# Methodology of Calculating Harmonic Distortion from Multiple Sources

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***Abstract*--This paper presents a methodology for studying and specifying incremental and aggregated harmonic distortion emission limits from multiple harmonic sources that are connected to the utility network in close proximity and/or electrically are close to each other. The methodology includes the development of combined equivalent circuits for the harmonic sources for evaluating their impact on power quality. This methodology has been used successfully to represent an electrified traction load and can be extended and applied to the assessment of multiple embedded generators such as from PV solar or wind farms.**

***Keywords*: Power quality, harmonics, harmonic distortion, multiple harmonic sources.**

## I. INTRODUCTION

As part of the requirements for Distribution Code and Grid Code compliance in the United Kingdom, harmonic voltage distortion must be kept below specified limits. The assurance of compliance is provided through undertaking of harmonic voltage distortion studies and measurements. The voltage distortion study method described in this paper is based on the harmonic load flow procedure. The harmonic load flow procedure calculates the propagation of harmonic currents at each harmonic frequency resulting in the combined harmonic current through branches of interest and more importantly in harmonic voltage distortion at each node of interest.

This paper presents a methodology for studying and specifying incremental and aggregated harmonic distortion emission limits from multiple harmonic sources that are connected to the utility network in close proximity and/or electrically are close to each other. This methodology has been developed to represent an electrified traction load with multiple feeder stations connected to the utility network in the same area and each connection consisting of several traction units supplied through the railway power distribution system. The methodology can be extended and applied to the assessment of multiple embedded generators such as from PV solar or wind farms.

The methodology includes the development of combined electrical equivalent circuits for the harmonic sources for evaluating their impact on power quality. The combined electrical equivalent circuits are based on the network electrical parameters and statistical aggregation of harmonic distortion contributions from different sources. This enables integration of all the harmonic sources such that harmonic load flow studies can be undertaken with a conservative approach without exaggerating their effects on the network. Furthermore, this enables the separation of the complex load modelling from the mainstream power system harmonic load flow analysis.

The system considered for the study consisted of three main parts: the traction system; the utility (transmission/distribution) network; and the interface between them which may consist of traction transformers, compensation equipment, harmonic filters, etc. The system representation has to be sufficiently detailed and include suitable model representation of the trains' operations. Detailed representation of the trains in harmonic studies can be very complex and it is common to employ simplified methods while maintaining accuracy.

A method based on aggregation of traction loads and construction of a Norton equivalent circuit and its integration to the utility network is described in this paper. The Norton equivalent accounts for the effects of the 25kV impedance on the harmonic propagation through the system and the aggregated traction load. The paper also describes the modification of utility network equipment to properly represent harmonic behaviour, according to recommended industry practices such as CIGRE.

## II. TRACTION SYSTEM

### A. Flow of harmonic currents

In an electric traction system, each train acts as a harmonic current source. The harmonic currents will propagate through the catenary system, traction transformer and into the EHV/HV system, as illustrated in Fig. 1. The harmonic current distribution will depend on the catenary system parameters, train parameters and train location.

The Norton equivalent has to adequately represent the traction system at each traction power supply point in order to account for all parameters. The Norton equivalent must account for a number of trains (potentially of different types) operating to a given timetable. Therefore, different Norton equivalents are required to model the different sections, modes of operation, timetables, rolling stock, etc. Based on previous experience, some averaging methods can be employed by the designer to simplify the process.

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The Norton equivalent circuit will depend on the type of electrification system such as autotransformer system, booster transformer system or rail return system etc. The traction system considered here is the rail return system (and it may include additional return conductors) with the related Norton equivalent circuit shown in Fig. 2. To derive the complete Norton equivalent circuit, both admittance matrix and equivalent current sources have to be determined. With reference to Figure 2 the following equations are used to derive the equivalent circuit parameters:

$$\begin{aligned} I_1 + I_{n1} &= V_1 \cdot Y_{n11} + (V_1 - V_2) \cdot Y_{n12} \\ I_2 + I_{n2} &= V_2 \cdot Y_{n22} - (V_1 - V_2) \cdot Y_{n12} \end{aligned} \tag{1}$$

