On the Influence of Earth Conduction Effects on the Propagation Characteristics of Aerial and Buried Conductors

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Abstract—In this paper, the propagation characteristics of transmission line configurations located above and below imperfect earth are investigated, by means on generalized and approximate earth formulations. The generalized formulations consider earth conduction effects both on series impedances and shunt admittances of the conductors, contrary to the approximate earth representations that most transient simulation software implement. Transient simulations are also performed to evaluate the impact of both earth approaches from a practical point of view.

Keywords: Aerial conductors, buried conductors, earth admittance, earth conduction effects, electromagnetic transients.

I. INTRODUCTION

Wave propagation characteristics of aerial and buried conductors are calculated using the per-unit-length (pul) series impedance and shunt admittance parameters, where a key factor is the accurate representation of earth conduction effects. Typically, in most pul calculation routines, the influence of imperfect earth is considered only by means of impedance earth correction terms for both aerial and buried conductor arrangements. The corresponding calculation of the pul shunt admittances only assume earth as perfect conductor.

Specifically, for overhead conductors extensive research has been carried out with the most known formulation being proposed by Carson in 1926 [1] and further extended by Wise [2] to include the influence of displacement currents. Wise proposed formulas to represent earth conduction effects also for shunt admittances in aerial conductor arrangements. Based on the same considerations, a generalized solution applicable also to multiconductor arrangements was proposed in [3], [4], using approximated logarithmic expressions. Although the above formulations significantly improved the representation of earth conduction effects, they rely on the assumption of the quasi-TEM field propagation. To overcome the quasi-TEM limitation, full-wave electromagnetic models based on the exact electromagnetic (EM) field solution have been proposed in [5]-[7].

On the other hand, research on buried conductors has been mainly focused on the improvement of numerical instability issues on the well-known Pollaczek’s formula [8] for the ground impedance, but also to propose more generalized formulations as Sunde did in 1968 with the inclusion of the influence of displacement currents [9]. Similar to overhead conductors, typically the influence of the imperfect earth is neglected on the calculation of the shunt admittance parameters. Thus, single-core (SC) cable shunt admittances are considered only by means of the cable insulation properties and consequently the mutual shunt admittances between adjacent cables are taken equal to zero. Although this assumption leads to satisfactory results at frequencies up to some kHz, several works have shown that significant deviations are observed at higher frequencies, challenging the validity of the specific imperfect earth formulation [10]-[14]. Recently, a new earth model for transient simulations involving underground power cables has been proposed in [12]. This approach is suitable for representing the influence of earth over a wide frequency range, by including earth conduction effects also on the calculation of shunt admittances.

Scope of this paper is to analyze the influence of earth conduction effects on wave propagation characteristics of power transmission lines, including most types of arrangements, i.e. overhead lines, aerial and underground cables. The wave propagation characteristics are analyzed considering the frequency dependent (FD) behavior of earth and results are further assessed by means of electromagnetic (EM) transient simulations.

This paper is organized as follows: In Section II, the problem formulation is analyzed. In Section III, the methodology for the analysis of the propagation characteristics of the transmission line configurations is described. In Sections IV, V and VI results of wave propagation characteristics and transient responses for the overhead conductor, aerial and underground cable configurations are presented. The paper is concluded in Section VII.

II. EARTH IMPEDANCE AND ADMITTANCE PARAMETERS

A. Problem formulation

Let us consider two thin wires situated in the topology of Fig. 1 with vertical distances from the air-earth surface, \( h_i \) and \( h_0 \), respectively. Air and earth correspond to media 1 and 2, respectively. Specifically, the EM properties of air are \( \varepsilon_0, \mu_0 \) and those of earth as \( \varepsilon \) and \( \mu \).
= 0) and of earth \( \varepsilon_i, \mu_i, \sigma_i \), while the corresponding propagation constants are defined as:

\[
\gamma_m = \sqrt{j \omega \mu_m (\sigma_m + j \omega \epsilon_m)},
\]

where index \( m \) can take values 0 and 1. The two wires can be located either in air or earth. The influence of earth conduction effects on the pul parameters of the wires are represented by means of the self and mutual earth impedance and admittance terms. The mutual earth impedance and admittance is given by the general form in (2) and (3), respectively.

