Parameter Estimation for a Jiles-Atherton based Current Transformer core model

Y. Chen, D. S. Ouellette, P. A. Forsyth, P.G. McLaren, Yi Zhang

Abstract-- The Jiles-Atherton (J-A) based current transformer (CT) core model provides accurate modelling of hysteresis and saturation effects and can effectively represent the remanence flux in CT cores. The disadvantage of the J-A CT is the relevant parameters are not easy to obtain. This paper develops a methodology to estimate the parameters for a J-A CT model from a B-H loop. The B-H loop is relatively easy to get from measurement or can be generated from an EMT digital simulation of a generic CT model using a B-H curve and other commonly available data. Validation of the proposed methodology has been performed by comparing the simulation results of the J-A CT model using the estimated parameters to the results using a generic CT model using the B-H loop as input.

Keywords: **Jiles-Atherton hysteresis, B-H loop, remanence flux**

I. INTRODUCTION

The Jiles-Atherton (J-A) based current transformer (CT) core model accurately represents the remanence flux [1][2], the effects of which can be critical to behavior of protective relays. The J-A CT model can also accurately represent long term saturation and hysteresis effects such as would occur with geomagnetic induced currents (GIC) entering a transformer [3]. Although its advantage has been well recognized and its simulation model has been developed in some simulation tools for over a decade, the J-A CT hasn't been widely used in Electromagnetic Transient (EMT) based simulations. The main restriction on the use of the J-A based CT is that the parameters required by the model are often not available. The parameters are based on the physics of the core and not on the electrical characteristics familiar to the power system engineers and the electrical community as a whole. The parameters are also interlinked in such a way that the set of possible combinations can be quite large. Thus significant changes in the shape of the hysteresis loop may result from small parameter variations.

The generic CT model that is widely used in EMT simulation tools is easy to model because it is not computationally intensive and is easy to use since the input parameters are more commonly known. The downside to this model is that it may not be accurate for very low currents, there is no long-term remanence, and it was not developed for the interconnection of CT secondaries such as in a busbar differential protection scheme. Therefore there is no possibility for interaction between multiple CT's.

The J-A CT would be used more in EMT simulations if the parameters needed by the model could be estimated from some parameters that are easier to obtain, such as the B-H or V-I curve for the core. These curves are normally provided by the manufacturer or can be created using measurements normally taken during on-site field commissioning.

II. SUMMARY OF THE MANUSCRIPT

This paper develops a methodology to estimate the J-A CT core model parameters from a measured *B-H* loop. The EMT simulations of a generic CT model, with B-H or V-I curves as input, are used to produce the B-H loop. The B-H loop is in turn used as input for a least square algorithm to estimate the parameters (including a_1, a_2, a_3, and M_s) of the J-A CT. The work in this paper was conducted on an RTDS® real time digital simulator. The paper is organized as follows: section III briefly introduces the J-A CT model on the RTDS Simulator. Section IV presents the detailed methodology to estimate the J-A CT parameters based on the B-H loop. Section V provides validation of the methodology. Finally section VI gives the conclusion.

The proposed method in this paper provides a method to easily obtain parameters for a superior CT model, thus allowing simulations to be undertaken which include a properly modeled long term remanence and interaction between CTs. The usefulness of this can be appreciated when testing high impedance bus differential protection schemes which require properly modeled interaction between coupled CT's [4]. The varistor or MOV placed across the neutral branch of a high impedance differential scheme can add additional non-linear effects. The "original" J-A CT model is the one used in reference [4] and shown to give excellent agreement with recordings (including remanent flux effects) taken from a synthetic test bed.

III. INTRODUCTION OF THE JILES-ATHERTON CT MODEL

Jiles and Atherton used a phenomenological based mathematical representation to accurately represent ferromagnetic material behavior in a soft magnetic material [1][5]. Equation (1) represents the relationship between *the M-H* loop and *B-H* loop.

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$$B=\mu_0(M + H) \qquad (1)$$

where B is magnetic flux density, M is magnetization, H is

$$\mu_0 = 4\pi * 10^{e-7}\ \frac{N}{A^2}$$

magnetic field intensity, and $\qquad$ (the permeability of free space).

The magnetization relationship between B and H is replaced by the anhysteretic magnetization curve (M_{an}) between H_e and M

$$M_{an} = M_s f(H_e) \qquad (2)$$

Where H_e *is* effective field and defined as $H_e = H + \alpha M$, α is the inter-domain coupling, M_s is the saturation magnetization.

J-A modeled the saturation characteristic using a modified Langevin function to produce the familiar sigmoid type curve for $f(H_e)$, giving the following equation.

$$M_{an} = M_s f(H_e) = M_s \left[\coth(\frac{H_e}{\alpha}) - \frac{a}{H_e}\right] \qquad (3)$$

where a is anhysteretic loss.

Equation (3) and J-A theory produces 2 terms in M as follows:

$$M = M_{irr} + M_{rev} \qquad (4)$$

The first term M_{irr} is due to the pinning of magnetic domains by discontinuities in the material structure. The second term M_{rev} is due to domain wall bending in an elastic manner. J-A built upon these fundamental relationships to arrive at the final set of equations to construct the B-H and M-H loops in (5).

$$\frac{dM}{dH} = (1-c)\frac{M_{an} - M_{irr}}{k\delta - \alpha(M_{an} - M_{irr})} + C\frac{dM_{an}}{dH} \qquad (5)$$

where k is irreversible loss, c is reversible/irreversible proportions, δ is sign of $\frac{dH}{dt}$.

