Current and Voltage Dependent Sources Modelling in MATE – Multi-Area Thévenin Equivalent Concept

Benedito D. Bonatto, Mazana L. Armstrong, José R. Martí, and Hermann W. Dommel

Abstract-- **The objective of this paper is to provide a clear derivation of the Multi-Area Thévenin Equivalent Concept (MATE) including current- and voltage dependent sources. The links concept in MATE is advantageous in representing branches connecting subsystems. MATE deviates from Diakoptics and from the Modified Nodal Analysis (MNA) methods in the way it is solved, by manipulating the submatrices in a form that preserves the individuality of the internal subsystems while solving their interdependences at the level of Thévenin Equivalents. The generalization presented in this paper expands the link branch equations to dependent, coupled, linear or nonlinear relations, thus resulting in unsymmetrical matrices. Its significance occurs when complex control systems and power system equations are simultaneously solved in an Electromagnetic Transients Program (EMTP). In this case, exact results can be achieved with less computational effort for power system dynamics studies. A test case with simulation results illustrates the main modelling concepts.**

***Keywords*: Electromagnetics Transients Program, Multi-Area Thévenin Equivalent, Current and Voltage Dependent Sources, Power System Dynamics, Power System Stability, Control, Nonlinear Modelling, Smart Grids.**

I. Introduction

THIS paper presents an extension of the Multi-Area Thévenin Equivalent Concept (MATE) [1]-[5] for electromagnetic transients program (EMTP)- based simulations [6],[7] including building functions for controllers and other asymmetrical branches. The application is useful in real-time power systems dynamics simulations [8].

The proposed MATE modelling easily incorporates dependent branches and control blocks in a simultaneous (no time delay decoupling) EMTP solution. In this paper, firstly, the mathematical conceptual derivation of the MATE [1] method is reviewed. Then, current and voltage dependent sources [9] are incorporated. Finally, control blocks, realized using one of these dependent sources [10] are proposed to be incorporated into the model in the future.

The MATE algorithm generalization presented in this paper expands the link branch equations to dependent, coupled, linear or non-linear relations, thus resulting in unsymmetrical matrices. A test case with simulation results from [2] illustrates the main modelling concepts especially with regards to the simultaneous solution of complex control systems and the power system equations. Simultaneous solution of the control system equations and the power network equations [2], [10]-[12] are particularly relevant in the development of distributed control strategies in the emerging smart grid real time supervisory control applications.

II. Mathematical Conceptual Derivation of the MATE Concept

The Multi-Area Thévenin Equivalent (MATE) concept provides an effective means for partitioning large systems of equations into subsystems connected through links. The subsystems are solved independently (even with different solution techniques and with parallel processing) and the overall solution is integrated at the level of the links. In the Multilevel MATE algorithm each subsystem becomes the basis for another level of MATE partitioning, thus improving solution efficiency [2], [3].

A. General Formulation of MATE

Consider initially, for didactic reasons, any power system composed of only two subsystems, A and B, and any set of dependent, independent, coupled, uncoupled, linear or nonlinear branch relations among them, as illustrated in Fig. 1.

Then, the Multi-Area Thévenin Equivalent concept allows that the two subsystems can be solved independently (even with different solution techniques and with parallel processing) and the overall solution is integrated at the level of the branch links with the compensation method. If the branch equations are nonlinear, then the fixed point iteration method or a Newton-Raphson type algorithm can be used for the solution, as illustrated in Fig. 2. The system of Fig. 1, with the “assumed conventions for voltage polarities and current directions”, can be represented by the matrix equations in (1):

$$\begin{bmatrix} [G_A] & [0] & [p] \\ [0] & [G_B] & [q] \\ [m] & [n] & -[z] \end{bmatrix} \cdot \begin{bmatrix} [v_A] \\ [v_B] \\ [i_\alpha] \end{bmatrix} = \begin{bmatrix} [h_A] \\ [h_B] \\ [V_s] \end{bmatrix} \quad (1)$$

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B. D. Bonatto is with UNIFEI – Federal University of Itajuba, Av. BPS, n. 1303, Bairro Pinheirinho, P.O. Box 50, Zip Code 37500-903, Itajuba-MG, BRAZIL. (bonatto@unifei.edu.br - corresponding author).

M. L. Armstrong is with BC Hydro for Generations, 6911 Southpoint Drive Burnaby, B.C., V3N 4X8, CANADA (mazana.armstrong@bchydro.com)

H. W. Dommel and J. R. Martí are with the Department of Electrical and Computer Engineering, UBC - The University of British Columbia, 2332 Main Mall, V6T 1Z4, Vancouver, B.C., CANADA. (hermannd@ece.ubc.ca, jrms@ece.ubc.ca)

