

Modal transformation matrices for phasor measurement based line parameter estimations

Kaur Tuttelberg and Jako Kilter

Abstract—This paper provides insight into choosing the most correct transformation matrices for the estimation of transmission line parameters from PMU measurements at both ends of a transmission line. The paper continues the study on a method of estimating the parameters of multi-conductor transmission lines by applying modal transformations that decouple the measured voltage and current phasors. For transposed lines, the unique preferred transformation matrix is derived, which results in the smallest errors in case of deviations from ideal conditions. Moreover, it is shown that for untransposed transmission lines the modal transformation matrices can be calculated once (from line design data) and assumed constant in PMU-based line parameter estimation applications.

Keywords: Transmission line measurements, Transmission line matrix methods, Modal analysis, Phasor measurement units

I. INTRODUCTION

THE increasing installation of phasor measurement units (PMU) is enabling simultaneous voltage and current phasor measurements, often at both ends of a transmission line. This has led to the suggestion of various applications enabling the online monitoring of transmission line parameters. Such applications are based on common line models; however, the equations are solved for a reverse problem. When usually the parameters of the line model are known, then in this case, the parameters themselves are estimated from the inputs and outputs of the model.

Transmission lines can be modelled with different degrees of approximation. Likewise, the calculation of line parameters from phasor measurements can be based on models with different accuracy. Most of the approaches have been based on modelling the line as independent conductors, disregarding any coupling between phases [1]–[3]. Kurokawa et al. have suggested to solve the problem by modal decoupling, avoiding such a simplification [4], [5]. In this case, a linear transformation is made to transform the phase quantities into a modal domain where they are uncoupled. After solving the line equations in the modal domain, the parameters in the phase domain can be estimated after an inverse transformation [6].

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K. Tuttelberg and J. Kilter are with the Department of Electrical Power Engineering, Tallinn University of Technology, Tallinn, Estonia (e-mail of corresponding author: kaur.tuttelberg@ttu.ee).

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In general, the transformation matrix, which is used in modal decoupling, is not known as it is itself dependent on line parameters. However, for an ideally transposed line, there exists an infinite set of suitable transformation matrices, independent of any line parameters. In [4], a variation of the Clarke transformation matrix (which coincides with one of the correct transformation matrices of ideally transposed lines) is suggested as the unique solution for a simplified estimation procedure. As a part of this paper, it is shown why this is, in fact, the preferred choice for realistic, imperfectly transposed lines.

In order to improve the accuracy of this method in the untransposed case, it is proposed to compute the transformation matrices of untransposed lines from line design data (geometry, conductor resistance, etc.) under known conditions. It is verified that environment-dependent changes affect the modal transformation matrices negligibly, and thus, they can be assumed constant for the line in such applications. This way, the conventional estimates of line parameters are used to find suitable transformation matrices, which can then be applied to estimate line parameters from measurements under changing conditions. This approach is more accurate than simply using the transformation matrix of transposed lines.

Improved knowledge about the modal transformation matrices can be used in estimating line parameters from phasor measurements, e.g. in applications based on PMUs. Such applications enable the parameters of the line model to be monitored in real-time. Contrary to common solutions, the studied approach models the lines more accurately and the resulting estimates include mutual quantities between phases. The measured parameters can be used as inputs in various models to study and plan the operation of the line, i.e. it would be possible to run simulations on models using a full set of actual measured parameters of transmission lines.

The paper is structured as follows. Section II describes the modelling of multi-conductor transmission lines and modal analysis of such models. Section III presents how the most correct modal transformation matrices can be found. Conclusions are summarised in section IV.

II. MATHEMATICAL BACKGROUND

A. Model of a three-phase transmission line

Let's consider the matrix notation of a three-phase distributed parameter transmission line [6], [7]. The phase volt-

ages and currents at any point of the line are given as

$$\mathbf{u}(l) = \begin{bmatrix} \underline{U}_a(l) \\ \underline{U}_b(l) \\ \underline{U}_c(l) \end{bmatrix}, \quad \mathbf{i}(l) = \begin{bmatrix} \underline{I}_a(l) \\ \underline{I}_b(l) \\ \underline{I}_c(l) \end{bmatrix}, \quad (1)$$

where l is the distance toward the beginning of the line and $\underline{U}_i(l)$ and $\underline{I}_i(l)$ are the voltage and current phasors in phase i .