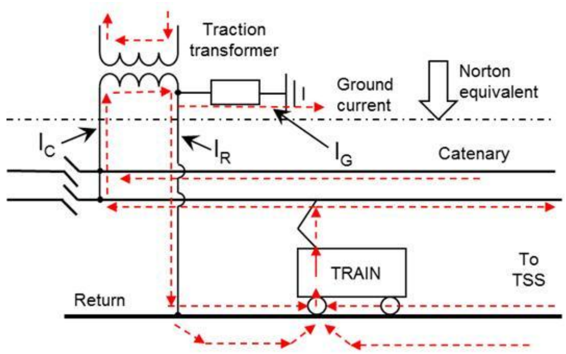


Fig. 1. Flow of harmonic currents through the traction system

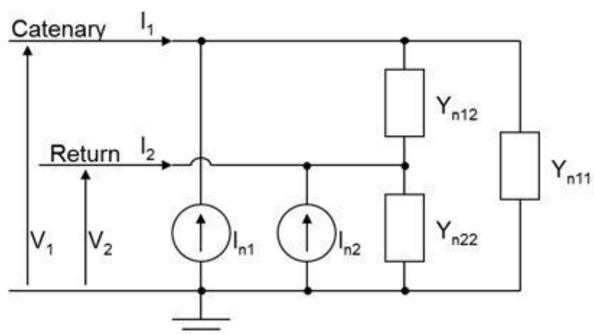


Fig. 2. Traction system Norton equivalent circuit

To calculate the Norton equivalent circuit admittance matrix for conductor "*i*", it is necessary to:

- eliminate all internal sources (loads, feeder stations, etc.),
- apply a known current $\boldsymbol{I}$ to the conductor "*i*" for all frequencies of interest,
- short-circuit all other conductors to earth,
- calculate voltage $\boldsymbol{V_i}$ and currents in all other conductors for all frequencies.

In this particular example case, the first step is to short the conductor 2 (return) to ground and inject a known current into the conductor 1 (catenary). This is followed with shorting the conductor 1 (catenary) to ground and injecting a known current into the conductor 2 (return).

From these it is possible to write:

$$I_1 = V_1 \cdot Y_{n11} + V_1 \cdot Y_{n12} \Rightarrow Y_{n11} + Y_{n12} = \frac{I_1}{V_1} \tag{2}$$

$$I_2 = -V_1 \cdot Y_{n12} \Rightarrow Y_{n12} = -\frac{I_2}{V_1} \tag{3}$$

$$I_2 = V_2 \cdot Y_{n22} + V_2 \cdot Y_{n12} \Rightarrow Y_{n12} + Y_{n22} = \frac{I_2}{V} \tag{4}$$

$$I_1 = -V_2 \cdot Y_{n12} \Rightarrow Y_{n12} = -\frac{I_1}{V_2} \tag{5}$$

This effectively determines the Norton equivalent admittances. These admittances must be determined for each harmonic frequency, each feeder station and for each of the feeding configurations, such as transformer outage, feeder station outage, etc.

To determine Norton equivalent source currents, a procedure similar to the one below has to be followed:

- all internal sources have to be re-activated,
- all conductors have to be short circuited to earth,
- the position of traction harmonic current generators (trains) must be changed in accordance with the traffic pattern (power demand and train position),
- train harmonic currents are injected at the location of the train and the currents flowing into the feeder station ($I_C$, $I_R$ and $I_G$ in Fig. 1) are calculated,
- harmonic current contributions from all trains in the sections are aggregated.

The steps above are repeated for each required frequency.