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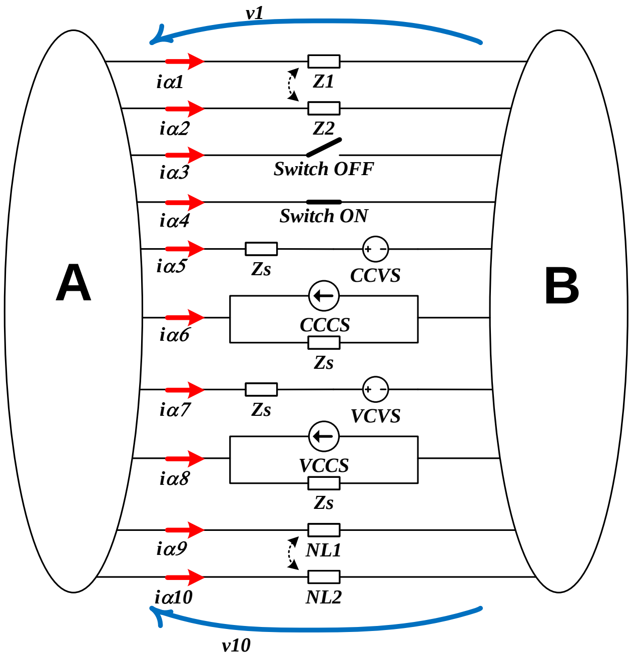

Fig. 1. MATE concept expansion considering link branch equations for dependent, coupled, linear or nonlinear relations.

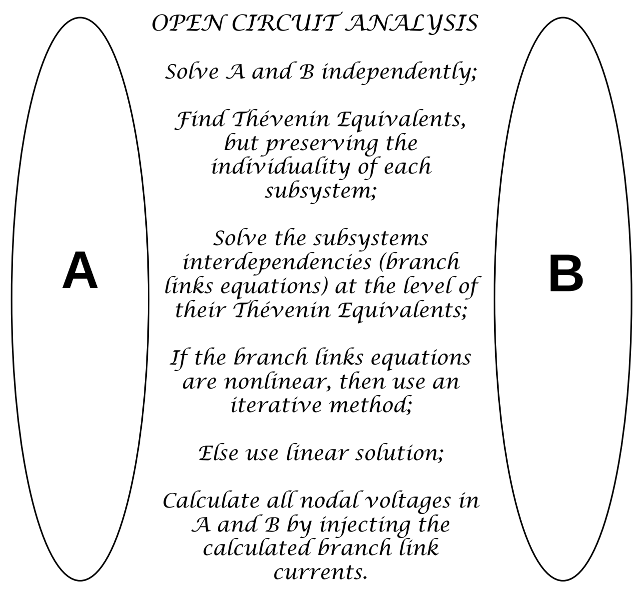

Fig. 2. MATE concept solution in partitioned subsystems with dependent, coupled, linear or nonlinear branch relations.

where:

$[G_A]$ is the EMTP conductance matrix of subsystem A;
$[v_A]$ is the vector of nodal voltages of subsystem A;
$[h_A]$ is the vector of current sources of subsystem A;
$[p]$, $[q]$, $[m]$, $[n]$ are "connectivity" submatrices needed to express the branch relations between the subsystems A and B;
$[z]$ is the submatrix of impedance relations of the branches connecting subsystems A and B;
$[V_s]$ is the vector of equivalent voltage sources in each branch connecting subsystems A and B.

Similar meaning stands for subsystem B, and eventually other subsystems (C , D , etc.), if more partitioning is realized, or if Multilevel MATE concept [2], [3] is used.

By pre-multiplying the first row in (1) by $[G_A]^{-1}$, and the second row by $[G_B]^{-1}$, results in (2). The third row in (1) becomes the third row in (2) as derived from (7)-(16).

$$\begin{bmatrix} [1] & [0] & [a] \\ [0] & [1] & [b] \\ [0] & [0] & [z_\alpha] \end{bmatrix} \cdot \begin{bmatrix} [v_A] \\ [v_B] \\ [i_\alpha] \end{bmatrix} = \begin{bmatrix} [e_A] \\ [e_B] \\ [e_\alpha] \end{bmatrix} \quad (2)$$

where:

$$[a] = [G_A]^{-1} \cdot [p] \quad (3)$$

$$[b] = [G_B]^{-1} \cdot [q] \quad (4)$$

$$[e_A] = [G_A]^{-1} \cdot [h_A] \quad (5)$$

$$[e_B] = [G_B]^{-1}.[h_B] \quad (6)$$

From (2) one can derive that:

$$[v_A] + [a] \cdot [i_\alpha] = [e_A] \quad (7)$$

$$[v_A] = [e_A] - [a] \cdot [i_\alpha] \quad (8)$$

$$[v_B] + [b] \cdot [i_\alpha] = [e_B] \quad (9)$$

$$[v_B] = [e_B] - [b] \cdot [i_\alpha] \quad (10)$$

Multiplying the third row of (1), but using (8) to express $[v_A]$, and (10) to express $[v_B]$, results in:

$$[m] \cdot [v_A] + [n] \cdot [v_B] - [z] \cdot [i_\alpha] = [V_s] \quad (11)$$

$$[m] \cdot [e_A] - [m] \cdot [a] \cdot [i_\alpha] + [n] \cdot [e_B] - [n] \cdot [b] \cdot [i_\alpha]$$

$$-[z] \cdot [i_\alpha] = [V_s] \quad (12)$$

$$\{[m] \cdot [a] + [n] \cdot [b] + [z]\} \cdot [i_\alpha] = [m] \cdot [e_A]$$

$$+[n] \cdot [e_B] - [V_s] \quad (13)$$

By defining $[z_\alpha]$ and $[e_\alpha]$ as:

$$[z_\alpha] = [m] \cdot [a] + [n] \cdot [b] + [z] \quad (14)$$

$$[e_\alpha] = [m] \cdot [e_A] + [n] \cdot [e_B] - [V_s] \quad (15)$$

the third row of (2), is obtained, as in (16):

$$[z_\alpha] \cdot [i_\alpha] = [e_\alpha] \quad (16)$$

The solution for link currents is now independent from the solution of the nodal subsystems' voltages. The interaction of the subsystems Thévenin equivalents is solved at the links level and returned to each subsystem in the form of injected link currents $[i_\alpha]$ at the linking nodes:

$$[i_\alpha] = [z_\alpha]^{-1} \cdot [e_\alpha] \quad (17)$$

The subsystems´ nodal equations are then solved independently of each other at the subsystem level as:

$$[v_A] = [e_A] - [a] \cdot [i_\alpha] \quad (18)$$

$$[v_B] = [e_B] - [b] \cdot [i_\alpha] \quad (19)$$

Now, the really important issues of this matrix partitioning and manipulation are the meaning of the new variables, such as:

$[e_A]$ is the Thévenin Equivalent Source vector of subsystem A;

$[e_B]$ is the Thévenin Equivalent Source vector of subsystem B;

$[a]$ is the Thévenin Impedance submatrix of subsystem A, as seen from the link nodes connecting the two subsystems;

$[b]$ is the Thévenin Impedance submatrix of subsystem B, as seen from the link nodes connecting the two subsystems;

The reduced system at the Thévenin linking nodes is given by:

$[e_\alpha]$ is the Thévenin Equivalent Source vector of the reduced system;

$[z_\alpha]$ is the Thévenin Impedance submatrix of the reduced system;

$[i_\alpha]$ is the links´ branch current vector solution;

The interaction between the two subsystems A and B occurs when injecting the link currents $[i_\alpha]$ into the corresponding nodes into subsystems A and B.

Therefore, the final solution for the node voltages in both subsystems is comparable to applying the superposition of the calculated voltages, due to all internal voltage and current sources for the open circuit solution (each subsystem solved separately), to the voltages obtained only due to the link current injections (compensation theorem as used in the M-phase Thévenin Equivalent [6]).

A particular formulation of MATE is achieved for the case of branch links formed only with impedances. In this case, the sub matrices needed to express the branch relations between the subsystems A and B, i.e., [p], [q], [m], [n], [z] have special properties such as $[m] = [p]^t$, $[n] = [q]^t$, and [z] contains only branch impedances. They usually contain unit and/or zero values. In the case of links representing ideal switches, either the branch current is equal to zero for an ideal opened switch (switch OFF in Fig. 1), or the branch voltage is equal to zero for an ideal closed switch (switch ON in Fig. 1).

Concluding, all advantages of using MATE are preserved in this general formulation, which can be used for the Multilevel MATE concept [2], [3] as well, where branch equations are treated as sublinks and contribute to the subsystems´ Thévenin equivalents obtained from nodal equations to form Modified Thévenin Equivalents (MTEs).

III. Current and Voltage Dependent Sources Modelling in MATE

If the branch equations are linear, as in the case of dependent sources, they can be represented in the form of a voltage source behind an impedance, or in the form of a current source in parallel with an impedance. In this paper, it is assumed that the branch impedances (Z_s) of the sources are not coupled and that they are resistive (R_{out}). This section presents the necessary equations for implementing current and voltage dependent sources in the MATE concept. The following assumptions are made:

- A Thévenin equivalent circuit can be calculated where the dependent source is to be connected, and also where the controlling current or voltage is to be measured. In cases where this calculation fails, the connection of large resistors in parallel may make a Thévenin equivalent circuit possible;
- Proper precautions are taken to handle extremely large numbers and zero values;

The following models are derived: Current Controlled Voltage Source (CCVS), Current Controlled Current Source (CCCS), Voltage Controlled Voltage Source (VCVS) and Voltage Controlled Current Source (VCCS).

A. Current Controlled Voltage Source (CCVS)

Assume that the controlling current is measured through a branch between nodes a and b in a circuit, such that v_j is its branch voltage and $i_{\alpha j}$ is its branch current, i.e.:

$$v_j = v_a - v_b \quad (20)$$

$$i_{\alpha j} = i_{ab} \quad (21)$$

and the dependent source, CCVS, is connected between nodes c and d, with branch voltage:

$$v_k = v_c - v_d \quad (22)$$

and branch current:

$$i_{\alpha k} = i_{cd}. \quad (23)$$

Then, based on Fig. 3 the following equations can be derived:

$$v_j = R_{in}.\, i_{\alpha j} \quad (24)$$

$$v_k = \Omega.\, i_{\alpha j} + R_{out}.\, i_{\alpha k} \quad (25)$$

From (20)-(25) follows that:

$$1v_a - 1v_b - R_{in}.\, i_{\alpha j} - 0 i_{\alpha k} = 0 \quad (26)$$

$$1v_c - 1v_d - \Omega.\, i_{\alpha j} - R_{out}.\, i_{\alpha k} = 0 \quad (27)$$

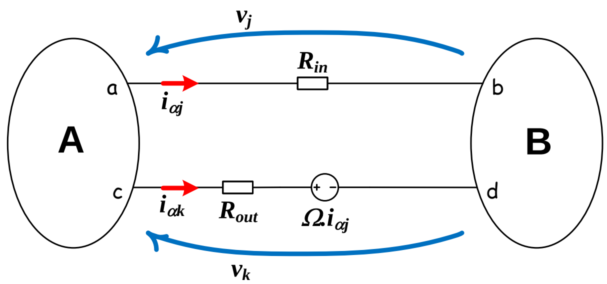

Fig. 3. Current Controlled Voltage Source (CCVS) in MATE concept.