When the J-A algorithm was implemented for a CT model, the CT core characteristics could not be accurately modeled using the Langevin function [2]. The J-A CT model on the RTDS Simulator uses an improved anhysteretic function instead of the Langevin function which accurately models the shoulder area of the anhysteretic magnetization [2]. The details of the improved J-A model equations used in the RTDS Simulator are described as follows:

$$\frac{dM}{dH} = \begin{cases} \dfrac{c\frac{dM_{an}}{dH_e} + \dfrac{\frac{M_{an-M}}{\delta k_{mod}} - \frac{\alpha(M_{an}-M)}{\mu_0}}{1-c}}{1 - \alpha c \frac{dM_{an}}{dH_e}} &, \text{if } (M_{an} - M)\delta \geq 0 \quad (6) \\[2em] \dfrac{c\frac{dM_{an}}{dH_e}}{1 - \alpha c \frac{dM_{an}}{dH_e}} &, \text{otherwise} \qquad (7) \end{cases}$$

Where M_{an} is anhysteretic magnetization, α *and* *c* are constants, k_{mod} is dynamic adjustment of the domain pining parameter, δ and k_{mod} are defined as follows:

$$\delta = sign(H - H_{prev}) \qquad (8)$$

$$k_{mod} = \begin{cases} k\left(1 - \beta\left(\frac{M}{M_s}\right)^2\right) &, \text{if } M\delta \geq 0 \qquad (9) \\ k &, \text{otherwise} \end{cases}$$

The improved anhysteretic magnetization function is given by the following:

$$M_{an} = sign(H_e)M_s \frac{a_1\left|H_e\right| + \left|H_e^2\right|}{a_3 + a_2\left|H_e\right| + \left|H_e\right|^2} \qquad (10)$$

The improved anhysteretic magnetization function must have the following properties and will be satisfied, provided that $a_1 > 0, a_2 > a_1, a_3 > 0$.

$$H_e \xrightarrow{\lim} 0\, M_{an} = 0,\ H_e \xrightarrow{\lim} \infty\, M_{an} = M_s,\ \frac{dM_{an}}{dH_e} \geq 0 \forall H_e$$

The derivative M_{an} with respect to H_e is given by the following:

$$\frac{dM_{an}}{dH_e} = M_s \frac{a_1 a_3 + b a_3\left|H_e\right| + (a_2 - a_1)\left|H_e\right|^2}{\left(a_3 + a_2\left|H_e\right| + \left|H_e\right|^2\right)^2} \qquad (11)$$

IV. DETERMINATION OF HYSTERESIS PARAMETERS

The object in this paper is to extract the parameters for a J-A CT model from a B-H loop. The B-H loop can be obtained from the manufacturer/field measurements or from digital simulation of a generic CT model using B-H curve input and plotting the B-H loop.

