On Tower Modeling using Fitting Techniques for Analysis of the Lightning Performance in Power Transmission Systems

E. C. M. Costa, A. J. G. Pinto, S. Kurokawa, B. Kordi and J. Pissolato

Abstract--**An approach using fitting techniques is proposed for modeling of tall steel towers for analysis of the lightning performance of transmission lines. The frequency-dependent surge impedance of a steel tower is modeled in the time domain using fitting techniques and an equivalent electric circuit composed of lumped elements. The proposed tower model is validated based on results obtained using the well-established Numerical Electromagnetic Code - NEC and inverse transforms. The contribution of the proposed model is the development direct in the time domain, enabling further interactions of the tower modeling with other time-variable power elements in the transmission system. Emphasizing that most of the transient events in power transmission systems are well-established in the time domain as time-variable elements during modeling and simulation processes. This same statement is not true in the frequency domain**.

***Keywords*: power transmission towers, time-domain modeling, atmospheric impulse, electromagnetic transients.**

I. Introduction

THE computational modeling of metallic towers is an important subject for analysis of lightning performance in power transmission lines. The analysis of the electromagnetic transients in tall metallic towers during an atmospheric surge is usually achieved based on the transmission line theory or antenna theory [1]. In the transmission line approach, currents and voltages are obtained from solving the well-established Telegrapher's equations of transmission lines as a function of the distributed electrical parameters of the tower. In the approach using the antenna theory, the towers are modeled based on the numerical analysis of full wave electromagnetic fields, *i.e.*, the tower structure is modeled as an antenna. These two categories provide a general classification for most of the models available in the technical literature on the modeling of tall tower/objects for simulation of electromagnetic transients.

In the modeling from the transmission line approach, tower is basically represented as a constant impedance, calculated from a simplified geometry of the tower structure [2], or a combination of a series of vertical and horizontal short transmission lines that represent each section and crossarm of the tower [3]. Modeling a tower as constant surge impedance is a simplistic solution but is not an accurate representation because the atmospheric discharge impulse is composed of a wide range of frequencies. Furthermore, the tower representation as constant impedance does not provide a detailed profile of voltages and currents through the tower. The currents and voltages at each segment can be calculated using the Telegrapher's equations that represent the transmission line model of the tower. However, representing a tower as a system of short line segments has some restrictions for tall towers/structures because it does not take into account the successive wave reflections between the sending and receiving ends of each segment and from the top to the tower foot. Thus, for tall towers, these equivalent line segments could be too long to neglect the current and voltage wave reflections between terminals [1]. Some possible restrictions were discussed in previous analyses on the lightning performance of towers with more than 180 meters height [4].

On the other hand, modeling towers using the antenna approach is based on the solution of the full wave Maxwell's equations, which is more general than transmission line approach where the transverse electromagnetic (TEM) is assumed to be the only propagation mode. The existing models of towers using transmission line or antenna theories are limited to simplistic geometries, such as cone or cylindrical forms. However, even with such simplification the results cannot be properly evaluated without an accurate numerical procedure [5].

The modeling and simulation process proposed in this paper combines the use of both the transmission line and antenna theories. In this paper, a full-wave-based electromagnetic solver is used for modeling and simulation of an arbitrary structure composed of thin wires that represents the metallic tower. This computational program is the Numeric Electromagnetic Code (NEC). In the NEC, the electromagnetic fields can be calculated for a simple conical or cylindrical structure as well as for a complex system of thin cylindrical conductors that represents the exact tower structure. NEC calculates the electromagnetic fields in the frequency domain using the method of moments and as such does not require discretizing the whole space [6]. In the proposed approach, the frequency-dependent surge impedance of the tower is calculated from an atmospheric impulse as an

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input signal at the top of the tower structure [7, 8]]. As a second step, the frequency-dependent surge impedance of the tower is fitted as a rational function, which can be represented as an equivalent *RLC* circuit. Thus, the frequency effect on the tower surge impedance can be represented directly in the time domain, without further numerical transformations [9]. The same frequency-domain current and voltage, calculated from the NEC, are used as reference values in order to validate the proposed time-domain modeling technique. The reference current and voltage are obtained in the time domain using Laplace transform.

