f}$  measured at the substation meets the following relation (Fig. 6a):

$$Z_{1f} = (1-m)Z_{1k}^{f1} + \frac{\left(mZ_{1k}^{f1} + \Delta Z_f\right)Z_{1k}^{f2}}{mZ_{1k}^{f1} + \Delta Z_f + Z_{1k}^{f2}} \quad (19)$$

2. After rearranging (19) the value for residuum impedance can be obtained as:

$$\Delta Z_f = \frac{(Z_{1k}^{f1} - Z_{1f}) \left( m Z_{1k}^{f1} + Z_{1k}^{f2} \right) - (m Z_{1k}^{f1})^2}{m Z_{1k}^{f1} - Z_{1k}^{f2} - (Z_{1k}^{f1} - Z_{1f})} \quad (20)$$

3. The impedance  $\Delta Z_f$  represents the scheme seen from the node  $k$  up to the fault place (Fig. 6b) what can be determined as:

$$\Delta Z_f = \frac{Z_{1k} \left( x Z_L + \frac{R_f \left( (1-x) Z_L + Z_{1(k+1)}^u \right)}{R_f + (1-x) Z_L + Z_{1(k+1)}^u} \right)}{Z_{1k} + x Z_L + \frac{R_f \left( (1-x) Z_L + Z_{1(k+1)}^u \right)}{R_f + (1-x) Z_L + Z_{1(k+1)}^u}} \quad (21)$$

4. Right-hand sides of (20) and (21) should be equal what leads to determination of the unknown fault resistance:

$$R_f = x^2 A - x B + C \quad (22)$$

where:  $A = \frac{Z_L^2 (Z_{1k} - \Delta Z_f)}{M}$ ,  $B = \frac{Z_L (M + 2 \Delta Z_f Z_{1k})}{M}$ ,  
 $C = \frac{\Delta Z_f Z_{1k} (Z_L + Z_{1(k+1)}^u)}{M}$ ,  
 $M = (Z_{1k} - \Delta Z_f) (Z_L + Z_{1(k+1)}^u) - \Delta Z_f Z_{1k}$ .

5. Distance to fault  $x$  can be obtained from (22) under condition that the fault resistance takes real value:

$$\text{Im}(R_f) = x^2 A_i - x B_i + C_i = 0 \quad (23)$$

where:  $A_i = \text{Im}(A)$ ,  $B_i = \text{Im}(B)$ ,  $C_i = \text{Im}(C)$

After rearranging one obtains:

$$x_1 = \frac{B_i + \sqrt{p}}{2A_i}, \quad x_2 = \frac{B_i - \sqrt{p}}{2A_i}, \quad (24)$$

where:  $p = B_i^2 - 4A_i C_i$ .

First root of (24) takes unrealistic value so, finally, a distance to fault is determined as  $x = x_2$ .

### B. Phase-to-ground fault

For phase-to-ground faults the criterion for distance to fault estimation, defined by (1) with respect to the second equation in (2) is equivalent to the following condition (for simplicity the equality is considered):

$$Z_{1k}^f = Z_{1N} - k_I Z_{0k}^f \quad (25)$$

where:  $k_I$  and  $Z_{1N}$  are defined by (3).

The parameters  $k_I$  and  $Z_{1N}$  in (25) can be calculated from measurements whereas  $Z_{1k}^f$  and  $Z_{0k}^f$  are actual positive- and zero-sequence impedance of the assumed fault loop and are available from steady-state conditions.

Equivalent scheme of the fault loop circuit, which satisfies the condition (25) is similar to the phase-to-phase one (Fig. 7). Instead of  $Z_{1f}$  now the impedance combination  $Z_{1N} - k_I Z_{0k}^f$  is used. Bearing this in mind, the algorithm for a distance  $x$  (p.u.) to the fault at section  $k, k+1$  can be derived by repeating points 1-5 from the previous section. Final relation is represented by (22), where:

$$A = \frac{Z_{eL}^2 (Z_{ek} - \Delta Z_f)}{M}, \quad B = \frac{Z_{eL} (M + 2 \Delta Z_f Z_{ek})}{M},$$

$$C = \frac{\Delta Z_f Z_{ek} (Z_L + Z_{e(k+1)}^u)}{M},$$

$$M = (Z_{ek} - \Delta Z_f) (Z_{eL} + Z_{e(k+1)}^u) - \Delta Z_f Z_{ek},$$

$$Z_{eL} = \frac{2Z_{1L} + Z_{0L}}{3}, \quad Z_{ek} = \frac{2Z_{1k} + Z_{0k}}{3}, \quad Z_e^u = \frac{2Z_{e1}^u + Z_{e0}^u}{3}$$

Index  $e$  relates to the equivalent impedance of the scheme in Fig. 7b. The distance to fault is also calculated according to (18).

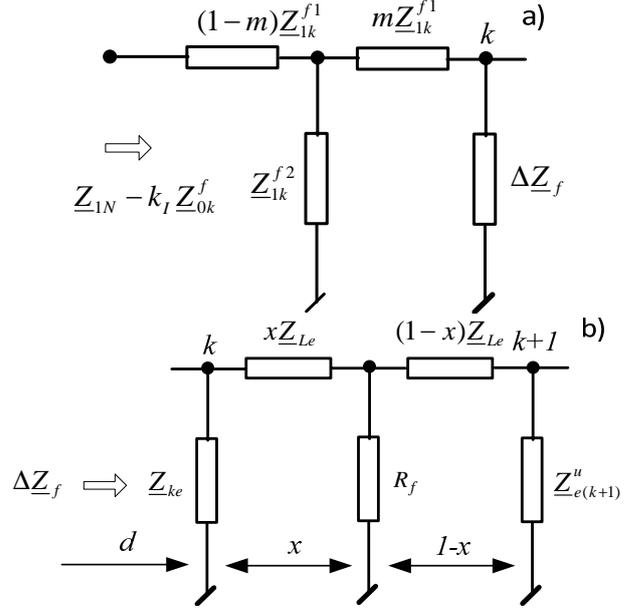


Fig. 7. Equivalent scheme for A-G fault-loop: a - from the substation up to the fault point, b - beyond the fault point.