Fig. 1 shows the flowchart of the estimation methodology used to obtain the J-A CT model parameters from a piecewise linear B-H curve. First, the B-H loop is obtained from the digital simulation of a generic CT model containing a 14 point B-H curve. Second, the parameters of the J-A CT are estimated with a Least Square fitting algorithm using data from the B-H loop.
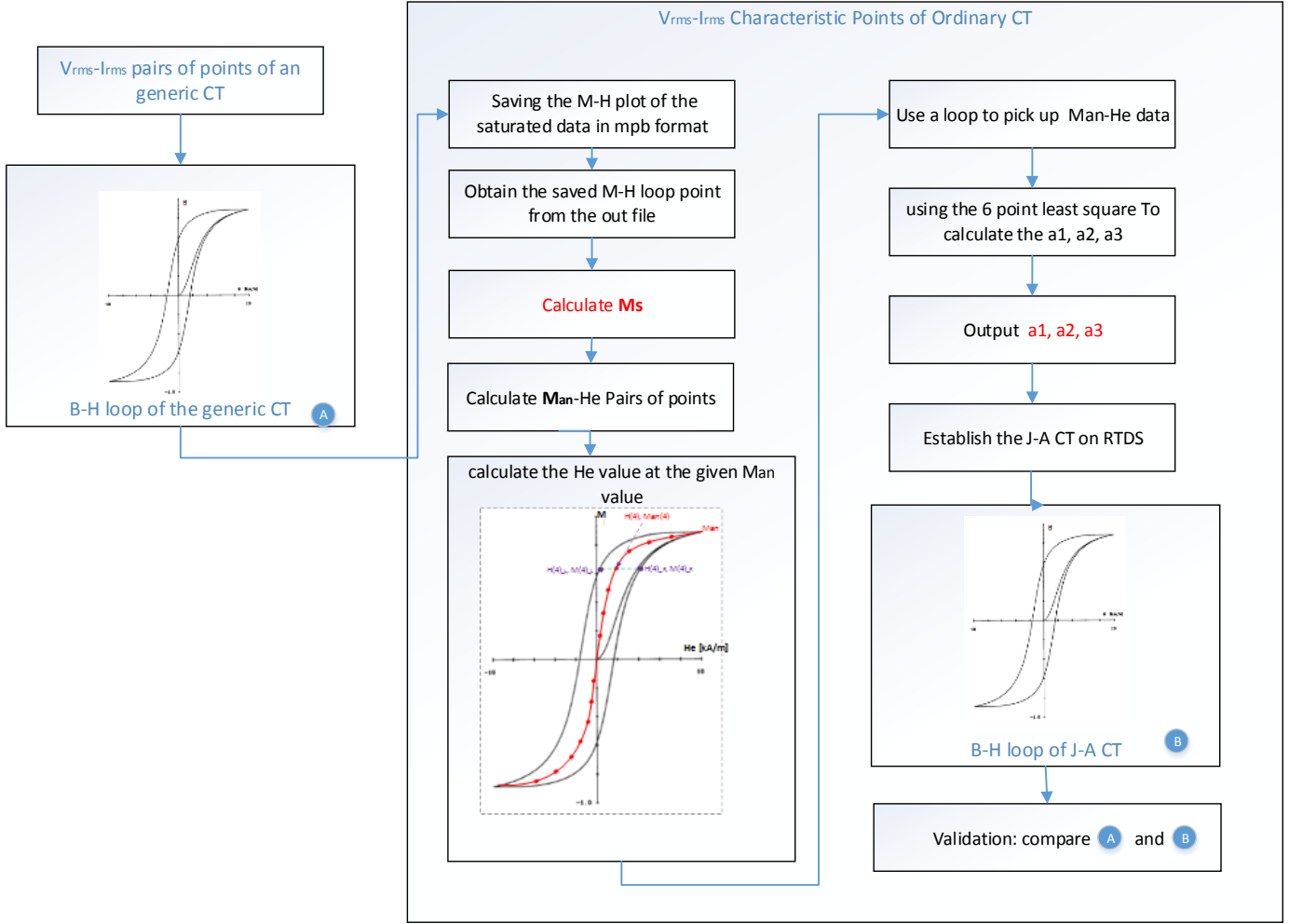
Fig. 1. Flowchart of the estimation methodology for parameters of a J-A CT model. It should be noted that the algorithm can also be performed using the B-H loop data from field measurements or manufacturer's data.

A. Obtaining the B-H Loop Parameters

In order to estimate the parameters for J-A CT, one needs to have a measured B-H loop as shown in Fig. 2. Ideally the B-H loop can be obtained from field measurements. An alternative way to obtain the B-H loop is from the digital simulation. It is relatively easy to get the point by point B-H curve or V-I reference curve from the manufacturer or by field measurements. Once we have the B-H or V-I curve of the CT to be modelled, EMT simulations can be co
nducted using the generic CT model. The B-H loop can be obtained as a result of the simulation. In this paper, the B-H loop is obtained by EMT simulation of the generic CT on the RTDS Simulator. The core saturation characteristic is represented by the dynamic solution of an integer power series equation given as follows:

$$H(t) = B_1 * B(t) + B_2 * B^{35}(t) \qquad (12)$$

where coefficients B_1 and B_2 are determined from curve fitting techniques using pairs of points from the reference curve.
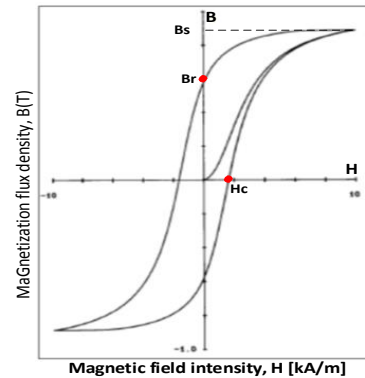

Fig. 2. B-H loop

B. Determine the Saturation Magnetization M_S

The easiest parameter to obtain is the saturation parameter M_S. It is often known for a particular material, so can be taken directly from the material data sheets. This parameter can also be obtained from the measured B-H (M-H) loop parameters.

Equation (1) is used as a basis to convert the B-H loop data to M-H loop data with the following:

$$M = \frac{B}{\mu_0} - H \qquad (13)$$

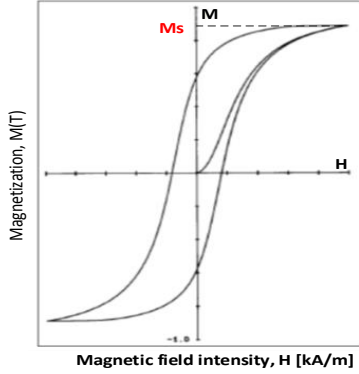M_S, saturation magnetization is the maximum value of M shown in Fig. 3.