\[
Z'_e = \frac{j \omega \mu_m}{2 \pi} J_m, \tag{2}
\]

\[
Y'_e = j \omega P_m^{-1}, \tag{3}
\]

with \( m \) determined by the location of wires, consequently defining complex functions \( J_m \) and \( P_m \), which are analyzed in detail in the following sub-sections. The corresponding self pul parameters are derived by replacing \( h_i \) and \( h_j \) with common distance \( h \) [12, 15].

Fig. 1. Two-wire configuration in two media space.

### B. Overhead conductors and aerial cables

In the case of overhead conductors, i.e. \( m = 0 \), \( J_0 \) and \( P_0 \) are given by (4) and (5), respectively [15].

\[
J_0 = \ln \left( \frac{D_0}{d_0} \right) + \int_0^\infty F_0(\lambda) \cos (y_0 \lambda) d\lambda, \tag{4a}
\]

\[
F_0(\lambda) = \frac{2 \mu_0 e^{-\alpha_i (h + h_i)}}{\alpha_i \mu_0 + a_0 \mu_i}, \tag{4b}
\]

\[
P_0 = \frac{1}{2 \pi \alpha_i} \left( \ln \left( \frac{D_0}{d_0} \right) + \int_0^\infty G_0(\lambda) \cos (y_0 \lambda) d\lambda \right), \tag{5a}
\]

\[
G_0(\lambda) = \frac{2 \mu_0 \gamma_0^2 (\mu_0 a_0 + a_0 \mu_0) e^{-\alpha_i (h + h_i)}}{(a_0 \mu_0 + \alpha_i \mu_i) (a_0 \gamma_0^2 \mu_0 + a_0 \gamma_0^2 \mu_i)}, \tag{5b}
\]

where \( D_0 = \sqrt{y_0^2 + (h_i + h_j)^2} \), \( d_0 = \sqrt{y_0^2 + (h_i - h_j)^2} \), \( a_0 = \sqrt{\lambda^2 + \gamma_0^2 + k_s^2} \) and \( k_s = \omega \sqrt{\mu_0 \epsilon_0} \) [2], [15]. The formulas of (4) and (5) were originally proposed by Wise [2] and Kikuchi [5] and due to their generalized form other approximate approaches can be derived as follows [15]:
- Neglecting \( \omega_0 \) and setting \( \mu_1 = \mu_0 \), (4) is reduced to Sunde’s impedance formula. This formula has been used for the simulation of fast-wave transients in overhead multiconductor configurations [16], while neglecting the influence of the imperfect earth on the conductor admittances, i.e. neglecting the semi-infinite integral term of (4b). This approach will be named as “Sunde’s” approach in this paper and can be implemented in ATP-EMTP, using the CABLE PARAMETERS routine [17].
- By further neglecting the influence of displacement currents on the pul impedances, i.e. setting \( \varepsilon_i = 0 \), (4) is reduced to the widely-used approach of Carson [1].

The above formulations apply both to overhead conductor and aerial cable configurations, using the methodology of [18], accordingly.

### C. Buried conductors

In the case of buried conductors, \( m \) is equal to 1, thus \( J_1 \) and \( P_1 \) take the form of (6) and (7), respectively [12].