### *B. Frequency dependency of admittance*

Dependency of the catenary system parameters on frequency is very important and needs to be accurately represented. Frequency dependency of a typical catenary system resistance and inductance is shown in Fig. 3 [1]. The conductor resistance dependency on frequency is very noticeable and very important. The change in the conductor inductance may not appear to be high, but it does have an effect and it should not be neglected. The dependency of the catenary system parameters on frequency can be approximated using some exponential functions. For the particular case shown in Fig. 3, a good match is obtained using the following approximations:

$$R(h) = \frac{R_1}{2} \cdot \left(1 + h^{0.735}\right) \tag{6}$$

$$L(h) = L_1 \cdot h^{-0.05} \tag{7}$$

where $\boldsymbol{h}$ is the harmonic order, $\boldsymbol{R_1}$ and $\boldsymbol{L_1}$ are the fundamental frequency resistance and inductance, respectively.


Fig. 3. Typical catenary system parameters as a function of frequency

### C. *Train currents*

The trains move along the route in line with the timetable and will accelerate, coast, brake and stop at stations in accordance with the timetable and required driving strategy. Therefore, to accurately represent the train harmonic generation, each train movement needs to be represented using an electrical network representation and assigned an appropriate harmonic current generation depending on the mode of operation and magnitude of fundamental current. This very complicated process of detailed assessment has limited feasibility considering uncertainties and statistical nature of the harmonic distortion assessment, so a less complex yet robust way of representing the train movement can be adopted. This simplified approach should:

- take into account the different modes of train operation – such as motoring, coasting and braking – as each mode of operation produces a different harmonic spectrum,
- take into account the number of trains in the section for a specific length of time,
- detect any harmonic resonance points which will depend not only on frequency but also on the traction unit location along the line.

The train harmonic generation is typically provided as a dependency on the train mode of operation and the magnitude of the fundamental current. The best information on the harmonic generation of a train is normally supplied by the train manufacturer who would have undertaken extensive harmonic and EMI tests. A typical train will have a main transformer, several converters (rectifiers) and inverters supplying traction motors.

Train converters may operate with different switching patterns thus generating different harmonic profile depending on the mode of operation, primary current, speed etc. For instance, when a train is accelerating out of a station, a converter feeding a motor may go through several modes of control. In the first instance, a converter may generate fixed frequency output voltage such that the current through the motor remains constant and the voltage is increased. Once the initial acceleration phase is completed and the voltage equals nominal voltage, the converter may start increasing the frequency to accelerate the motors further. Higher speeds can yet be achieved by reducing the motor field. This acceleration phase may cause an overall different harmonic generation profile from the train. Furthermore, modern trains are equipped with regenerative braking capability to recover some of the train kinetic energy back to the grid. This operation will cause further change to the harmonic generation profile. Auxiliary systems are typically fed through a separate set of converters (to maintain constant voltage) and have a different harmonic generation profile. Because different train converter loadings as well as different converter modes of operation result in a different harmonic generation profile, the level of harmonic generation needs to be defined by an area; an example of which is shown in Fig. 4. Furthermore, different rolling stock may have different harmonic generation profiles. The harmonic generation profile may be different for different harmonic frequencies.

Each train travelling through the section will encounter different catenary system impedance determined by frequency and train location. It is even possible to reach a state of resonance causing harmonic current amplification. To account for this, it is necessary to move the train model along the catenary system model, carry out the harmonic current propagation calculations and determine the corresponding harmonic current at the feeder station.

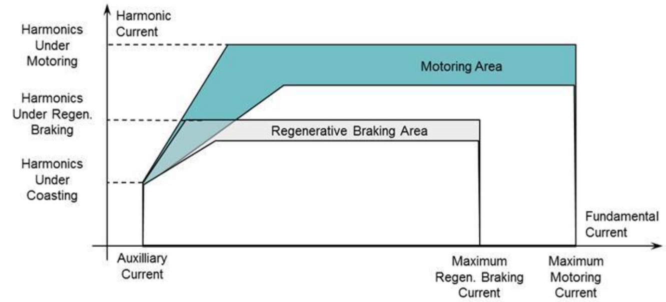


Fig. 4. Train Harmonic Current Generation

To represent a number of trains in a section on both up and down lines, the process of representing a train movement must be repeated taking into account the location of each train and the distance between the trains. The process must result in the harmonic currents at the feeder station for each time step.