II. Calculation of the Surge Impedance Using the NEC

The Numerical Electromagnetic Code (NEC) is a computer program applied for analyzing the electromagnetic response of antennas, steel structures and conductor surfaces. The code is based on the numerical solution of integral equations by the method of moments, combining an electrical-field integral equation for modeling thin wires with a magnetic-field integral equation for closed perfectly conducting surfaces [6].

The basic elements for modeling using NEC are short and straight segments for modeling wires and flat patches for modeling surfaces. A given system and any other conducting objects in its vicinity must be modeled with strings of segments following the paths of wires. Thus, the adequate choice of the segments and patches for modeling is the most critical step to obtaining accurate results. Furthermore, the number of segments and patches should be the minimum required for the proper representation of the given system [6].

In the NEC, a wire segment is defined by the coordinates of its two end points and radius of the transversal section. Modeling a wire structure with segments involves both geometrical and electrical characteristics. Geometrically, the segments should follow the paths of conductors as closely as possible, using a piece-wise linear fit on curves. The main electrical consideration is the segment length Δ relative to the wavelength λ at every frequency of the analysis spectrum. Usually, Δ should be less than 0.1λ, however, somewhat longer segments may be acceptable on long wires with no abrupt changes while shorter segments, less than 0.05λ, may be needed in the modeling of critical regions of an antenna or any other structure. The size of the segments determines the resolution in solving for the current of the model, since the current is computed at the center of each segment. The wire radius *r* relative to the wavelength λ is limited by the approximations used in the kernel of the electric field integral equation. The NEC uses the thin-wire approximation, neglecting transverse currents on wires and assuming that the axially directed current is uniformly distributed around the segment surface.

Following the boundary conditions for wire modeling in the NEC, an example of a generic metallic structure represented by wire segments is shown in Fig. 1.

The simplified structure in fig. 1 is representing a generic tower using twelve wires, s_1 to s_{12}. A current source is applied at point *T* at the top of the structure whereas the base of the tower is grounded at point *G*. The current source provides an input current signal *I*(ω), resulting a potential *V*(ω) between the points *T* and *G*.

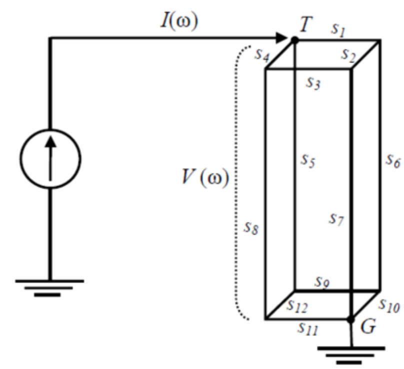

Fig. 1. Generic structure modeled as wire segments using the NEC.

The frequency-domain surge impedance of the tower *Z*(ω) is defined as [1]:

$$Z(\omega) = \frac{V(\omega)}{I(\omega)} \tag{1}$$

A frequency-domain alternative to the surge impedance is the harmonic impedance [8]. It is not dependent on the excitation signal. The harmonic impedance depends solely on the geometry and electromagnetic characteristics of the tower. This approach for the surge impedance is not suitable for modeling of nonlinear phenomena. On the other hand, this method is suitable for modeling frequency-dependent characteristics, such as the soil conductivity [1].

The harmonic impedance is obtained considering the input current signal as an unitary impulse. Since the impulse function is composed of a wide and constant range of frequencies, the harmonic impedance can be obtained by determining the voltage *V*(ω) as a response to the input signal *I*(ω) = 1 A. Thus, the surge impedance, or harmonic impedance, represents a frequency range up to the highest frequency of interest for the transient analyses. The harmonic impedance is expressed as [8]:

$$Z(\omega) = Z_h(\omega) = \frac{V(\omega)}{I_h(\omega)} \tag{2}$$

Terms $Z_h(\omega)$ and $I_h(\omega)$ are the harmonic impedance and impulse current signal.