\[
J_1 = \int_0^\infty F_1(\lambda) \cos (y_0 \lambda) d\lambda, \tag{6a}
\]

\[
F_1(\lambda) = e^{-\alpha_i h_i} e^{-\alpha_j (h + h_j)} + \frac{2 \mu_0}{\alpha_i^j + a_0 \mu_j} \tag{6b}
\]

\[
P_1 = \frac{j \omega}{2 \pi (\sigma_1 + j \omega \epsilon_1)} \int_0^\infty \left( F_1(\lambda) + G_1(\lambda) \right) \cos (y_0 \lambda) d\lambda, \tag{7a}
\]

\[
G_1(\lambda) = \frac{2 \mu_0 \alpha_i (\gamma_0^2 - \gamma_0^2) e^{-\alpha_i (h + h_j)}}{(a_0 \mu_0 + \alpha_i \mu_j) (a_0 \gamma_0^2 \mu_0 + a_0 \gamma_0^2 \mu_j)}, \tag{7b}
\]

where \( \alpha_i = \sqrt{\lambda^2 + \gamma_0^2 + k_s^2} \) and \( k_s' = \omega \sqrt{\mu_0 \epsilon_0} \). From this generalized formulation, the following approximate homogeneous earth approaches for buried cables can be reproduced [12, 19]:
- Following the assumption of wave propagation at low-frequencies (LF), i.e. the propagation constant is equal to zero \( (k_s' = 0) \) [19], (6) results in the impedance formula proposed by Sunde for the case of buried conductors. Typically, for the calculation of the pul admittance parameters, earth is considered as a perfect conductor, thus the shunt admittance term of (7) is neglected. This approach will be also named as “Sunde’s” approach in this paper, regarding buried configurations.
- By further neglecting the effect of displacement currents on the pul impedances, (6) is reduced to the well-known formula proposed by Pollaczek [8].

The total pul impedance and admittance matrices of underground cable systems are derived by means of [12], [18].

### III. Wave Propagation Characteristics Analysis

The propagation characteristics of the examined two-wire configurations are decomposed to the corresponding modal properties [20]. The two-wire arrangements include overhead conductors, aerial and underground cable systems. The ratio of the propagation characteristics defined in (8) is calculated to compare the results between different earth approaches.

\[
\text{ratio} = \frac{\text{propagation characteristics}_{\text{generalized}}}{\text{propagation characteristics}_{\text{approximate}}}, \tag{8}
\]
where, in the numerator, the wave propagation characteristics are calculated by means of the generalized earth formulations of (4)-(5) and (6)-(7), respectively, while, in the denominator, the approximate earth approaches are considered. Additionally, the propagation characteristics are analyzed by taking into account the FD behavior of earth that is described using the critical frequency $f_{cr}$, defined by:

$$f_{cr} = \frac{\epsilon_1}{2\pi\sigma_1\epsilon_0}.$$  \hfill (9)

In brief, for frequencies below $f_{cr}$, earth behaves as a conductor, while as the frequency increases the displacement and resistive currents become comparable and the earth behaves both as conductor and insulator. Finally, at higher frequencies the displacement currents are predominant and the earth behaves as an insulator [15].

**IV. OVERHEAD CONDUCTORS**

In the examined overhead two-wire configuration, the resistivity of the conductors is $\rho_c = 3.8610^{-08}$ $\Omega\cdot m$ and $h_1 = h_2 = 10$ m. The propagation characteristics ratios are calculated by means of the generalized approach proposed by Wise and the approximate of Carson. In Figs. 2 and 3, the ratio of the mode attenuation constant and velocity is presented for different values of earth resistivity, while $\epsilon_1 = 10\epsilon_0$.

![Fig. 2. Overhead conductor attenuation constant ratios of a) mode #1 and b) mode #2. Influence of earth resistivity.](image)

![Fig. 3. Overhead conductor velocity ratios of a) mode #1 and b) mode #2. Influence of earth resistivity.](image)

Differences in the modal characteristics are observed for both modes, whereas higher ratios are calculated for the attenuation constant term. Deviations in the mode velocities start at slightly higher frequencies than the corresponding of the attenuation constants. Specifically, the mode attenuation constants calculated by the two earth approaches deviate at frequencies higher than 10% of $f_{cr}$.