Fig. 3. M-H hysteresis loop

C. Determination of a_1, a_2, a_3

The most important procedure of the estimation is to calculate a_1, a_2, and a_3 used in (10). In fact, *the M_{an}-H_e curve can be determined if M_s, a_1, a_2, a_3 are known.*

From $H_e = H + \alpha M$ the M_{an}-H_e curve can be determined as shown in Fig. 4. Assuming α is at a default value, one can obtain the M_{an}-H_e curve, in which H_e is the averaging point of the left side and right side of curve H_e as in Equation (14) and (15):

$$H_e(n) = 0.5 * \left(H_e(n)_{\_R} + H_e(n)_{\_L} \right) \qquad (14)$$

$$M_{an}(n) = M(n) \qquad (15)$$

Where $M_{an}(n)$ is the value of anhysteretic magnetization M_{an} at point n; $H_e(n)_{\_R}$ is the H value of M-H_e curve(right side) at point n; $H_e(n)_{\_L}$ is the H value of M-H_e curve(left side) at point n.
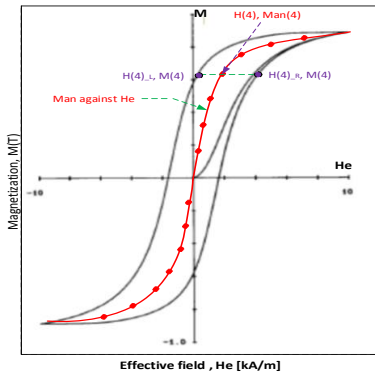

Fig. 4. M_{an}-H_e hysteresis loop (n=4)

Equation (16) can be obtained from (10), with a_1, a_2 and a_3 as unknowns, when M_{an} and H_e is known:

$$\left(|H_e| \right) a_1 - \left(\frac{M_{an}}{M_s} |H_e| \right) a_2 + \left(\frac{M_{an}}{M_s} \right) a_3 = sign(H_e) \left(\frac{M_{an}}{M_s} |H_e|^2 - |H_e|^2 \right) \qquad (16)$$

And in which the following restriction needs to be satisfied: $a_1 > 0, a_2 > a_1, a_3 > 0$.

Now estimates of a_1, a_2 and a_3 are computed with a least square method. Six samples of M_{an} and H_e data are used to compute a_1, a_2 and a_3 as follows:

Equation (17) is established with the six samples of M_{an} and H_e in which there are 3 unknowns but 6 equations.

$$\begin{bmatrix} |H_e|_{(1)} & -\left(\frac{M_{an}}{M_s}|H_e|\right)_{(1)} & \left(\frac{M_{an}}{M_s}\right)_{(1)} \\ \vdots & \vdots & \vdots \\ |H_e|_{(6)} & -\left(\frac{M_{an}}{M_s}|H_e|\right)_{(6)} & \left(\frac{M_{an}}{M_s}\right)_{(6)} \end{bmatrix}_{6x3} \times \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}_{3x1}$$

$$= \begin{bmatrix} \left(sign(H_e) \left(\frac{M_{an}}{M_s}|H_e|^2 - |H_e|^2 \right) \right)_{(1)} \\ \vdots \\ \left(sign(H_e) \left(\frac{M_{an}}{M_s}|H_e|^2 - |H_e|^2 \right) \right)_{(6)} \end{bmatrix}_{6x1} \qquad (17)$$

The 6 equations are defined into X and Y terms and the 3 unknowns are solved using the least square solution.

Define X and Y:

$$X = \begin{bmatrix} |H_e|_{(1)} & -\left(\frac{M_{an}}{M_s}|H_e|\right)_{(1)} & \left(\frac{M_{an}}{M_s}\right)_{(1)} \\ \vdots & \vdots & \vdots \\ |H_e|_{(6)} & -\left(\frac{M_{an}}{M_s}|H_e|\right)_{(6)} & \left(\frac{M_{an}}{M_s}\right)_{(6)} \end{bmatrix}_{6x3}$$

$$Y = \begin{bmatrix} \left(sign(H_e) \left(\frac{M_{an}}{M_s}|H_e|^2 - |H_e|^2 \right) \right)_{(1)} \\ \vdots \\ \left(sign(H_e) \left(\frac{M_{an}}{M_s}|H_e|^2 - |H_e|^2 \right) \right)_{(6)} \end{bmatrix}_{6x1}$$

We have:

$$X_{6x3} * \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}_{3x1} = Y_{6x1}$$

Using the least square solution we have:

$$X^T_{(3x6)} * X_{(6x3)} * \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}_{3x1} = X^T_{(3x6)} * Y_{(6x1)}$$

$$\begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}_{3x1} = \left(X^T{}_{(3x6)} * X_{(3x6)} \right)^{-1} * \left(X^T{}_{(3x6)} * Y_{(6x1)} \right) \quad (18)$$

The solution of (18) will be further checked by the restriction of $a_1 > 0, a_2 > a_1, a_3 > 0$. If this restriction is met the solution is considered to be optimal. If the restriction is not met, 6 new pairs of data points are chosen and equation 18 is solved until the restriction $a_1 > 0, a_2 > a_1, a_3 > 0$ is satisfied.

D. Determination of Hysteresis Parameters c, α, k

Reference [1] provided detailed descriptions on how to calculate the hysteresis parameters c, α, k in equation (6). According to [1], the coercivity is determined by the amount of pinning and hence by the parameter k. Modification of the k parameter will change the coercivity point H_c (the width of the hysteresis loop).