The amplification of harmonic currents by the catenary system can be accounted for by first determining the transfer gain between the train and the feeder station. This is done by assigning the train a unity harmonic current generation and moving it along the section. The currents at the feeder station equal the transfer gain factor, remembering that the gain will be a complex number. This gain is a function of the location and therefore it is necessary to convert it into time dependence so that the time aggregation of harmonic generation can be performed as required for the derivation of the Norton equivalent circuit. The conversion process must take account of the train movement through the section, as illustrated in Fig. 5. The train speed is known and hence the train location can be determined, thus giving the train location as a function of time. Using the harmonic current gain function and the train location as a function of time, it is possible to map the harmonic current gain function to time as illustrated in Fig. 5.

It is generally sufficient to use discrete time periods, such as one minute, to do the conversion. Furthermore, it is sufficient to use average values for train position, harmonic generation, gain, etc. In any given period of time, a train will be at the (average) location determined from the train travel diagram. For this location, there is an average harmonic current gain that can be assigned to this location and so to this period of time. The process is repeated for all time periods the train travels through the section. The gain may be different for different currents (currents $\boldsymbol{I_C}$ , $\boldsymbol{I_R}$ and $\boldsymbol{I_G}$ in Fig. 1).

It is necessary to further determine the actual train harmonic currents as a function of time when the train passes through the section. This dependency is determined from the train movement diagram and the knowledge of the magnitude of the train fundamental current. The magnitude of the fundamental current is typically obtained from multi-train simulation studies or some other methodology. Different harmonic current generation values are assigned to the train based on the mode of operation and magnitude of the fundamental current. This results in the profile of train harmonic generation against the time as determined from the train location. To obtain the harmonic currents at the feeder station, the train harmonic currents must be multiplied by the gain factor determined previously for each time period.