Based on the definition of harmonic impedance, the time-domain voltage *v*(*t*), resulted from the excitation current *i*(*t*), can be obtained by inverse Fourier transform:

$$v(t) = \mathbf{F}^{-1}\{V(\omega)\} = \mathbf{F}^{-1}\{I(\omega).Z(\omega)\} \tag{3}$$

Since the surge impedance obtained from an unitary impulse is independent of the excitation input signal, the surge impedance can be expressed for any input signal as:

$$Z(\omega) = R(\omega) + X(\omega) \tag{4}$$

The surge impedance in general is composed of a frequency-dependent resistance *R*(ω) and an imaginary part that represents a reactance *X*(ω).

III. Fitting the Surge Admittance in the Time Domain

There are several fitting techniques available in the technical literature [9, 10, 11]. However, the Vector Fitting algorithm [9] has presented a great accuracy for smooth and resonant responses with high order in wide frequency bands. Furthermore, Vector Fitting has been widely used for development of frequency-dependent transmission line models in the time domain for simulation of electromagnetic transients in power systems [12, 13, 14] and transformers [15].

Vector Fitting approximates a general function $F(\omega)$ in the frequency domain by a rational function based on tabulated values of residues and poles. Thus, since the surge impedance $Z(\omega)$ of a tower is obtained from a given input current or voltage signal, it can be approximated by a rational function $F(\omega)$ in terms of an admittance function $Y(\omega)$ as follows:

$$Z^{-1}(\omega) = Y(\omega) \approx F(\omega) = \sum_{n=1}^{N} \frac{c_n}{j\omega - a_n} + d + j\omega h \tag{5}$$

The rational function in (5) is composed of a sum of N terms, where c_n and a_n denote the n-th residue and pole, respectively. The N residues and poles of $F(\omega)$ can be both real quantities or complex conjugate pairs. The constant and proportional terms are expressed by d and h, respectively. The number of residues and poles is directly proportional to the fitting frequency range of $F(\omega)$ and the required fitting accuracy.

A circuit whose terminal admittance is described in (5) can be synthesized as shown in Fig. 2 [9, 10].

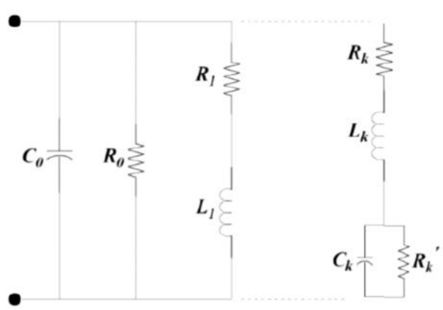

Fig. 2. Representation of the surge admittance $Y(\omega)$ by electric circuit.

In this figure, the constant capacitance C_0 is given by the proportional term h and the resistance R_0 is obtained from the constant term as $1/d$. The remaining branches of the equivalent circuit are obtained from the N residues and poles that can be summarized as two cases; 1) real residues and poles; and 2) complex conjugate pairs [10].

In the first case, if the residues and poles are real, the following analogy is given:

$$\frac{c_n}{j\omega - a_n} = \frac{{}^{1}/_{L_1}}{j\omega - {}^{R_1}/_{L_1}} \tag{6}$$

which results in

$$c_n = \frac{1}{L_1} \quad ; \quad a_n = -\frac{R_1}{L_1} \tag{7}$$

Thus, if both c_n and a_n are real, R_1 and L_1 can be calculated for $n = 1$ as follows:

$$R_1 = -\frac{a_n}{c_n} \quad ; \quad L_1 = \frac{1}{c_n} \tag{8}$$

The elements R_1 and L_1 form a series RL branch, as shown in Fig. 2. If the Vector Fitting algorithm calculates more than one real residue and pole, the equivalent circuit in fig. 2 will have more than one RL branches, as many as the number of real residues and poles.