Figure 5 shows the B-H loop with different k parameter values. Increasing k value in a reasonable range results in a larger coercivity point. One can obtain the k value approximately by trial and error based on the comparison between the EMT simulation results of the J-A CT model and the known B-H loop.
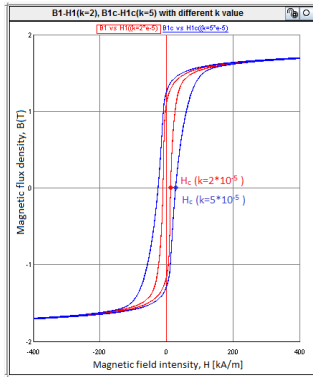

Fig. 5. B-H loopwith different k value: Blue – B-H loop with K=2*10^-5, Red – B-H loop with K=5*10^-5

According to [1], the remanence point M_r is dependent on α and other parameters. If the other parameters are known, the remanence can be used to calculate α [1]. Modification of the parameter α will change the remanence point and the inclination of the hysteresis loop.

Fig. 6 shows the M-H curve with different α values. Basically a larger α value corresponds to a larger remanence point. Similarly the value can be adjusted according to the remanence point of the hysteresis loop of the known M-H loop.
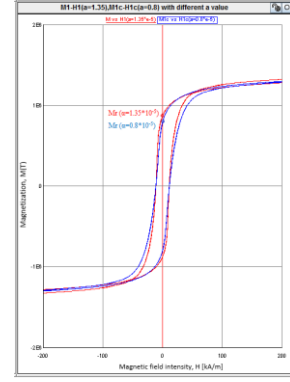

Fig. 6. M-H loop with different α value: Red - M-H loop with α=1.35*10^-5, Blue - M-H loop with α=0.8*10^-5

The reversible component of magnetization due to reversible wall bending and reversible translation is determined in the model by the coefficient c. This can be calculated from the ratio of the initial normal susceptibility to the initial anhysteretic susceptibility [6]. According to the test results, parameter c does not contribute much to the B-H loop when the CT is saturated. So one can perform the above test and get the a_1, a_2, a_3 , k and α with c set at the default value, then adjust the value of c by comparing the known B-H loop and B-H loop from the simulation. Fig. 7 shows the $B\text{-}H$ loops are almost identical with different c values.
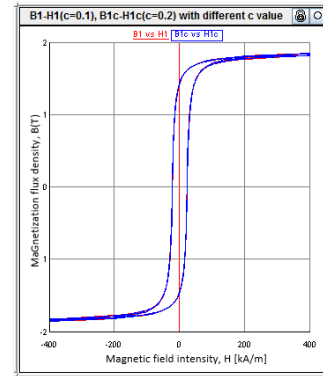

Fig. 7 B-H loop with different c value: Blue - B-H loop with c=0.1, Red - B-H loop with c=0.2

V. VALIDATION

The proposed methodology is validated by comparing the RTDS simulation results of J-A CT model with the estimated parameters (section IV.A) with the generic CT (section IV.B) with the B-H curve input. The simulation results demonstrated that the proposed methodology can effectively estimate parameters for a J-A CT.

A. Validate the calculations with a J-A based CT

Fig. 8 shows the test case used for validation consisting of a source, bus, breaker, load and two J-A CT models used for comparison. The top J-A CT is used with the original data and the bottom J-A CT model is used with the estimated data. The 3 phase breaker current is used as the CT primary current.
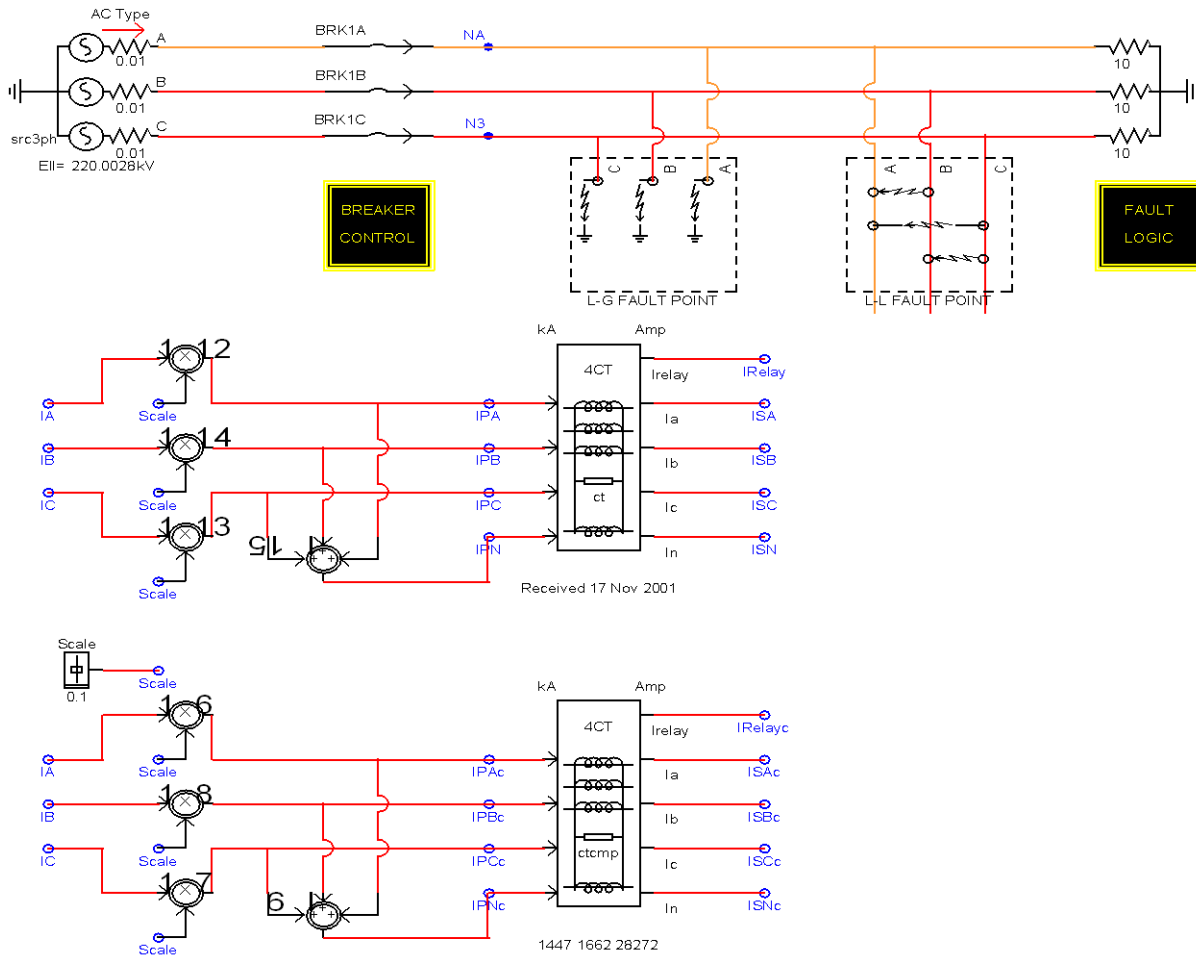
Fig. 8. Test case for parameter Comparison