According to Kirchhoff's laws these quantities are related by [6]

$$\frac{d\mathbf{u}(l)}{dl} = \mathbf{Z}\mathbf{i}(l), \quad (2)$$

$$\frac{d\mathbf{i}(l)}{dl} = \mathbf{Y}\mathbf{u}(l), \quad (3)$$

which can be further differentiated into

$$\frac{d^2\mathbf{u}(l)}{dl^2} = \mathbf{Z}\mathbf{Y}\mathbf{u}(l), \quad (4)$$

$$\frac{d^2\mathbf{i}(l)}{dl^2} = \mathbf{Y}\mathbf{Z}\mathbf{i}(l). \quad (5)$$

The matrices in the equations above are the reduced impedance and admittance matrices, denoted as

$$\mathbf{Z} = \begin{bmatrix} \hat{z}_{aa} & \hat{z}_{ab} & \hat{z}_{ac} \\ \hat{z}_{ba} & \hat{z}_{bb} & \hat{z}_{bc} \\ \hat{z}_{ca} & \hat{z}_{cb} & \hat{z}_{cc} \end{bmatrix}, \quad \mathbf{Y} = \begin{bmatrix} \hat{y}_{aa} & \hat{y}_{ab} & \hat{y}_{ac} \\ \hat{y}_{ba} & \hat{y}_{bb} & \hat{y}_{bc} \\ \hat{y}_{ca} & \hat{y}_{cb} & \hat{y}_{cc} \end{bmatrix}. \quad (6)$$

These matrices are obtained from the self and mutual impedances and admittances of phase and neutral conductors by Kron's reduction [7]. For example, an untransposed three-phase transmission line with two neutral wires would be described by an impedance matrix of the form

$$\mathbf{Z}_0 = \begin{bmatrix} z_{aa} & z_{ab} & z_{ac} & z_{au} & z_{aw} \\ z_{ba} & z_{bb} & z_{bc} & z_{bu} & z_{bw} \\ z_{ca} & z_{cb} & z_{cc} & z_{cu} & z_{cw} \\ z_{ua} & z_{ub} & z_{uc} & z_{uu} & z_{uw} \\ z_{wa} & z_{wb} & z_{wc} & z_{wu} & z_{ww} \end{bmatrix} = \begin{bmatrix} \mathbf{Z}_1 & \mathbf{Z}_2 \\ \mathbf{Z}_3 & \mathbf{Z}_4 \end{bmatrix} \quad (7)$$

where z_{ii} is the self impedance of conductor i and z_{ik} is the mutual impedance of conductors i and k , both given per unit length, with a, b, c being the phases and u and w the neutral conductors. The reduction is carried out as [7]

$$\mathbf{Z} = \mathbf{Z}_1 - \mathbf{Z}_2\mathbf{Z}_4^{-1}\mathbf{Z}_3. \quad (8)$$

Admittances are found via the potential coefficient matrix

$$\mathbf{P}_0 = \begin{bmatrix} \mathbf{P}_1 & \mathbf{P}_2 \\ \mathbf{P}_3 & \mathbf{P}_4 \end{bmatrix}, \quad (9)$$

which has the same structure as \mathbf{Z}_0 [7]. The potential coefficient matrix is similarly reduced as

$$\mathbf{P} = \mathbf{P}_1 - \mathbf{P}_2\mathbf{P}_4^{-1}\mathbf{P}_3, \quad (10)$$

yielding the admittance matrix

$$\mathbf{Y} = j\omega\mathbf{C} = j\omega\mathbf{P}^{-1}. \quad (11)$$

Under known conditions, the reduced impedance and ad-

mittance matrices \mathbf{Z} and \mathbf{Y} can be approximated by the following (or similar) equations based on line design data. The impedances are computed as