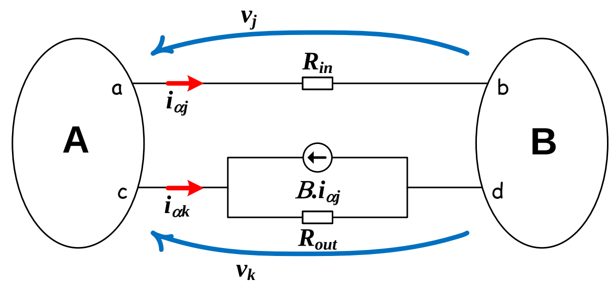

Fig. 4. Current Controlled Current Source (CCCS) in MATE concept.

where:

R_{in} = input resistance of branch j;

R_{out} = output resistance of the dependent source in branch k;

Ω = gain over the controlling or measured current, applied as a dependent source at branch k;

For an ideal current controlled voltage source, $R_{in} = 0$, and $R_{out} = 0$, which leads to:

$$1v_a - 1v_b - 0i_{\alpha j} - 0i_{\alpha k} = 0 \tag{28}$$

$$1v_c - 1v_d - \Omega . i_{\alpha j} - 0i_{\alpha k} = 0 \tag{29}$$

Equations (26)-(27) or (28)-(29) can easily be inserted into the matrix formulation presented in (1), as illustrated in (30):

$$\begin{bmatrix} [G_A] & & [0] & & \cdots & +1 & 0 \\ \vdots & & \vdots & & \cdots & 0 & +1 \\ & & & & \vdots & \vdots & \vdots \\ [0] & & [G_B] & & \cdots & -1 & 0 \\ \vdots & & \vdots & & \cdots & 0 & -1 \\ & & & & \vdots & \vdots & \vdots \\ +1 \quad 0 & \cdots & -1 & 0 & \cdots & 0 & 0 \\ 0 \quad +1 & \cdots & 0 & -1 & \cdots & -\Omega & 0 \end{bmatrix} \begin{bmatrix} v_a \\ v_c \\ \vdots \\ v_b \\ v_d \\ \vdots \\ i_{\alpha j} \\ i_{\alpha k} \end{bmatrix} = \begin{bmatrix} i_{ha} \\ i_{hc} \\ \vdots \\ i_{hb} \\ i_{hd} \\ \vdots \\ 0 \\ 0 \end{bmatrix} \tag{30}$$

B. Current Controlled Current Source (CCCS)

Based on Fig. 4, the necessary equations for the implementation of a current controlled current source into the MATE concept are:

$$v_j = R_{in} . i_{\alpha j} \tag{31}$$

$$v_k = R_{out} . B . i_{\alpha j} + R_{out} . i_{\alpha k} \tag{32}$$

From (20)-(23) and (31)-(34) it follows that:

$$1v_a - 1v_b - R_{in} . i_{\alpha j} - 0i_{\alpha k} = 0 \tag{33}$$

$$1v_c - 1v_d - R_{out} . B . i_{\alpha j} - R_{out} . i_{\alpha k} = 0 \tag{34}$$

where:

R_{in} = input resistance of branch j;

R_{out} = output resistance of the dependent source in branch k;

B = gain over the controlling or measured current, applied as a dependent source at branch k;

For an ideal current controlled current source, $R_{in} = 0$, and $R_{out} \to \infty$. When dividing (34) by R_{out} it follows that:

$$1v_a - 1v_b - 0i_{\alpha j} - 0i_{\alpha k} = 0 \tag{35}$$

$$0v_c - 0v_d - B . i_{\alpha j} - 1i_{\alpha k} = 0 \tag{36}$$

Equations (33)-(34) or (35)-(36) can easily be inserted into the matrix formulation presented in (1).

C. Voltage Controlled Voltage Source (VCVS)

Based on Fig. 5, the necessary equations for the implementation of a voltage controlled voltage source into the MATE concept are:

$$v_j = R_{in} . i_{\alpha j} \tag{37}$$

$$v_k = A . v_j + R_{out} . i_{\alpha k} = A . R_{in} . i_{\alpha j} + R_{out} . i_{\alpha k} \tag{38}$$

From (20)-(23) and (37)-(38) it follows that:

$$1v_a - 1v_b - R_{in} . i_{\alpha j} - 0i_{\alpha k} = 0 \tag{39}$$

$$1v_c - 1v_d - A . R_{in} . i_{\alpha j} - R_{out} . i_{\alpha k} = 1v_c - 1v_d - A . v_a + A . v_b - 0i_{\alpha j} - R_{out} . i_{\alpha k} = 0 \tag{40}$$

where:

R_{in} = input resistance of branch j;

R_{out} = output resistance of the dependent source in branch k;

A = gain over the controlling or measured voltage, applied as a dependent source at branch k;

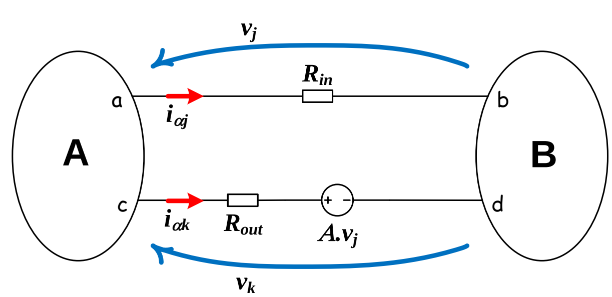

Fig. 5. Voltage Controlled Voltage Source (VCVS) in MATE concept.