One can get the estimated hysteresis parameters through a RTDS RunTime Script programmed with the estimation algorithm. (Other platforms will have to program the estimation algorithm.) Table I shows the comparison between the original parameters and estimated parameters.

TABLE I
COMPARISON BETWEEN ORIGINAL PARAMETERS AND ESTIMATED PARAMETERS

	a_1	a_2	a_3	Ms
Original	2784	3274	20978	1.72
Estimate	2792.182	3291.53	20866.66	1.72

It is obvious that the calculation result is very close to the original data. The simulation results of CTs with the original data and the estimated data in table I are compared. Fig. 9 and Fig. 10 show the waveform comparison of an original J-A CT model against a test J-A CT model using estimated parameters derived by the proposed method. The black curve is the original J-A CT model and the red curve is the estimated J-A CT model. The B-H loop, secondary current (Is), B(t) and H(t) are almost identical in the comparison.
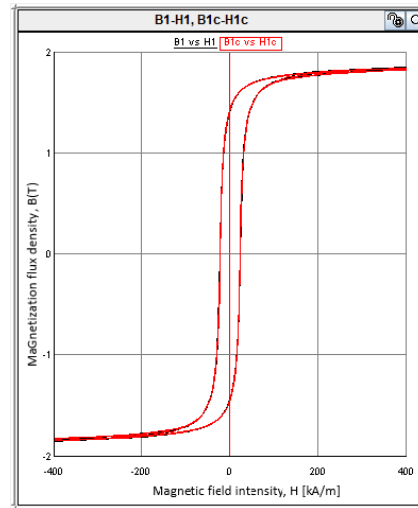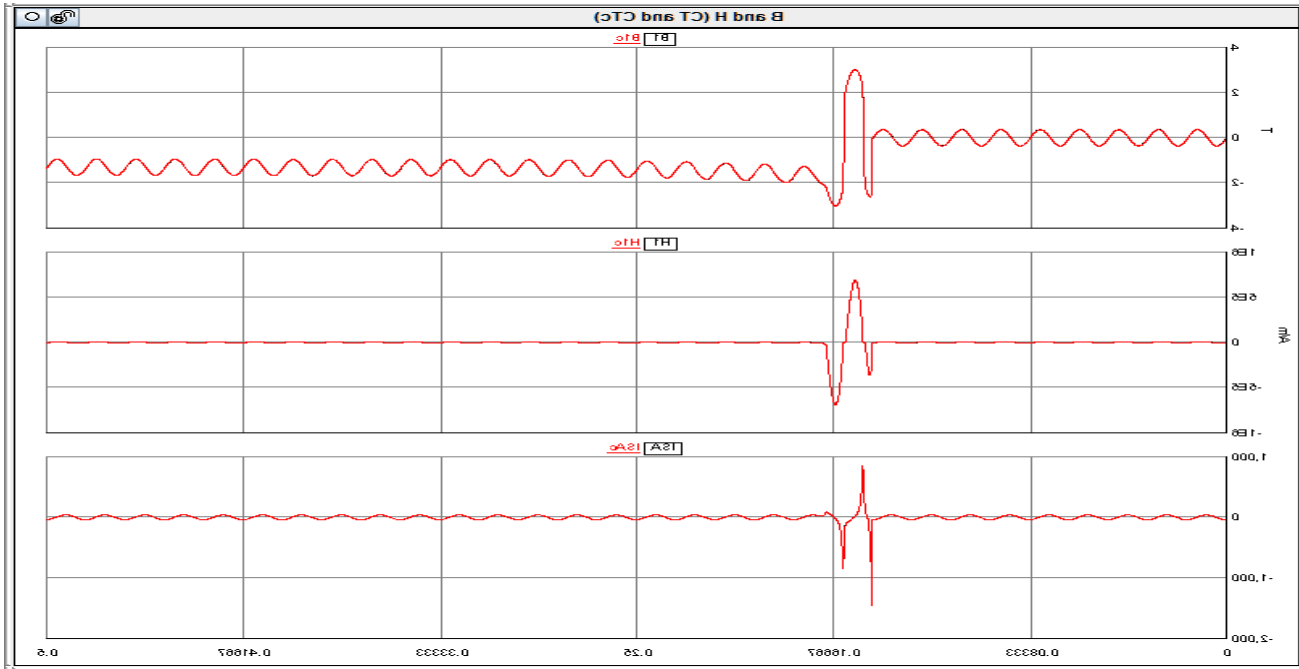