$$z_{ik} = r_{ik} + r_d + j\omega \frac{\mu_0}{2\pi} \ln \frac{D_e}{D_{ik}} \left[\frac{\Omega}{\text{m}} \right], \quad (12)$$

where r_{ik} is the resistance of the conductor if $i = k$ and zero when $i \neq k$. Carson's earth return corrections [8] are included in $r_d = 9.869 \cdot 10^{-7} \Omega/\text{m}$ and $D_e = 658.5 \sqrt{\rho/f} \text{ m}$, where ρ is ground resistivity and f is the fundamental frequency [7]. D_{ik} is the distance between two conductors if $i \neq k$ and the GMR of the conductor when $i = k$ [7]. The GMR (geometric mean radius) of a conductor depends on its radius and geometry of the bundle, if it is bundled [9].

Elements of the potential coefficients matrix p_{ik} are computed as

$$p_{ik} = \frac{1}{2\pi\epsilon_0} \ln \frac{H_{ik}}{D'_{ik}} \left[\frac{\text{m}}{\text{F}} \right], \quad (13)$$

where D'_{ik} is like D_{ik} but with the GMR of an individual conductor taken equal to its radius; H_{ik} is the distance from conductor i to the image of k . The image of a conductor is an imaginary conductor located at the same depth under ground as the real conductor is above the ground (in average height) [7]. For $i = k$ the distance $H_{ik} = H_{ii}$ is the double average height of the conductor.

These kinds of calculations are always based on some simplifications; moreover, under varying operating conditions and over time factors such as the resistance of conductors, geometry of lines, resistivity of earth, etc. change. However, having measured values of voltage and current phasors available from both ends of the line would enable the estimation of line parameters, even as they vary in time.

Equations (4) and (5) are theoretically solvable with respect to the matrices $\mathbf{Z}\mathbf{Y}$ and $\mathbf{Y}\mathbf{Z}$, however, the solutions are found as matrix exponents and the number of unknowns exceeds the number of equations [10]. Due to the coupling between phases, there are no general solutions with respect to phase quantities

$$\frac{d^2\underline{U}_a(l)}{dl^2} = (\underline{ZY})_{11}\underline{U}_a(l) + (\underline{ZY})_{12}\underline{U}_b(l) + (\underline{ZY})_{13}\underline{U}_c(l), \quad \text{etc.} \quad (14)$$

This means that the data from phasor measurements is insufficient for the estimation of the parameters of a three-phase transmission line, as long as the phases are considered to be coupled.

The system of equations given by (4) and (5) would have a unique solution if the matrices $\mathbf{Z}\mathbf{Y}$ and $\mathbf{Y}\mathbf{Z}$ were diagonal. The line parameter identification problem can be solved by applying modal transformations. This, however, requires that the correct transformation matrices are known [6].

B. Modal analysis

A modal transformation produces linear transformations of voltage and current phasors [11]. Phase quantities are replaced

by modal quantities; the voltage and current vectors become

$$\mathbf{u}'(l) = \begin{bmatrix} \underline{U}_0(l) \\ \underline{U}_1(l) \\ \underline{U}_2(l) \end{bmatrix}, \quad \mathbf{i}'(l) = \begin{bmatrix} \underline{I}_0(l) \\ \underline{I}_1(l) \\ \underline{I}_2(l) \end{bmatrix}, \quad (15)$$

where indices 0, 1, and 2 denote the modes.