## IV. SIMULATION RESULTS

The considered 10 kV substation is supplied from 150 kV system [6]. The cable network is operated in a radial way. Measurements of current are available at the supplying transformer or at the feeders. Cable shield is considered as grounded only at the load points.

For a distance to fault calculation the each feeder should be represented by the detail scheme with adequate line and load models. In the cable networks grounding system has different structure than feeders have (open cables may have connected grounding circuits), what should be also represented in the model. This requires representing all feeders connected in a given substation in the general simulation model. However, for proper post-fault transient analysis some simplifications can be introduced: - supplying system is described by steady-state parameters; - analyzed feeder is represented in detail; - all other feeders are represented by equivalent schemes with reproducing only the grounding system connections.

Example of the analyzed network is presented in Fig. 8. Cable sections are represented by appropriate  $\pi$ -schemes, while loads and equivalent circuits are reflected by  $R-L$  or  $R-L-C$  scheme.

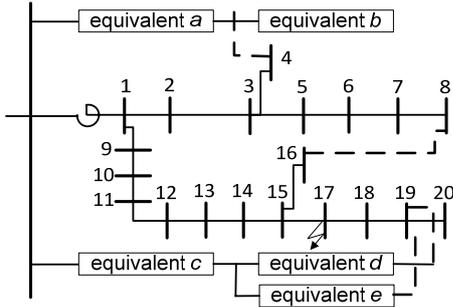


Fig. 8. Equivalent scheme of the analyzed network; dotted lines are for grounding system connection

EMTP/ATP model [8] of the analyzed network has been extensively used for investigation of the proposed fault location algorithm. The MV network consists of 16 feeders which, except of the particular analyzed feeder, are represented by their equivalent schemes. Let us consider an example of A-G (phase A to ground) fault at node 17 in the analyzed feeder (actual distance to fault - 8138 m, Fig. 8) with assuming the fault resistance  $R_f = 0.1 \Omega$ . Phase voltages and currents observed at the substation are presented in Fig. 9.

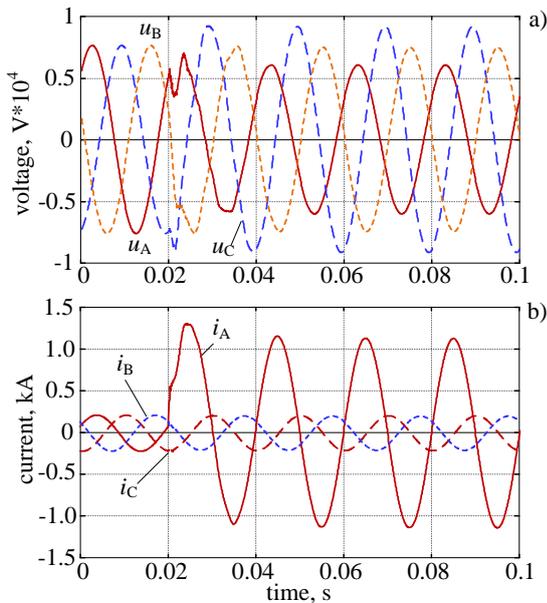


Fig. 9. Phase voltages (a) and currents (b) during A-G fault at node 17

Results of distance to fault estimation for 3-phase and phase-to-ground fault are gathered in Table 1. It can be seen that the algorithm gives quite good results.

## V. IMPACT OF DISTRIBUTED GENERATION ON FAULT LOCATION ESTIMATION

The MV Distribution Networks are supplied from HV/MV transforming substations. The electrical energy is then transported via a series of underground-cable and overhead line circuit to the customers. In some cases a Distributed Generation (DG) can be connected to the network. The particular line sections are of length from a few hundred meters to some dozens of kilometers. Although due to different loads and the network development, separate feeders are made up of sections

with different technical data: varying cable cross-section and OH line parameters. The configuration layout and complexity of distribution networks vary widely depending on the application [7]. In order to analyze thoroughly the effects of distributed generation on the requirements for the protection and fault location of distribution networks, detailed simulation studies including dynamic modeling of various types of DG technologies are necessary [9,10].

Table 1.

Type of fault	Fault resistance, $\Omega$	Obtained result, m	Error, m
3-phase	0.1	8145	+7
	2.0	8135	-3
A-G	0.1	8131	-7
	2.0	8081	-57

## VI. CONCLUSIONS

The presented fault location algorithm for MV Distribution Networks (DN) is based on impedance measurement principle and uses three-phase voltage and current phasor estimation as well as the network parameters. The algorithm was investigated and proved on the basis of voltage and current data obtained from versatile EMTP/ATP simulations.

The developed algorithm utilizes the current measurements delivered from the faulty feeder or from the substation. In the latter case the estimation error depends on accuracy of pre-fault condition determination in the MV substation. The distance to fault estimation error depends on the accuracy of measurements as well as cable parameters. Extensive tests by utilizing fault data from EMTP/ATP detailed model confirmed the accuracy and validity of the method presented in the paper.

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