For residues and poles are also obtained from complex conjugate pairs:

$$G_k(\omega) = \frac{c_k}{j\omega - a_k} + \frac{c_k^*}{j\omega - a_k^*} \tag{9}$$

The complex conjugate in (9) represents the RLC branch in Fig. 2 (R_k, R'_k, L_k and C_k). A relationship of the rational function in (9) and the referred R, L and C terms can be expressed as follows [9, 10]:

$$G_k(\omega) = \frac{1}{L_k} \frac{\left(j\omega + \frac{1}{R'_k C_k}\right)}{\left(\omega^2 + \left(\frac{R_k}{L_k} + \frac{1}{R'_k C_k}\right) j\omega + \left(\frac{R_k}{L_k}\frac{1}{R'_k C_k} + \frac{1}{L_k C_k}\right)\right)} \tag{10}$$

Analogously in (8), the lumped elements of the equivalent circuit in Fig. 2 and in (10), for the case of a pair of complex conjugates, are expressed as follows:

$$R_k = \frac{1}{c_1 + c_2}\left[-(a_1 + a_2) + \frac{c_1 a_2 + c_2 a_1}{c_1 + c_2}\right] \tag{11}$$

$$R'_k = -\frac{1}{C_k}\frac{c_1 a_2 + c_2 a_1}{c_1 + c_2} \tag{12}$$

$$C_k = \frac{c_1 + c_2}{a_1 a_2 + \left[-(a_1 + a_2) + \frac{c_1 a_2 + c_2 a_1}{c_1 + c_2}\right] \times \frac{c_1 a_2 + c_2 a_1}{c_1 + c_2}} \tag{13}$$

$$L_k = \frac{1}{c_1 + c_2} \tag{14}$$

Therefore, the equivalent circuit in Fig. 2 can be established from (5), based on the constant terms and on the type of residues and poles, which can be both real or complex conjugates.

IV. Calculation of the Tower Circuit Using the NEC

As described in the previous section, the surge admittance $Y(\omega)$ of a tower can be approximated by a rational function and synthesized with an equivalent RLC circuit (Fig. 2). Thus, for validation of the proposed model using lumped elements of electric circuits, the reference surge admittance is obtained from the NEC.

The tower modeling using the NEC was established in the technical literature based on field measurements and experimental results. The tower is represented as a grid composed of thin wires with variable position, orientation and radius [5, 15, 16]. The surge current and voltage in the tower are calculated and the surge impedance is obtained in the frequency domain, as denoted in (1).

In addition, the time-domain currents and voltages in the tower can be also calculated using the NEC and inverse transforms. These time-domain results are also considered as reference values for validation of time-domain simulations using the proposed tower model.

Frequency- and time-domain results are obtained considering a double-circuit power transmission tower with 120 meters high [15, 16]. The geometrical characteristics of the tower modeled in the NEC are described in Fig. 3(a) and the geometry of the actual tower is shown in Fig. 3(b).

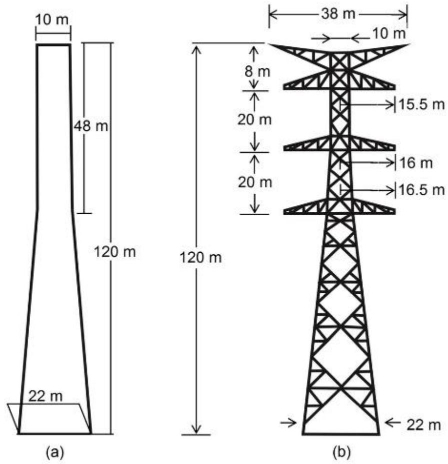

Fig. 3. Tower structure: modeled in the NEC (a) and real dimension (b).

In modeling the tower using NEC the horizontal elements have a small influence on the tower surge characteristics. Nevertheless, the crossarms result in some distortions in the voltage waveform at the top of the tower that are smoothed by the slant elements in the tower representation, as demonstrated in experimental results obtained from real towers [16].

All wire segments and ground are treated as ideal conductors. Frequencies from 19 kHz up to 10 MHz are considered in the analyses. This frequency band covers all electromagnetic transients that power transmission systems are subject to [5, 15].

The tower modeling and simulation are carried out in the NEC, as prior discussed, and the proposed tower model is developed in the *Electromagnetic Transient Program* (EMTP) using the equivalent *RLC* circuit shown in Fig. 2. The principal difference between the two models is that the first one is developed in the frequency domain and the time-domain results are obtained using inverse transforms whereas the proposed model is developed directly in the time domain, without inverse transforms.