For an ideal voltage controlled voltage source, $R_{in} \to \infty$, and $R_{out} = 0$. When dividing (39) by R_{in}, it follows that:

$$0v_a - 0v_b - 1i_{\alpha j} - 0i_{\alpha k} = 0 \quad (41)$$

$$1v_c - 1v_d - A.v_a + A.v_b - 0i_{\alpha j} - 0i_{\alpha k} = 0 \quad (42)$$

If $A \to \infty$, $R_{in} \to \infty$, and $R_{out} = 0$, VCVS can be used to model ideal operational amplifiers. By dividing (42) by the gain A, results in the following equations:

$$0v_a - 0v_b - 1i_{\alpha j} - 0i_{\alpha k} = 0 \quad (43)$$

$$0v_c - 0v_d - 1v_a + 1v_b - 0i_{\alpha j} - 0i_{\alpha k} = 0 \quad (44)$$

Equation (43) implies that $i_{\alpha j} = 0$, and (44) implies that $v_j = 0$, thus meaning that ideal operational amplifiers have, at the same time, an “open circuit” and a “virtual ground”, respectively, at their input terminals.

Equations (39)-(40) or (41)-(42) or (43)-(44) can easily be inserted into the matrix formulation presented in (1). Equation (45) illustrates the modelling of an ideal operational amplifier, which can be used for transfer functions control modelling as proposed in [8], resulting in unsymmetrical matrices:

$$\begin{bmatrix} [G_A] & & & [0] & & \cdots & +1 & 0 \\ \vdots & & & \vdots & & \cdots & 0 & +1 \\ & & & & & \vdots & \vdots & \vdots \\ & & & & & \cdots & -1 & 0 \\ [0] & & & [G_B] & & \cdots & 0 & -1 \\ \vdots & & & \vdots & & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & 0 & \cdots & +1 & 0 \\ +1 & 0 & \cdots & -1 & 0 & \cdots & 0 & 0 \end{bmatrix} \begin{bmatrix} v_a \\ v_c \\ \vdots \\ v_b \\ v_d \\ \vdots \\ i_{\alpha j} \\ i_{\alpha k} \end{bmatrix} = \begin{bmatrix} i_{ha} \\ i_{hc} \\ \vdots \\ i_{hb} \\ i_{hd} \\ \vdots \\ 0 \\ 0 \end{bmatrix} \quad (45)$$

D. *Voltage Controlled Current Source (VCCS)*

Based on Fig. 6, the necessary equations for the implementation of a voltage controlled current source into the MATE concept are:

$$v_j = R_{in}.i_{\alpha j} \quad (46)$$

$$v_k = R_{out}.\Gamma.v_j + R_{out}.i_{\alpha k} = R_{out}.\Gamma.R_{in}.i_{\alpha j} + R_{out}.i_{\alpha k} \quad (47)$$

From (20)-(23) and (46)-(47) it follows that:

$$1v_a - 1v_b - R_{in}.i_{\alpha j} - 0i_{\alpha k} = 0 \quad (48)$$

$$1v_c - 1v_d - R_{out}.\Gamma.R_{in}.i_{\alpha j} - R_{out}.i_{\alpha k} = 1v_c - 1v_d - R_{out}.\Gamma.v_a + R_{out}.\Gamma.v_b - 0i_{\alpha j} - R_{out}.i_{\alpha k} = 0 \quad (49)$$

where:

R_{in} = input resistance of branch j;
R_{out} = output resistance of the dependent source in branch k;
Γ = gain over the controlling or measured voltage, applied as a dependent source at branch k;

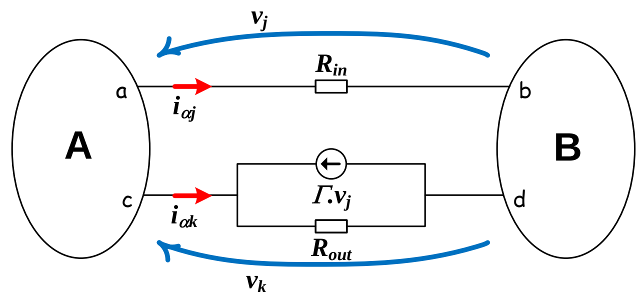

Fig. 6. Voltage Controlled Current Source (VCCS) in MATE concept.

For an ideal voltage controlled current source, $R_{in} \to \infty$, and $R_{out} \to \infty$.