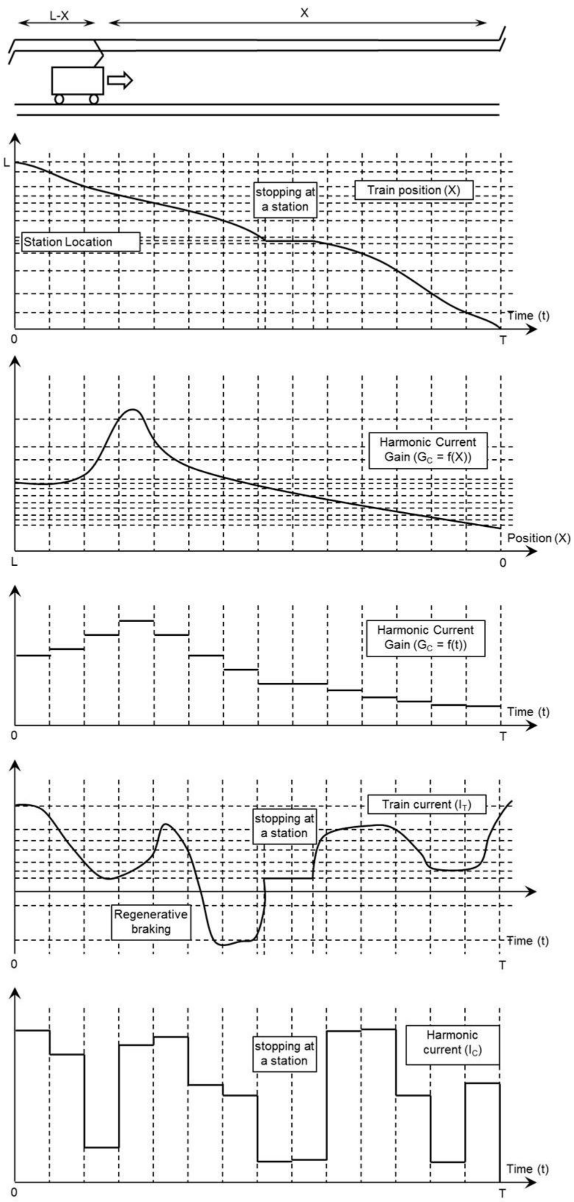


Fig. 5. Determining harmonic currents and gain as a function of time

### *D. Aggregation of harmonic currents*

The process described so far determines the harmonic current generation of a single train only. To complete the process, it is necessary to take account of multiple trains in the section, in line with the train service timetable. The process used to do this is illustrated in Fig. 6. For illustration purposes it is assumed that the timetable is such that one train enters the section every 5 minutes. This means in minute 6 there will be two trains in the section and the total harmonic current at the feeder station over the period of time between minutes 5 and 6 is a combination of the current from train 1 in minute 6 and the current of train 2 in minute 1. The process is repeated for a number of trains and it must include trains moving in the opposite direction. For the trains moving in the opposite direction, the process is the same but the train speed and so the location will be different and must be appropriately incorporated.

In this example it is assumed that the train converters operate as PWM controlled rectifiers and inverters and so the combined harmonic generation of different trains at different locations and under different modes of operation should not be arithmetically added. Instead, the harmonics from different trains are added as vectors. With reference to the IEC61000-3-6 [2], the harmonic summation law is based on the vector summation in the form of:

$$I_h = \sqrt[\alpha]{\sum_j I_{hj}^{\alpha}} \tag{8}$$

where $\boldsymbol{I_h}$ is the magnitude of the resulting harmonic current (order $\boldsymbol{h}$), for all trains in the section;

$\boldsymbol{I_{hj}}$ is the magnitude of each individual train harmonic generation (depending on time and train position) at feeder station (order $\boldsymbol{h}$);

$\boldsymbol{\alpha}$ is the summation exponent dependent on the harmonic order as shown in Table 1.

Different converters or train types may require a different summation law.

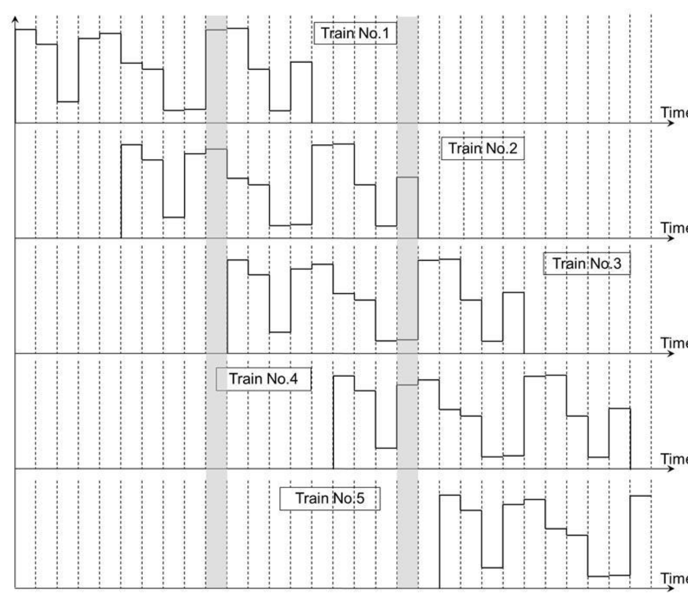


Fig. 6. Aggregation of Harmonic Currents from Multiple Trains

TABLE I
SUMMATION EXPONENT

| Harmonic Order | Exponent α |
|---|---|
| h < 5 | 1 |
| 5 ≤ h ≤ 10 | 1.4 |
| h > 10 | 2 |

Following this process it is possible to aggregate the contributions from multiple train units and so complete the Norton equivalent circuit representation of the electrical section. The process is repeated for each electrical section (if they are different) and under different operational scenarios, as required. The process may appear complex but once the calculations are set up, for example in a spread sheet program, the process is relatively quick. Further simplifications are possible depending on the train parameters, timetable and other requirements.

## III. Utility System

Information concerning system harmonic impedance is critically important during calculation of harmonic emission limits. This is because harmonic voltage distortion caused by any non-linear load connection is characterised not only by the harmonic current injection from that load but also by the network harmonic impedance it is connected to.