The simulations obtained from the proposed model take into account a transient current source with a wave front of 1.2 μs and tail of 50 μs [7]. This wave shape can be modeled using the Heidler's function that is capable of synthesizing transient waveforms with continuous derivatives and wave front inclination similar to field measurements [6, 7]. The Heidler's function is expressed as:

$$I(t) = \frac{F_0}{\eta}\frac{(t/\tau_1)^n}{1+(t/\tau_1)^n}e^{(-t/\tau_2)} \tag{15}$$

The correction factor denoted by η is expressed as follows:

$$\eta = exp\left[-(\tau_1/\tau_2)(n\tau_2/\tau_1)^{(1/n)}\right] \tag{17}$$

The term F_0 is the signal magnitude of 3.5 kV , τ_1 is a wave-front constant of 1.25 μs, τ_2 is a wave-tail constant of 0.2 μs and n is a constant exponent of 2.5 [6, 7]. From the function in (15) and a lumped resistance of 5 kΩ connected in series with $I(t)$, an atmospheric impulse is modeled as a 1.2/50 μs current source at the top of the tower [7]. Another attribution of the proposed modeling methodology is that the soil conductivity is taken into account in the NEC during the surge impedance calculation, which means that the fitting procedure also considers the soil characteristics in the tower parameters.

V. Calculation of the Tower Circuit Using the NEC

The frequency-domain surge impedance calculated from the NEC is described in real and imaginary terms in Figs. 4 and 5 (solid curves), respectively. The frequency-domain response for a very wide range is possible using antenna theory and numerical electromagnetic methods (NEC). However, models based on transmission line theory are not appropriated for very high frequencies, *e.g.* an impulse with front time lower than 1 μs, typical of subsequent return strokes [17].

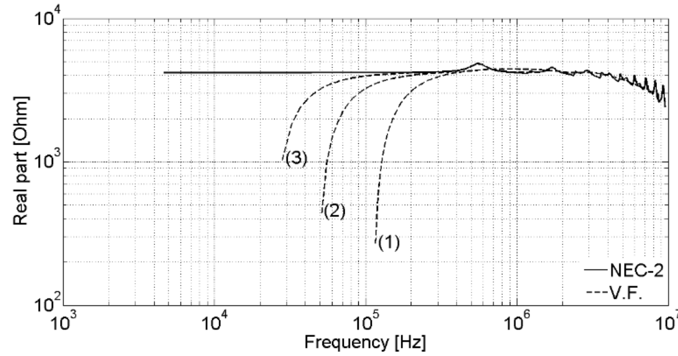

Fig. 4. Real part of the surge impedance fitted with 10 (curve 1), 30 (curve 2) and 50 (curve 3) poles.

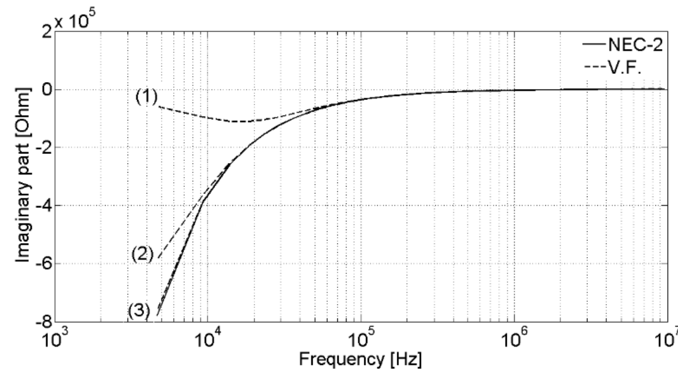

Fig. 5. Imaginary part of the surge impedance fitted with 10 (curve 1), 30 (curve 2) and 50 (curve 3) poles.

The fitting of the real and imaginary parts of the surge impedance are also shown in Figs. 4 and 5, respectively, for a variable number of poles. The dotted curves labeled as 1, 2, and 3 are fitted by 10, 30 and 50 poles, respectively, whereas the solid curve represents the reference surge impedance obtained using NEC.