Phase quantities and modal quantities are associated by a transformation matrix, so that $\mathbf{u}(l) = \mathbf{T}_u \mathbf{u}'(l)$ and $\mathbf{i}(l) = \mathbf{T}_i \mathbf{i}'(l)$ [6]. Substituting these into Eqs. (4) and (5) and assuming the transformation matrices to be independent of l , it can be written that

$$\frac{d^2 \mathbf{u}'(l)}{dl^2} = \mathbf{T}_u^{-1} \mathbf{Z} \mathbf{Y} \mathbf{T}_u \mathbf{u}'(l), \quad (16)$$

$$\frac{d^2 \mathbf{i}'(l)}{dl^2} = \mathbf{T}_i^{-1} \mathbf{Y} \mathbf{Z} \mathbf{T}_i \mathbf{i}'(l). \quad (17)$$

Modal decoupling is based on the eigendecomposition of the $\mathbf{Z} \mathbf{Y}$ and $\mathbf{Y} \mathbf{Z}$ matrices. This means that the transformation matrices \mathbf{T}_u and \mathbf{T}_i are chosen so that [6], [11]

$$\langle \lambda_u \rangle = \mathbf{T}_u^{-1} \mathbf{Z} \mathbf{Y} \mathbf{T}_u, \quad (18)$$

$$\langle \lambda_i \rangle = \mathbf{T}_i^{-1} \mathbf{Y} \mathbf{Z} \mathbf{T}_i, \quad (19)$$

where $\langle \lambda_u \rangle$ and $\langle \lambda_i \rangle$ are diagonal matrices containing eigenvalues of the $\mathbf{Z} \mathbf{Y}$ and $\mathbf{Y} \mathbf{Z}$ matrices, respectively. It is known that both the \mathbf{Z} and \mathbf{Y} matrices are symmetrical, so that $\mathbf{Z} \mathbf{Y} = (\mathbf{Y} \mathbf{Z})^T$ and $\langle \lambda_u \rangle = \langle \lambda_i \rangle$.

Following that, the propagation matrix is defined as $\mathbf{\Gamma} = (\mathbf{Z} \mathbf{Y})^{1/2}$ with eigenvalues denoted as γ_0 , γ_1 , and γ_2 [6]. According to properties of eigenvalues, it can be seen that $\langle \gamma^2 \rangle = \langle \lambda_u \rangle = \langle \lambda_i \rangle$. The modal characteristic impedance matrix is defined as $\langle \mathbf{z}_c \rangle = \langle \gamma \rangle^{-1} \mathbf{T}_u^{-1} \mathbf{Z} \mathbf{T}_i$ [6]. The diagonality of $\langle \mathbf{z}_c \rangle$ can be verified based on previous definitions.

Eqs. (16) and (17) can be rewritten as

$$\frac{d^2 \mathbf{u}'(l)}{dl^2} = \langle \gamma^2 \rangle \mathbf{u}'(l), \quad (20)$$

$$\frac{d^2 \mathbf{i}'(l)}{dl^2} = \langle \gamma^2 \rangle \mathbf{i}'(l). \quad (21)$$

Solving these differential equations yields the equations for the three-phase distributed parameter line, written in matrix form as [6]

$$\mathbf{u}'(l) = \cosh\langle \gamma l \rangle \mathbf{u}'(0) + \langle \mathbf{z}_c \rangle \sinh\langle \gamma l \rangle \mathbf{i}'(0), \quad (22)$$

$$\mathbf{i}'(l) = \langle \mathbf{z}_c \rangle^{-1} \sinh\langle \gamma l \rangle \mathbf{u}'(0) + \cosh\langle \gamma l \rangle \mathbf{i}'(0), \quad (23)$$

where

$$\cosh\langle \gamma l \rangle = \begin{bmatrix} \cosh(\gamma_0 l) & & \\ & \cosh(\gamma_1 l) & \\ & & \cosh(\gamma_2 l) \end{bmatrix}, \text{ etc.} \quad (24)$$

Equations (22) and (23) can be solved for modal quantities γ_m and \underline{z}_{cm} (elements of $\langle \gamma \rangle$ and $\langle \mathbf{z}_c \rangle$) as [4]

$$\gamma_m = \frac{1}{L} \operatorname{arccosh} \left[\frac{\underline{U}_m(L) \underline{I}_m(L) + \underline{U}_m(0) \underline{I}_m(0)}{\underline{U}_m(0) \underline{I}_m(L) + \underline{U}_m(L) \underline{I}_m(0)} \right], \quad (25)$$

$$\underline{z}_{cm} = \frac{\underline{U}_m(0) \sinh(\gamma_m L)}{\underline{I}_m(L) - \underline{I}_m(0) \cosh(\gamma_m L)}. \quad (26)$$

where m denotes the mode (0, 1, and 2) and L is the length of the transmission line.