Due to variation between different power systems, it is extremely difficult to generalise a procedure that can be applied to all systems unilaterally. However, there are certain areas that need to be taken into account whilst undertaking harmonic load flow studies. These include:

- variation of system demand level,
- variation of generation merit order dependent on system demand,
- operational status of reactive power plant,
- network contingencies.

Although not a strict requirement, it is highly desirable to have a full network model for any harmonic analysis. In cases where this is not possible, a portion of the system is modelled and the part that is not modelled explicitly needs to be included in the analysis by way of utilising an equivalent system representation. The resistive part of impedance and/or susceptance of individual power system elements play a critical part in harmonic voltage distortion calculations as these are components introducing damping to the system. Therefore, their frequency dependency has to be captured properly and to achieve this recommendations provided by CIGRE can be followed [3, 4].

Frequency dependent data of overhead lines and cables are relatively well established. However the case for transformers, generators and loads is rather different. A resistance equal to 10% of the subtransient reactance corresponding to a subtransient time constant of 32ms is recommended for the conventional generators. The skin effect associated with the resistance should then be taken into account by:

$$R_h = \sqrt{h} R_1 \tag{9}$$

It is suggested that a similar skin effect correction is used for any system equivalents. It is of course preferable to have the frequency dependency of such an equivalent model by way of proper system reduction techniques. Frequency dependency of transformer resistance is an area where caution is advised as certain models used may introduce optimistic damping. For steady-state harmonic distortion studies, transformer resistance is made up of a resistance $\boldsymbol{R_s}$ in series with an assembly consisting of a reactance $\boldsymbol{X_h}$ in parallel with a resistance $\boldsymbol{R_p}$ as shown in Fig. 7. $\boldsymbol{X_h}$ is equal to $\boldsymbol{hX_1}$ and $\boldsymbol{R_s}$ and $\boldsymbol{R_p}$ are constant whatever the frequency and are given by:

$$R_s = \frac{X_1}{\tan \psi_1} \tag{10}$$

$$R_p = 10 * X_1 * \tan \psi_1 \tag{11}$$

with $\boldsymbol{\psi_1}$ taken from the following table if better knowledge is not available.

TABLE II
Coefficients for Transformer Models

| Transformer Rating S (MVA) | $\tan \psi_1$ |
|---|---|
| 1 | 2 |
| 10 | 10 |
| 100 | 32 |

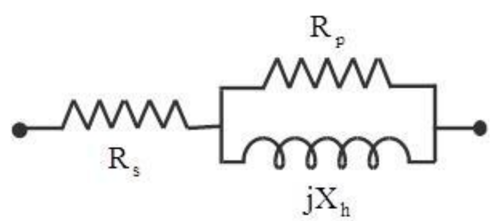


Fig. 7. CIGRE-recommended transformer model for harmonic studies [4]

Fig. 8 depicts a typical frequency sweep for a node with and without the resistance values for generators and transformers only. It is of interest to note that although the resonant frequencies are not altered, the amplitudes show large variation when no damping is considered.

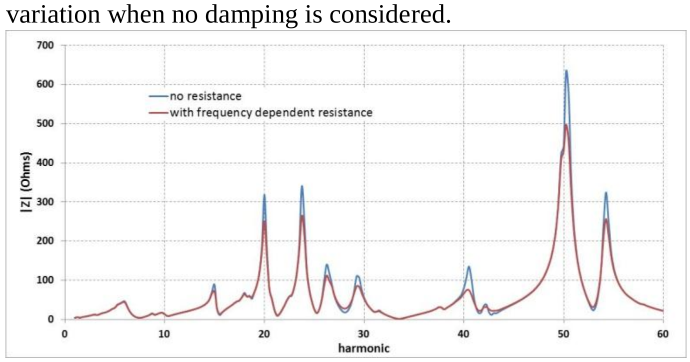


Fig. 8. The effect of damping on resonances

Modelling of loads is an area of difficulty as detailed knowledge into the composition of residential, commercial and industrial load is required for accurate representation. This type of information is normally not available in a straightforward fashion and also the composition of one particular load type may differ from country to country. As an example, residential load in United Kingdom is predominantly resistive in nature due to lighting and heating and can provide damping to any distribution system resonances. However, in other geographies, residential load may be dominated by inductive motor load due to air conditioning and hence less damping. In the absence of detailed information it is always preferable to represent loads by simple models consisting of parallel combination of *R* and *X*.