Fig. 9. Zoomed Simulated B-H Loop Comparison

Fig. 10. Waveform Comparison of J-A CT with estimated parameters and Original J-A CT

B. Validate the calculations with the Generic CT

The test case shown in Fig. 11 is similar to the test case shown in Fig. 8. The only difference between Fig. 8 and Fig. 11 is the top CT is a generic CT with B-H/V_{rms}-I_{rms} data instead of the J-A CT with original data.
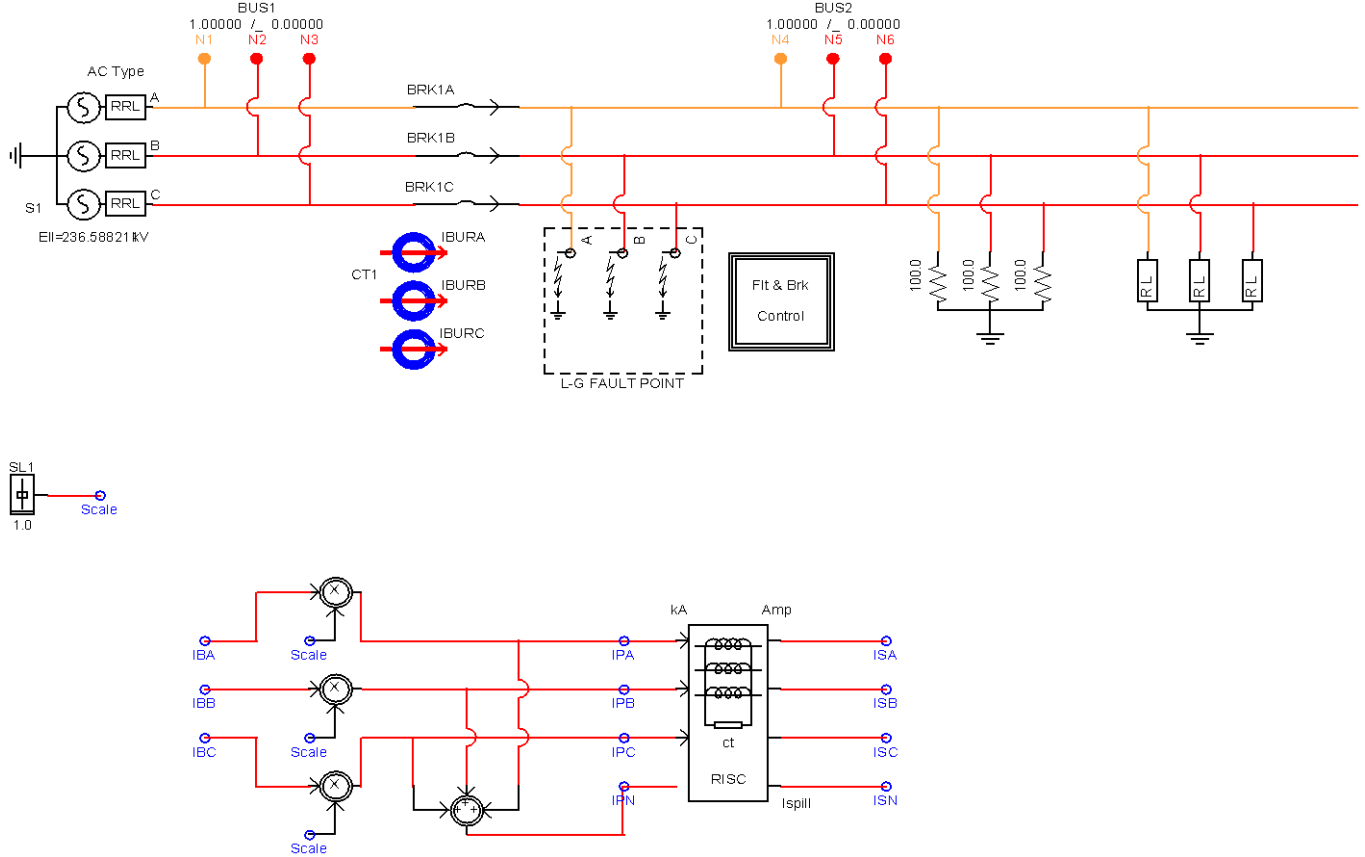

Fig. 11. Test case for parameter Comparison with Generic CT

One can get the estimated hysteresis parameters through a RTDS RunTime Script programmed with the estimation algorithm. Parameter k, α and c are determined according to

section IV.D (i.e., the coercivity, remanence point and B-H loop). Table II shows estimated J-A parameters with $k=5.8X10^{-5}$, $\alpha=1.35x10^5$, and $c=0.1$.