The reduced impedance and admittance matrices in the phase domain can then be found by an inverse transform [6]

$$\mathbf{Z} = \mathbf{T}_u \langle \gamma \rangle \langle \mathbf{z}_c \rangle \mathbf{T}_i^{-1}, \quad (27)$$

$$\mathbf{Y} = \mathbf{T}_i \langle \gamma \rangle \langle \mathbf{z}_c \rangle^{-1} \mathbf{T}_u^{-1}. \quad (28)$$

III. MODAL TRANSFORMATION MATRICES

A. Transposed lines

Modal transformations have mostly been studied for applications in frequency-dependent modelling of transmission lines [11]–[14]. It is well known that a variation of the Clarke transformation matrix (sometimes referred to as the travelling-wave transformation matrix) coincides with one of the possible transformation matrices of an ideally transposed line [15], [16]. It is also known that for untransposed lines, the unique transformation matrices closely resemble the Clarke transformation matrix [6], [12]. However, to the authors' knowledge, no analytical treatment has been given to why this one matrix should be the preferred choice.

In order to analyse the transformation matrices, we return to the eigendecomposition specified in Eqs. (18) and (19); we denote $\mathbf{A} = \mathbf{Z} \mathbf{Y}$, and multiply (18) by \mathbf{T}_u from the left to write

$$\mathbf{T}_u \langle \lambda \rangle = \mathbf{A} \mathbf{T}_u. \quad (29)$$

This is the eigenvalue equation for \mathbf{A} with eigenvalues λ_0 , λ_1 , and λ_2 ; the eigenvectors \mathbf{v}_0 , \mathbf{v}_1 , and \mathbf{v}_2 form the columns of the matrix \mathbf{T}_u . Thus, the modal transformation matrix \mathbf{T}_u is specified by the eigenvectors of \mathbf{A} [6]. Similarly, the other transformation matrix \mathbf{T}_i is specified by the eigenvectors of $\mathbf{Y} \mathbf{Z} = \mathbf{A}^T$.

As a first approach in analysing the eigenvectors, it would be reasonable to start off from a perfectly transposed (balanced) system, i.e. we specify

$$\mathbf{A}''' = \begin{bmatrix} \underline{z}_i & \underline{z}_k & \underline{z}_k \\ \underline{z}_k & \underline{z}_i & \underline{z}_k \\ \underline{z}_k & \underline{z}_k & \underline{z}_i \end{bmatrix} \cdot \begin{bmatrix} \underline{y}_i & \underline{y}_k & \underline{y}_k \\ \underline{y}_k & \underline{y}_i & \underline{y}_k \\ \underline{y}_k & \underline{y}_k & \underline{y}_i \end{bmatrix} = \begin{bmatrix} q & s & s \\ s & q & s \\ s & s & q \end{bmatrix}, \quad (30)$$

where q and s are still complex numbers but the underline is omitted for simpler notation. The matrix \mathbf{A}''' has only two unique eigenvalues $\lambda_0''' = q + 2s$ and $\lambda_1''' = \lambda_2''' = q - s$, which yield an infinite set of transformation matrices formed by the eigenvectors $[1, 1, 1]^T$ and $[1, x, -1 - x]^T$, $x \in \mathbb{C}$. All of these possible matrices are essentially correct for transformations with ideally transposed lines.

In practical cases, the transposition is not ideal. Thus, it should be determined if there is a transformation matrix that is most accurate when the transposition is realistic, i.e. it is only close to ideal. To study this, a modified, or perturbed, matrix is assumed

$$\mathbf{A}'' = \begin{bmatrix} q & s & s' \\ s & q & s \\ s' & s & q \end{bmatrix}, \quad (31)$$

where $s' = s + \Delta s$ is s perturbed by a small quantity Δs .