More significant variations are observed in the real term of the impedance at lower frequencies, as observed in Fig. 4. However, these deviations in the real term could be mitigated with more than 50 poles for fitting the surge impedance. On the other hand, the imaginary part described in Fig. 5 is fitted with good accuracy with 50 poles.

The errors between the module of the surge impedances, calculated using the NEC $Z(\omega)$ and using fitting technique $Z_{fit}(\omega)$, are described in Fig. 6 considering 10, 30 and 50 poles in the fitting.

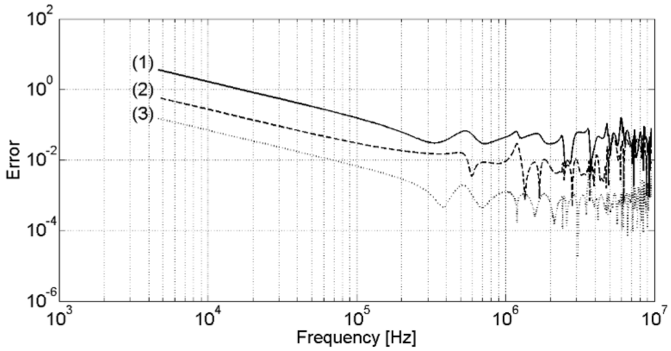

Fig. 6. Errors in the tower impedance module using the vector fitting with 10 (curve 1), 30 (curve 2) and 50 (curve 3) poles.

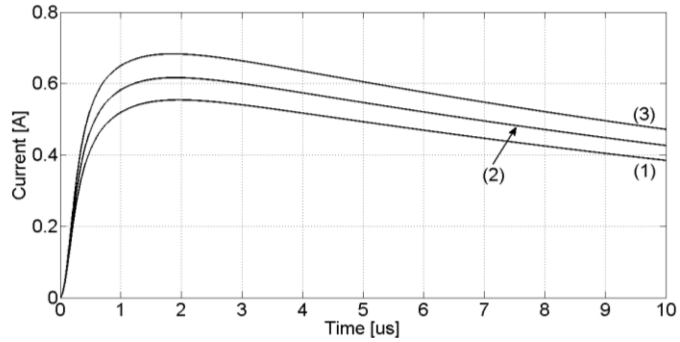

Fig. 7. Surge current through the tower simulated using the proposed time-domain model with 10 (curve 1), 30 (curve 2) and 50 (curve 3) poles.

Figure 6 proves that the imaginary term is predominant in the surge impedance magnitude. More expressive errors are observed in the impedance fitting for 10 poles at frequencies no greater than 10 kHz. On the other hand, the vector fitting produces accurate results for the entire range of frequencies when 30 or 50 poles are considered, as demonstrated in the curves 2 and 3 in Fig. 6. The imaginary term, represented by the curve 3 in fig 5, is practically overlapped on the reference curve obtained from the NEC. The resistance values presented inaccuracies even for 50 poles for frequencies up to 100 kHz. However, as the imaginary part is significantly predominant over the entire range of frequencies, low relative errors in the modulus of $Z_{fit}(\omega)$ are observed for 30 and 50 poles in Fig. 6.

Figures 4 and 5 show that the surge impedance of the tower has capacitive characteristics for the range of frequencies from 20 kHz up to 200 kHz, inductive behavior over 200 kHz and describes a resistive characteristic for the entire range of frequencies. The real part represents the losses due to the current wave propagation through the metal structure of the tower.

The variations observed in the frequency domain can be better evaluated in the time domain, from transient simulations considering an atmospheric impulse applied at the top of the tower that is modeled based on the geometrical and technical descriptions in Fig. 3.

VI. Transient Simulations in the Time Domain

The time-domain model using vector fitting is validated in the time domain based on results obtained direct from the NEC. Since the transient current of the tower is obtained using NEC in the frequency domain, the time-domain values can be calculated by using inverse Fourier transform [18]. Thus, the time-domain results, obtained using the NEC and inverse transform, can be considered as reference values for validation of the proposed modeling using fitting techniques.