When dividing (48) by R_{in}, and (49) by R_{out}, it follows that:

$$0v_a - 0v_b - 1i_{\alpha j} - 0i_{\alpha k} = 0 \quad (50)$$

$$0v_c - 0v_d - \Gamma.v_a + \Gamma.v_b - 0i_{\alpha j} - 1i_{\alpha k} = 0 \quad (51)$$

Equations (48)-(49) or (50)-(51) can easily be inserted into the matrix formulation presented in (1).

IV. Control Blocks Modelling with MATE and Multilevel MATE

The implementation of current and voltage dependent sources in the MATE concept expand its capabilities for modelling many electric and electronic circuits and devices. With a voltage controlled voltage source, for example, it becomes easy to simulate ideal operational amplifiers. These can then be used to set up control circuits with analog-computer block-diagrams.

As long as the equations of the dependent sources are linear, one direct solution is performed using a solver for linear unsymmetrical matrices and pivoting techniques. Nonlinear effects arise with the inclusion of saturation or limits in the dependent sources. The fixed point iteration method or a Newton-Raphson type algorithm can be used in these and other nonlinear cases.

A circuit approach for the computer modelling of control transfer functions is given in [9], originally developed for EMTP-based simulations. Its implementation in the MATE concept is possible, based on the work presented here, especially with the inclusion of models for ideal operational amplifiers. Linear coupled branches, such as ideal transformers, or nonlinear coupled branches can also be modelled in MATE.

Therefore, the application of the enhanced MATE and Multilevel MATE [2]-[3] concepts allows the modeling, with more computational efficiency, of complex engineering systems and control devices. Fig. 7 presents an example system to demonstrate the “multilevel MATE” concept, which allows that any system can be partitioned using links and sublinks connecting systems or subsystems, respectively.

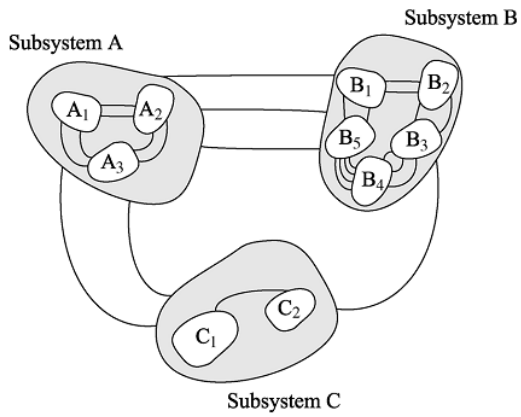

Fig. 7. Example system to demonstrate the "multilevel MATE" concept [2]-[3].

V. TEST CASE SIMULATIONS

To illustrate the application of the Multilevel MATE to a power system dynamics simulation, including control modelling as sublinks branch equations, Fig. 8 shows a schematic of a double fed induction generator (DFIG) wind turbine system. Fig. 9 presents the stator-side converter controller and Fig. 10 presents the rotor-side converter controller. Details of the modelling and other data are available in [2]:

Rated power = 7.5 kW
Stator voltage = 415 V
Rotor voltage = 440 V
Rated stator current = 19 A
Rated rotor current 11 A
Pole pairs = 3
Rated speed 970 rpm
Base frequency = 50 Hz
Ns/Nr = 1.7
$J = 7.5\ kgm^2$
Stator connection = delta
Rotor connection = wye
Machine resistances and inductances per phase:
$R_s = 1.06\ \Omega$
$R_r = 0.80\ \Omega$
$L_s = 0.0664\ H$
$L_0 = 0.0810\ H$
$L_r = 0.0320\ H$

Control Parameters:

Stator-side converter
$K_v = 0.12$
$a_v = 0.9248$
$K_i = 4.72$
$a_i = 0.96$

Rotor-side converter
$K_\omega = 0.49$
$a_\omega = 0.988$
$K_{ir} = 20$
$a_{ir} = 0.985$

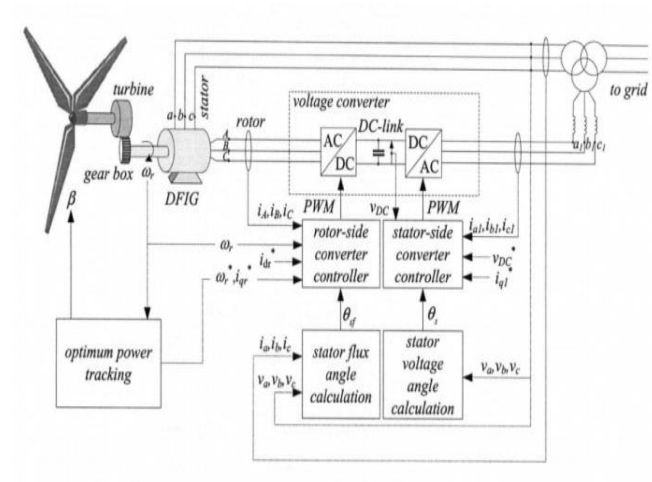

Fig. 8. Schematic of a double fed induction generator (DFIG) wind turbine system [2].