The eigenvalues of matrix \mathbf{A}'' are expressed as

$$\begin{aligned}\lambda_0'' &= q - s', \\ \lambda_1'' &= \frac{1}{2} \left(2q + s' - \sqrt{8s^2 + s'^2} \right), \\ \lambda_2'' &= \frac{1}{2} \left(2q + s' + \sqrt{8s^2 + s'^2} \right).\end{aligned}\quad (32)$$

The corresponding eigenvectors are

$$\begin{aligned}\mathbf{v}_0'' &= [-1, 0, 1]^\top, \\ \mathbf{v}_1'' &= \left[1, \frac{3ss' - s\sqrt{8s^2 + s'^2}}{2s^2 + s'^2 - s'\sqrt{8s^2 + s'^2}}, 1 \right]^\top, \\ \mathbf{v}_2'' &= \left[1, \frac{3ss' + s\sqrt{8s^2 + s'^2}}{2s^2 + s'^2 + s'\sqrt{8s^2 + s'^2}}, 1 \right]^\top.\end{aligned}\quad (33)$$

It can be seen that the perturbed matrix \mathbf{A}'' has sets of three unique eigenvalues and eigenvectors.

In the definition of \mathbf{A}'' we assumed $s' = s + \Delta s$, so we examine the limits when Δs approaches zero (keeping in mind that s is complex). We see that for the middle element of \mathbf{v}_1''

$$\lim_{\Delta s \rightarrow 0} \frac{3ss' - s\sqrt{8s^2 + s'^2}}{2s^2 + s'^2 - s'\sqrt{8s^2 + s'^2}} = \begin{cases} -2 & \text{if } s > 0 \\ 1 & \text{if } s \leq 0 \end{cases} \quad (34)$$

and for \mathbf{v}_2''

$$\lim_{\Delta s \rightarrow 0} \frac{3ss' + s\sqrt{8s^2 + s'^2}}{2s^2 + s'^2 + s'\sqrt{8s^2 + s'^2}} = \begin{cases} 1 & \text{if } s \geq 0 \\ -2 & \text{if } s < 0 \end{cases}. \quad (35)$$

Also, it is clear that for ideally transposed lines the two transformation matrices are equal $\mathbf{T}_i = \mathbf{T}_u$.

When $\Delta s \rightarrow 0$, the eigenvectors form a unique modal transformation matrix which, if scaled and ordered accordingly, becomes a variation of the Clarke transformation matrix [12]

$$\mathbf{T}_C = \begin{bmatrix} 1/\sqrt{3} & 1/\sqrt{6} & 1/\sqrt{2} \\ 1/\sqrt{3} & -2/\sqrt{6} & 0 \\ 1/\sqrt{3} & 1/\sqrt{6} & -1/\sqrt{2} \end{bmatrix}. \quad (36)$$

This is a simple way to show that in cases when the line is imperfectly transposed, the Clarke transform matrix is close to the correct solution and the preferred choice out of all the generally applied transformation matrices of an ideally transposed line.

It appears that for an ideally transposed transmission line a single preferred modal transformation matrix (nevertheless, still arbitrarily scaled) can be chosen. This matrix remains constant even when line parameters themselves change in time due to environmental or other changes, as long as the line is transposed.

The importance of this can be demonstrated on a simple example. To form the modal transformation matrix for transposed lines, the first two eigenvectors are taken as $[1, 1, 1]^\top$ and $[1, 0, -1]^\top$ ($x = 0$), like described earlier. The third column of the matrix can be $[1, 1, -2]^\top$ ($x = 1$), $[1, -1, 0]^\top$ ($x = -1$), $[1, -2, 1]^\top$ ($x = -2$), etc. When any of these transformation matrices are applied on a balanced matrix (as in (30)), the

matrix is successfully diagonalised. However, when the matrix to be diagonalised differs from the balanced case (as in (31)), it is the third transformation matrix (equivalent to (36)) that yields smaller off-diagonal elements and, thus, smaller errors than other options.