To investigate the influence of displacement currents on wave propagation, the ratios of the propagation characteristics are calculated, considering the approximate earth formulation of Sunde in (8). Earth resistivity is 500 $\Omega\cdot m$, while earth permittivity is varying. Results are analyzed for both mode attenuation constants in Figs. 4a and 4b, respectively. It is shown that as earth permittivity increases, smaller differences are observed between the generalized and the approximate formulation [21]. Earth impedance calculations are improved using Sunde’s approach compared to Carson’s by taking into account the influence of displacement currents. However, deviations with the generalized formulation still remain observable at frequencies higher than 1 MHz, revealing the significance of the influence of imperfect earth on shunt admittances.

![Fig. 4. Overhead conductor attenuation constant ratios of a) mode #1 and b) mode #2. Influence of earth permittivity.](image)

![Fig. 5. Transient responses at the receiving ends of overhead conductors, a) excited conductor and b) induced voltage.](image)
Finally, the configuration of the two overhead conductors is assumed with length equal to 1000 m. At the sending end of the conductor \( i \) a lightning impulse (LI) 1.2/50 \( \mu \)s of 1 pu amplitude voltage source is connected, whereas all remaining ends of the two conductors are open. The transient responses are simulated by means of the earth approaches of Wise, Sunde and Carson, considering \( \rho_1 = 500 \ \Omega \cdot m \) and \( \varepsilon_{r1} = 10 \), using the FD model of [22]. In Figs. 5a and 5b, the responses at the receiving ends of the excited conductor \( i \) and of conductor \( j \) are presented. In general, results obtained following Sunde’s and Carson’s earth approaches are similar. Therefore, displacement currents might have minor effect on the simulation of HF phenomena. Comparing results obtained by the generalized and the approximate formulations reveal that all simulated responses in Fig. 5a are practically identical, while for the induced voltages small differences are observed in Fig. 5b. In cases of poor earth conductivity that typically present also low permittivity [10], the influence of earth effects on the earth admittance must be taken into account if high accuracy is required.

V. AERIAL CABLES

In the aerial cable configuration, two similar cables are considered. Details on the cable geometry and properties can be found in [23]. First, the influence of the cable height is investigated, assuming both cables located at varying heights \( h_1 = h_2 \) above earth. The attenuation constant ratio of mode \#1 and \#2 is presented in Fig. 6, by means of the approaches of Wise and Carson. In all cases, earth resistivity and relative permittivity are 500 \( \Omega \cdot m \) and 10, respectively. Differences between the two earth approaches increase as the cable height decreases and become more evident in the case where cables are laid on the ground surface, i.e. for \( h = 0.1 \) m.

In Figs. 7 and 8 mode \#1 attenuation constant and velocity is analyzed, respectively, considering as reference Carson’s and Sunde’s earth approaches. Higher deviations are observed between the generalized and Carson’s approach, since the influence of displacement currents on cable impedances is neglected. However, for frequencies higher than 10% of \( f_{cr} \), the influence of earth on the cable admittances must be also taken into account. The above remarks are also verified in the simulations of the transient responses of Fig. 9. In Fig. 9a the LI response of the open-ended terminal of the excited conductor \( i \) is presented, while in Fig. 9b, the induced voltage at the end of conductor \( j \) is shown. A 100-m long cable arrangement is considered, while both cables are laid on the ground surface. Differences between Wise’s and the approximate formulations are observed in both responses.

Fig. 6. Aerial cable attenuation constant ratios of a) mode \#1 and b) mode \#2.

Fig. 7. Aerial cable mode \#1 attenuation constant ratio. Comparison of the generalized and the approaches of a) Carson and b) Sunde.

Fig. 8. Aerial cable mode \#1 velocity ratio. Comparison of the generalized and the approaches of a) Carson and b) Sunde.

Fig. 9. Transient responses at the receiving ends of aerial cables, a) excited conductor and b) induced voltage.
VI. UNDERGROUND CABLES

In this case, the same cables as in the aerial cable configuration are considered, though cables are buried 1-m below earth surface. The propagation characteristics of the buried configuration are calculated by means of the different earth approaches described in Section II.C.

Fig. 10. Modal attenuation constants of underground cables for different earth approaches.