TABLE II
ESTIMATED J-A HYSTERESIS PARAMETERS OF GENERIC CT

	a_1	a_2	a_3	Ms
Estimate	986.5068	1167.663	12097.973	1.55

Fig. 12 and Fig 13. show the waveform comparison of a generic CT model and the test J-A CT model using estimated parameters derived from the B-H curve of the generic CT model. The black curve is the generic CT model and the red curve is the test J-A CT model. The B-H loop is almost identical except at the shoulder (knee point) where improved anhysteretic function is used in RTDS J-A CT model. The B-H loop, secondary current (Is), B(t) and H(t) are almost identical in comparison.
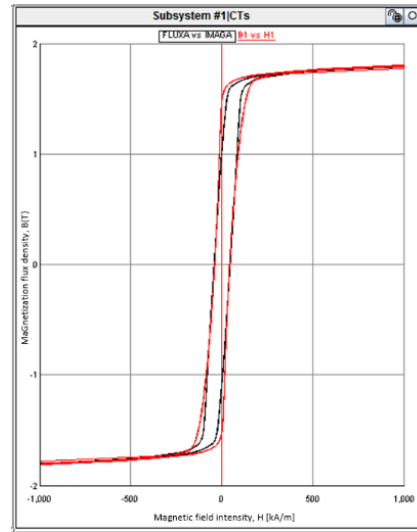

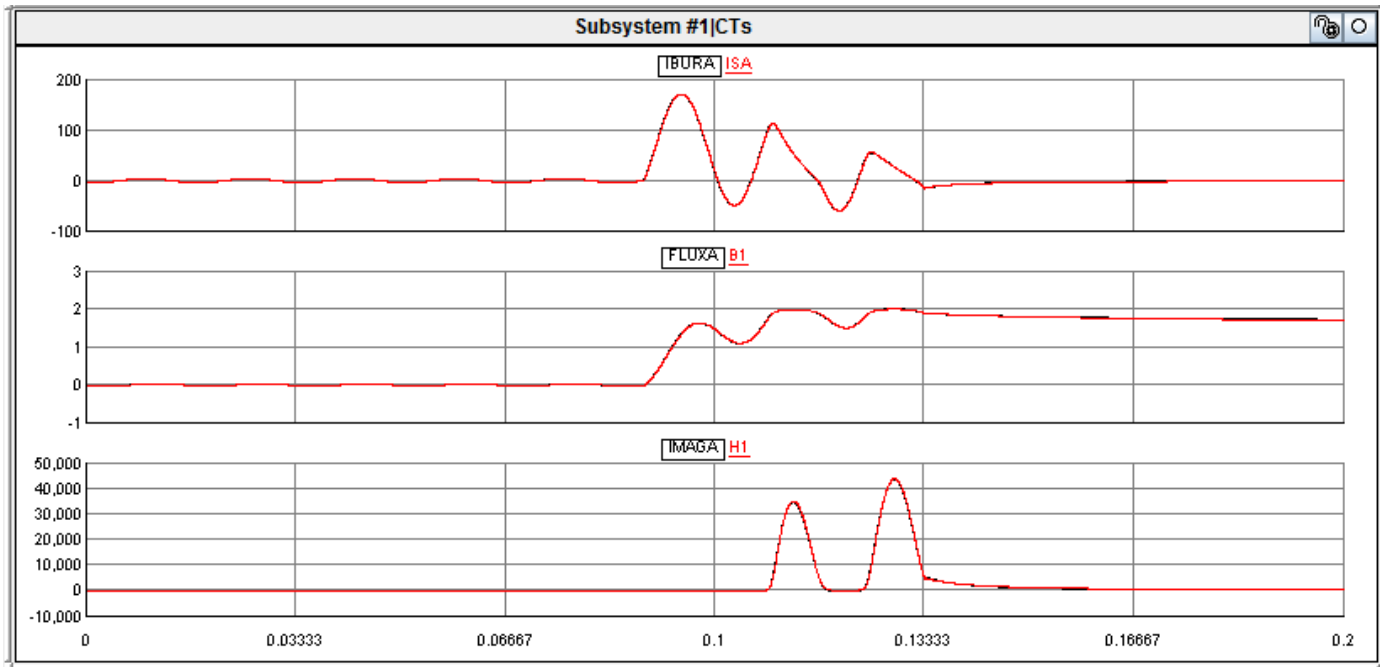Fig. 12. Zoomed Simulated B-H Loop Comparison of Generic CT

Fig. 13. Waveform Comparison of J-A CT with estimated parameters and Generic CT

VI. CONCLUSIONS

A new methodology has been developed to estimate the parameters for a J-A CT model. The method derives the parameters of J-A CT based on some commonly available B-H or V-I curves. The method has been implemented on the RTDS Simulator but can be implemented on other platforms. The simulation results demonstrated the proposed estimation algorithm gives accurate results, thus making it possible for the J-A CT to be more widely used in studies and simulations involving CTs [4] or other types of transformers [3].

VII. REFERENCES

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