B. Untransposed lines

For an untransposed line, there are two unique modal transformation matrices \mathbf{T}_u and \mathbf{T}_i , dependent on its parameters. The asymmetry of the transmission line is carried over to the \mathbf{ZY} and \mathbf{YZ} matrices and the corresponding transformation matrices. However, since the eigenvectors of a matrix are arbitrarily scaled, there are many cases when changes in the \mathbf{ZY} and \mathbf{YZ} matrices leave the transformation matrices unchanged.

Section II-A presents one approximation for the calculation of the reduced impedance and admittance matrices \mathbf{Z} and \mathbf{Y} . It can be assumed that during the operation of a transmission line its geometry and the physical properties of conductors are mostly only affected by changes in temperature. The quantity with the highest uncertainty and variability along the length of the line is earth resistivity. All other variables affecting \mathbf{Z} and \mathbf{Y} are assumed to be known accurately enough and change negligibly in time.

Based on these considerations, the modal transformation matrices of an untransposed transmission line are analysed in a sensitivity analysis. The temperature of the conductor and earth resistivity are varied and the transformation matrices re-evaluated for each change to observe any variations from results based on initial input values.

Calculations described in section II-A are modified to include changes in conductor temperature. The temperature dependent resistance is taken as

$$r_{ii} = r_{\text{ref}}(1 + \alpha\Delta T), \quad (37)$$

where r_{ii} is the series resistance given in Eq.(12), r_{ref} is the reference value of resistance at a certain temperature, ΔT is the change in conductor temperature, and α is the temperature coefficient of resistance.

The temperature dependent average sag of the line is estimated in a simplified way as

$$h = \sqrt{\frac{3}{8}d \left[\left(d + \frac{8h_{\text{ref}}^2}{3d} \right) (1 + \xi\Delta T) - d \right]}, \quad (38)$$

where d is the average span, h_{ref} is the average sag calculated from design data, and ξ is the thermal elongation coefficient [17].

The calculations are carried out on a common untransposed transmission line with bilateral symmetry. The geometry of the line is specified in Fig. 1. The sag of all conductors is 9 m, the diameter of phase conductors is 2.77 cm and for neutral conductors the value is 1.43 cm, the resistances are 0.072 Ω/km and 0.35 Ω/km , respectively. Earth resistivity in the base case is assumed to be 100 $\Omega \cdot \text{m}$.

The transformation matrices \mathbf{T}_u and \mathbf{T}_i can always be scaled in such a manner that only two elements differ from the

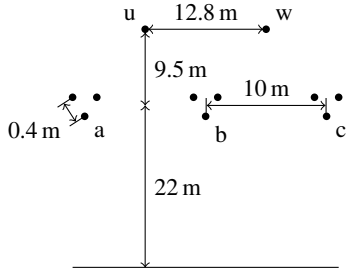


Fig. 1. Geometry of the line used in the sensitivity analysis of modal transformation matrices. A vertical plane of symmetry passes through the bundle of phase 'b'.

Clarke transformation matrix given in (36). In the following, only the middle elements of the first two columns of the transformation matrix are analysed. The third column is the same for both transposed and untransposed lines.

Results of the sensitivity analysis are presented in Fig. 2 and Fig. 3. The plots show that large changes in conductor temperature and large errors in specifying earth resistivity cause very small changes in the transformation matrices. Based on the data it can be suggested that for a specific transmission line the transformation matrices can be calculated once and assumed constant for measurement based estimation of reduced matrices of line parameters.