The existence of background harmonic voltage, in particular on the LV system, and the possibility of resonances within the system and between the new development and the existing system means that voltage magnification should also be checked. This is usually extended to location(s) remote from the point of common coupling. Usually distribution system capacitance and LV consumer load capacitance produces a network with a characteristic such that harmonic voltage produced at a transmission voltage level can be magnified at the LV, often appearing in the region of the $5^{th}$ harmonic. This is the dominant LV background harmonic in most systems and hence greater care is needed such that planning and/or compatibility levels are not exceeded. Voltage magnification is also a major issue with offshore renewable generation development where long cables are used to export the generated power onto the onshore system.

## IV. Estimation of Harmonic Voltage Distortion

Estimated harmonic currents from the source (in this case the traction system) are then injected into the utility transmission system model at all the connection points. The injected harmonic currents result in harmonic voltage distortion and this is normally termed incremental harmonic distortion. This calculation is normally undertaken for all combinations of identified scenarios.

There is also the voltage magnification component where the new connection alters the system impedance at the point of connection by effectively forming a voltage divider that could modify existing background harmonic voltage distortion. The combination of the two (i.e. the modified background harmonic distortion and the load injected harmonic distortion) is termed as aggregate harmonic distortion. The alteration of existing background harmonic distortion is not of major importance in the case of traction system electrification as the effect of traction system onto the larger transmission system impedance is not of major significance. However, calculated harmonic voltage distortion values were added onto the applicable background distortion to establish an aggregate distortion. This was achieved as described in ER G5/4-1; for the particular single harmonic which will have the largest summated magnitude, the measured and calculated values of voltage distortion were assumed to peak at the same time and to be in phase and for the other harmonics, an average phase difference of 90° was assumed.

## V. Conclusions

The paper presented a methodology of aggregating multiple harmonic current emitting loads for the purpose of harmonic voltage distortion studies. The case presented here was for a railway traction system consisting of multiple traction units, each treated as a separate load with variable harmonic current emission.

The methodology provides a simplified, yet sufficiently accurate, way of representing a “customer” installation so that the utility system designers/planners can undertake suitable compliance calculations and assessments. The methodology for the traction system side utilises Norton equivalent which can be developed with no knowledge of the utility system. This enables parallel modelling of the “customer” installation and the development of the utility network model. If the utility itself is undertaking the compliance studies, the procedure eliminates the need for the customer to consider the utility network.

The procedure described a traction power system as a classic case of a single development with multiple grid connections, dynamic loads and variable harmonic generation. It can be equally extended and applied to multiple wind farms or solar PV farms (or a combination of such loads) which are being developed at roughly the same time and need to be assessed simultaneously.

The methodology was extended to cover briefly the utility side of the modelling also, where great emphasis should be given to the availability of damping on the system. Finally, a short explanation is provided on the estimation of harmonic voltage distortion. The methodology thus explained has been successfully applied to a real traction system electrification.

## VI. References


[1] ITU Directives concerning the protection of telecommunication lines against harmful effects from electric power and electrified railway lines, Volume V, Inducing currents and voltages in power transmission and distribution systems, 2008.

[2] IEC TR 61000-3-6 Electromagnetic Compatibility, Limits – Assessment of emission limits for the connection of distorting installations to MV, HV and EHV power systems, Ed. 2, 2008.

[3] CIGRE Electra “AC System modelling for AC filter design – An overview of impedance modelling” No 164, February 1996.

[4] CIGRE Electra “Guide for assessing the network harmonic impedance” No 167, August 1996.

[5] ER G5/4-1 Planning Levels for Harmonic Voltage and the Connection of Non-linear Equipment to Transmission Systems and Distribution Networks in the United Kingdom, Energy Networks Association, October 2005.