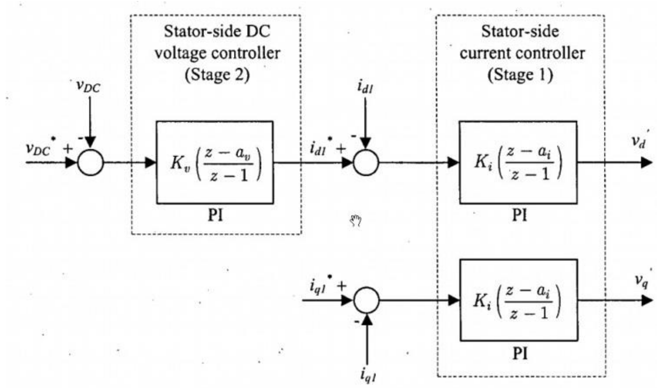

Fig. 9. Stator-side converter controller [2].

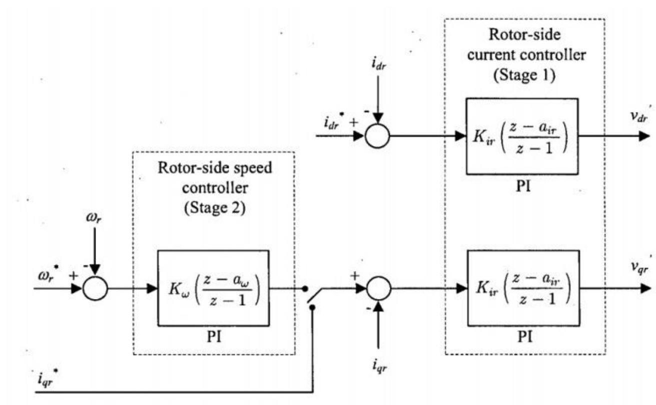

Fig. 10. Rotor-side converter controller [2].

The double fed induction generator wind turbine system model consists of the three-phase domain induction generator discrete model [2], the discrete phase-domain model of the voltage converter and the voltage-converter controllers operating in the dq reference frame. The phase-domain voltage converter model is composed of the voltage transfer characteristics of the stator and rotor side converters plus the differential equation of the DC link.

"An experimental setup from the literature [13] was replicated and simulated for two types of disturbances: decrease in wind velocity and a three-phase fault in the connecting double-circuit transmission line, illustrated in Fig. 11. The results were successfully compared against the results in the literature and against a traditional stability simulation tool [14]. The comparison has shown the advantages of using more detailed modelling, especially when control and protection devices plays a major role in the system´s response" [2].

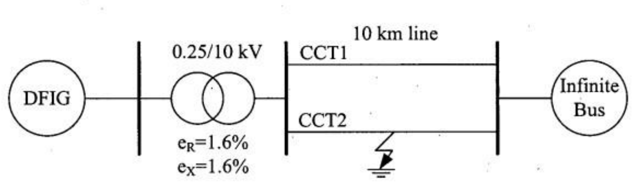

Fig. 11. Double fed induction generator (DFIG) wind turbine test case [2].

Fig. 12 presents the transient response of a double fed induction generator (DFIG) wind turbine to a step decrease in wind velocity [2].

"Fig. 12 depicts the DIFG currents and voltages in the pahse domain. Figs. 12 (c) and Fig. 12 (d) show the smooth operation of the DFIG through synchronous speed. By examining Fig. 12 (f) we note the change of "direction" of the stator-side converter current, indicating the change in converter power from generation to consumption" [2].

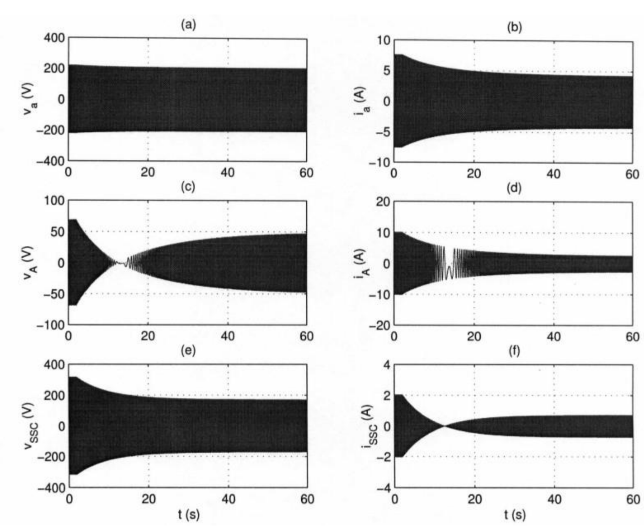

Fig. 12. Transient response of a double fed induction generator (DFIG) wind turbine to a step decrease in wind velocity [2]: (a) stator phase to neutral voltage, (b) stator phase current, (c) rotor phase to neutral voltage, (d) rotor phase current, (e) stator-side converter phase voltage, (f) stator-side converter phase current.

Fig. 13 presents the transient response of a double fed induction generator (DFIG) to a three-phase short circuit.