The same 1.2/50 µs current source, modeled using the Heidler's function in (15), is applied at the top of the tower. The transient current is simulated directly from the frequency-domain parameters calculated using NEC and from the proposed time-domain model using 10, 30 and 50 poles for fitting of the surge impedance of the tower. The equivalent electric circuit of the proposed model is modeled in the EMTP. Figure 7 shows the transient current through the tower using the proposed model with a variable number of poles: 10, 30, and 50.

The number of residues and poles considered in the fitting has significant influence on the waveform of the surge current, as observed in the peak and tail magnitudes of the curves 1, 2 and 3. With increasing the number of residues and poles, the transient currents are characterized with a larger current peak, as observed in Fig. 7.

In Fig. 8, the transient current simulated by NEC is compared to the transient simulated using the proposed model considering 50 poles in the fitting of the tower parameters.

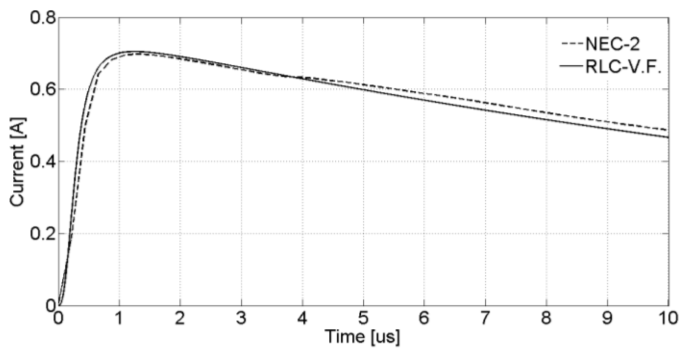

Fig. 8. Surge current through the tower simulated using the proposed model with 50 poles.

The transient current profile simulated using the proposed model is very similar to the same current simulated from the NEC by using inverse transform. The peak of both curves are approximately similar as well as the tail profile. A few variations are observed in the tail profile of the two curves, probably because of inaccuracies during the fitting of the real part of the surge impedance for frequencies lower than 100 kHz, as shown in Fig. 4. However, this problem can be easily solved increasing the number of residues and poles during the surge impedance fitting, as demonstrated in other technical references [9, 14].

VII. Conclusion

A methodology for modeling steel towers was proposed for the evaluation of lightning performance of transmission lines. The proposed tower model was developed directly in the time domain based on fitting techniques to approximate the surge impedance of the tower as an equivalent rational function. From the residues and poles of the equivalent rational function, the surge impedance of the tower can be represented as an equivalent lump-element electric circuit directly in the time domain, without use of inverse transforms.

The performance of the proposed model depends on the number of residues and poles considered for fitting the surge impedance as well as the bandwidth of input signal on the tower. If the input signal is composed of a wide range of frequencies, such as the atmospheric impulse considered in this research, more than 50 residues and poles are required for fitting the surge impedance. Otherwise, for transient signals composed of a smaller range and/or low frequencies (e.g. a switching transient), not more than 30 poles are required. Thus, the proposed model can be applied for simulation of any electromagnetic transient on power transmission towers by selecting an adequate number of residues and poles from the fitting procedure.

The performance evaluation of power transmission and distribution systems under lightning involves several other power components such as power transformers, line conductors, insulators and metal-oxide surge arresters for lightning protection. The modeling in the time domain of these power components are well established in the technical literature. On the other hand, due to the nonlinearity of such components, they are not usually modeled in the frequency domain. This leads to restrictions for applicability of power transmission lines towers in the frequency domain using the NEC or any other approach in the frequency domain. The direct representation in the time domain is the major advantage of the proposed model, enabling the inclusion of any other power device together with the tower modeling and thus a global analysis of the power transmission system as a whole.

Further research could be proposed including the frequency-dependent conductor modeling (phases and ground wires) for a line section composed of several towers, where the lightning performance could be evaluated considering the inclusion of line surge arresters. The design of transmission lines with surge arresters connected directly at the towers has been an emergent and alternative technology in lightning protection and could be investigated with more details using the proposed tower model.

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