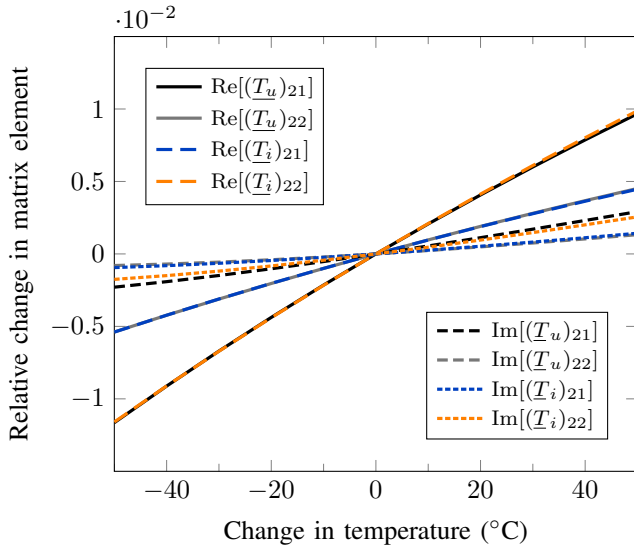


Fig. 2. Relative (per unit) changes in the elements of the transformation matrices of an untransposed transmission line under changing conductor temperature.

The results can be explained based on the following considerations. Firstly, a similar conclusion can be made for transposed lines, as their transformation matrices are constant under the same circumstances. Secondly, the difference from the Clarke transformation matrix in the transformation matrices of untransposed lines are caused by the asymmetry between phases. The quantities, in which the changes were analysed, affect all phases equally and, thus, it can be expected that

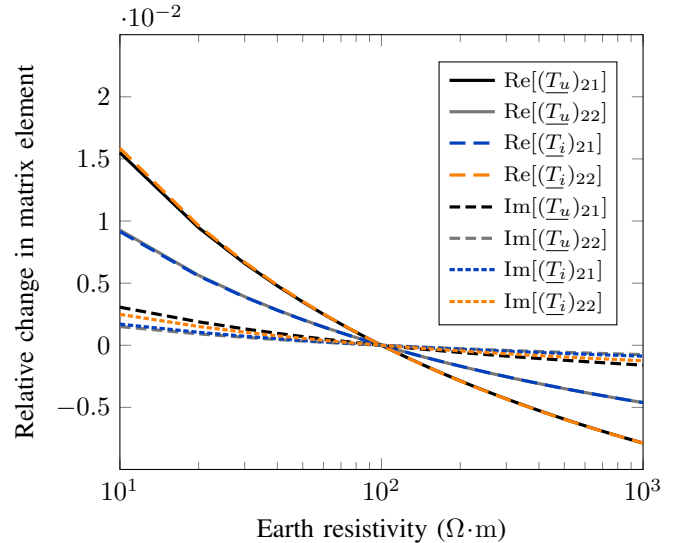


Fig. 3. Relative (per unit) changes in the elements of the transformation matrices of an untransposed transmission line under varying values of earth resistivity.

the changes in the asymmetry between phases themselves is negligible. This in turn means that the changes in the transformation matrices are also expected to be negligible, as seen from the sensitivity analysis.

IV. CONCLUSIONS

The paper presents how the most accurate modal transformation matrices can be chosen for the estimation of the parameters of transposed or untransposed transmission lines from phasor measurements. These transformation matrices are found for use in estimating the reduced impedance and admittance matrices of a three-phase transmission line, considering the coupling of phase quantities.

Firstly, it is analysed why there exists a unique preferred modal transformation matrix for use on transposed lines, even though theoretically, there is an infinite number of correct modal transformation matrices for ideally transposed lines. Using perturbation analysis, it is shown that when the system deviates from the ideal case, the single preferred transformation matrix is closest to the correct one.

Secondly, it is shown that in case of untransposed lines it is not necessary to make the simplification of using the transformation matrix of transposed lines, but instead, more correct modal transformation matrices can be calculated from line design data and assumed constant. In a sensitivity analysis it is demonstrated that modal transformation matrices are negligibly affected by changes in environment-dependent quantities. Thus, for applications estimating line parameters from phasor measurements, the transformation matrices can be calculated once and assumed constant for a certain transmission line.

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