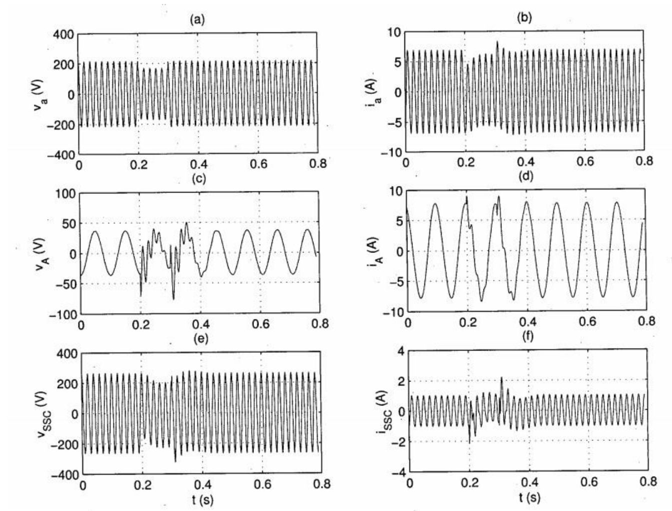

Fig. 13. Transient response of a double fed induction generator (DFIG) to a three-phase short circuit: (a) stator phase-to-neutral voltage, (b) stator phase current, (c) rotor phase-to-neutral voltage, (d) rotor phase current, (e) stator-side converter phase voltage, (f) stator-side converter phase current.

VI. CONCLUSIONS

This paper has provided a clear derivation of the Multi-Area Thévenin Equivalent Concept (MATE) [1], also including current- and voltage dependent sources. The generalization presented in this paper expands the link branch equations to dependent, independent, coupled, uncoupled, linear or nonlinear relations.

The links concept in MATE is advantageous in representing branches connecting subsystems. The subsystems can be solved independently (even with different solution techniques and with parallel processing) and the overall solution is then integrated at the level of the links. In the Multilevel MATE algorithm each subsystem becomes the basis for another level of MATE partitioning, thus improving solution efficiency [2], [3].

The algorithm has to take into account the particular system´s topology and characteristics (fast or slow dynamics, linear, nonlinear, etc.) to choose the partitioning. Optimal network partitioning algorithm is needed in future work, especially for very large power systems simulations.

Therefore, the application of the enhanced MATE and Multilevel MATE concepts allows the modelling and the simultaneous solution of control and power systems equations with more computational efficiency for complex engineering systems and control devices.

VII. REFERENCES

[1] J. R. Martí, L. R. Linares; J. A. Hollman; F. A. Moreira, "OVNI: Integrated Software/Hardware Solution for Real-Time Simulation of Large Power Systems", in Proceedings of the 14th Power Systems Computation Conference (PSCC 2002), Seville, Spain, June 24-28, 2002.

[2] M. L. Armstrong, "Multilevel MATE Algorithm for the Simulation of Power System Transients with the OVNI Simulator", *Ph.D. Thesis*, University of British Columbia, Vancouver, BC, Canada, 2006. Available for download in: https://circle.ubc.ca/handle/2429/30708.

[3] M. L. Armstrong, J. R. Marti, L. R. Linares, and P. Kundur. "Multilevel MATE for Efficient Simultaneous Solution of Control Systems and Nonlinearities in the OVNI Simulator", IEEE Transactions on Power Systems, 21(3):1250–1259, 2006.

[4] M. A. Tomim, "Parallel Computation of Large Power System Networks Using the Multi-Area Thévenin Equivalents", *Ph.D. Thesis*, University of British Columbia, Vancouver, BC, Canada, 2009.

[5] M. A. Tomim, J. R. Martí, and L. Wang. "Parallel Computation of Large Power System Network Solutions using the Multi-Area Thévenin Equivalents (MATE) Algorithm", in 16th Power Systems Computation Conference (PSCC 2008), Glasgow, Scotland, 2008.

[6] H. W. Dommel, "Digital Computer Solution of Electromagnetic Transients in Single- and Multiphase Networks", *IEEE Transactions on Power Apparatus and Systems*, vol. PAS-88, April 1969, pp. 388-399.

[7] H. W. Dommel, *EMTP Theory Book*, Second Edition, Microtran Power Systems Analysis Corporation, Vancouver, British Columbia, Canada, 1992, latest update 1996.

[8] P. Kundur, Power *System* Stability and Control, New York: McGraw-Hill, 1994.

[9] B. D. Bonatto and H. W. Dommel, "A Circuit Approach for the Computer Modelling of Control Transfer Functions", in *Proceedings* of the 14th Power Systems Computation Conference (PSCC 2002), Seville, Spain, June 24-28, 2002.

[10] B. D. Bonatto and H. W. Dommel, "EMTP Modelling of Control and Power Electronic Devices – Electromagnetic Transients Programs helping the analysis of the power interaction on either the load or the network side", LAP – Lambert Academic Publishing AG & Co, 2010, ISBN:978-3-8383-2790-7, 175p.

[11] J. Mahseredjian, L. Dube, Ming Zou, S. Dennetiere, and G. Joos, "Simultaneous Solution of Controls System Equations in EMTP", IEEE Transactions on Power Systems, 21(1):117-124, 2006.

[12] C. Dufour, J. Mahseredjian, J. Bélanger, J. L. Naredo, "An Advanced Real-Time Electro-Magnetic Simulator for Power Systems with a Simultaneous State-Space Nodal Solver", IEEE/PES Transmission and Distribution Conf. and Exposition: Latin America, pp. 349-358, 2010.

[13] R. Pena, J. C. Clare, and G. M. Asher, "Double-Fed Induction Generator Using Back-to-back PWM Converters and its Application to Variable-Speed Wind-Energy Generation", *IEE Proceedings - Electric Power Applications*, 143(3):231-241, May 1996.

[14] Power Tech Labs Inc. TSAT – Transient Security Assessment Tool, Version 5.1, 2005.