First, in Fig. 10, mode #1 and #2 attenuation constants are compared. Considering the results obtained by the generalized approach, the two modes tend asymptotically at almost the same value at HF. This asymptotic value is equal to the attenuation constant of $\gamma_1$. As mode #1 expresses the influence of earth on the conductors and mode #2 the coupling between the cables that takes place through earth [24], results obtained with the generalized formulation are consistent with the frequency dependent behavior of earth. On the contrary, following the approximate earth approaches that consider earth as a perfect conductor in the whole frequency range for the calculation of the cable admittances, thus acting practically as an outer shield to the cable, mode #2 behavior is exclusively determined by the EM characteristics of the cable outer insulation. However, this assumption is only valid in the LF range as also results verify. Additionally, by further analyzing mode #1 propagation constant considering different values of earth resistivity in Fig. 11, it is shown that propagation characteristics calculated by means of the generalized formulation present significant sensitivity on the EM properties of earth. On the contrary the corresponding characteristics obtained by the two approximate earth formulations are practically identical in the whole frequency range.

Fig. 11. Mode #1 attenuation constant of underground cables using different earth approaches and for different values of earth resistivity.

In Figs. 12 and 13, the attenuation constant and velocity ratios are presented for different values of earth resistivity.

Fig. 12. Underground cable mode #1 attenuation constant ratios. Comparison of the generalized and the approaches of a) Sunde and b) Pollaczek.

Fig. 13. Underground cable mode #1 velocity ratios. Comparison of the generalized and the approaches of a) Sunde and b) Pollaczek.

Fig. 14. Transient responses at the receiving ends of buried cables, a) excited conductor and b) induced voltage.

In Figs. 12 and 13, the attenuation constant and velocity ratios are presented for different values of earth resistivity,
respectively, considering as reference Sunde’s and Pollaczek’s earth approaches. It can be generally observed that above 0.1\% of \( f_c \), the propagation characteristics between the generalized and the approximate approaches present significant differences. Moreover, as also shown in Fig. 10, results obtained by the two LF approximations are practically similar [15]. The above are also verified in the transient responses of Fig. 14, where the same test case as in the aerial cable configuration is examined. It is generally shown that the earth admittance term is the most significant on the accurate calculation of wave propagation characteristics of underground cables at HF, while the accuracy of the earth impedance term has trivial influence.

### VII. Discussion & Conclusions

The propagation characteristics and transient responses of transmission line conductors located above and below earth are evaluated using generalized earth formulations that take into account earth conduction effects on both series impedance and shunt admittances. Results are also obtained using approximate formulations, typically used in transient simulation programs. From the conducted analysis, the main remarks are summarized as follows:

- Considering overhead lines and aerial cables, it is shown that the influence of imperfect earth on the shunt admittances must be generally taken into account when investigating phenomena at frequencies higher than 10\% of \( f_c \). This is more evident in cases of cables lying on the earth surface, where earth conduction effects are maximized. However, a more detailed definition of the frequency limit for the case of aerial cables is needed, taking into account also the influence of the cable height above ground.

- For buried cables, the conductor propagation characteristics calculated by means of the generalized formulation are consistent with the frequency dependent behavior of earth, while the corresponding using the approximate approaches follow the perfect conductor assumption in the whole frequency range. Therefore, significant differences are observed for frequencies higher than 0.1\% of \( f_c \).

- Generally, it can be concluded that differences between the generalized and the approximate earth formulations are higher in the mode attenuation constant terms than the corresponding mode velocities. Improvements on the earth impedance formulations result in practically negligible differences in the propagation characteristics and transient responses. Accordingly, in the HF range, the influence of imperfect earth is mainly expressed via the self and mutual earth admittance parameters. From the analysis of the transient responses it is shown that the generalized formulations must be considered, specifically for the calculation of the induced responses.

In summary, this work highlights the need to update current earth formulations and the corresponding calculation routines for transmission lines implemented in most EM transient programs with new ones, allowing a more accurate simulation of high frequency phenomena and fast front transients.

### VIII. References