Implementation of Smart DFT-based PMU Model in the Real-Time Digital Simulator

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***Abstract*-- Many commercial phasor measurement units (PMUs) extract phasors using the standard discrete Fourier transform (DFT) that has some inherent drawbacks such as leakage and picket fence effects. The standard DFT based PMUs employ additional filtering to overcome these effects, but the smart DFT (SDFT) algorithm is capable achieving the required accuracy with minimum or no additional filtering. The benefits of SDFT can outweigh the additional computational complexity in real-time implementation. This paper investigates the implementation of a SDFT based PMU model in a real-time simulator. Various aspects relevant to real-time implementation such as data sampling, time synchronization, and measurement streaming according to C37.118 format are addressed. Performances of the developed SDFT PMU model are evaluated according to the latest IEEE synchrophasor standard and compared with the reference M-class algorithm [1].**

***Keywords*: Discrete Fourier Transform (DFT), Smart DFT, Phasor Estimation, Phasor Measurement Unit, Real-time Digital Simulator.**

I. Introduction

PHASOR measurement unit (PMU) is the core component in synchrophasor based wide area monitoring, protection and control (WAMPaC) systems. PMUs provide phasor values of the voltage and current waveforms as well as the frequency and the rate of change of frequency. The measurements are time synchronized, and dispatched to the synchrophasor network at a specified reporting rate, with a timestamp indicating the measurement time. Many power utilities all over the globe install PMUs at suitable locations on the network, targeting various applications ranging from simple monitoring algorithms to advanced response based WAMPaC systems. It is important to test and verify these synchrophasor based WAMPaC applications before they are deployed in the field. It is practically difficult to evaluate the performance of a given synchrophasor application on a real power system, as some of the test conditions cannot be created without disrupting the normal operation of the power system. The more feasible solution for validation of real-time performance is to use real-time power system simulators. Some of the real-time simulators such as RTDS® have provided emulated software/hardware models of PMUs. The ability of these PMU models to output synchrophasor data streams through a network connection, similar to a real PMU, make them extremely useful for testing WAMPaC applications. These reference P-class and M-class PMUs are implemented according to the current IEEE synchrophasor standards C37.118.1-2011 [1] and C37.118.1a-2014 [2].

Many commercial PMU devices extract phasors using the standard discrete Fourier transform (DFT). The DFT based phasor estimation techniques are accurate when the sampling process is coherent with the fundamental tone's frequency. However, in reality, the system frequency can deviate from its nominal value, leading to erroneous phasor estimates caused by the leakage effect. In addition, the presence of noise and harmonics as well as power system dynamic events can further reduce the accuracy of the DFT algorithm. These effects can be mitigated by utilizing window functions and backend performance class filters [1]. However, implementation of these additional filtering requires storing of a large number of signal samples; thus, computationally expensive specifically for real-time implementations [3].

In this paper, recent research to enhance the accuracy and computational efficiency of a real-time PMU model is presented. The paper specifically examines the application of a novel phasor estimation algorithm referred to as smart DFT (SDFT). The SDFT algorithm proposed in [4]-[6] appears to be a better alternative for the commonly used standard DFT algorithm to estimate phasors in the presence of off-nominal frequencies, noise and harmonics. The algorithm requires additional computational steps compared to the standard DFT, but the benefits of reduced filtering requirements can outweigh the additional computational burden. Use of SDFT algorithm for implementing a PMU or a PMU model has not been previously reported, as per the authors' knowledge.

The objective of this paper is to implement a SDFT based PMU model in a real-time simulator, which is an electromagnetic transient (EMT) simulation based platform, and to evaluate its performances according the current synchrophasor standards [1], [2]. The main contribution of this paper is the adaption of SDFT to calculate synchrophasors defined according to IEEE C37.118.1-2011 [1]. This includes data sampling, time synchronization, and measurement streaming according to C37.118 formats defined in IEEE C37.118-2005 [7] and IEEE C37.118.2-2011 [8]. These aspects are not addressed in the original proposals of SDFT [4]-[6]. In the synchrophasor standard [1], the frequency is defined as the rate of change of estimated phase angles, in contrast, SDFT relies on accurately estimated system frequency for phasor estimations, thus reconciliation is necessary. In addition, the paper will highlight the challenges in implementing SDFT in a real-time

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environment. The simulations are performed to highlight the performances of the developed PMU model in comparison to the reference M-class PMU model in the RTDS simulator. Finally, an application of the developed PMU model is demonstrated by simulating a PMU network installed on the New England 10-machine 39-bus benchmark power system.

II. SMART DFT (SDFT) BASED PMU ALGORITHM

A. Standard SDFT Algorithm

A pure sinusoidal signal sampled at discrete instants in the time-domain can be represented as,

$$x(k) = X_m \cos\left(\frac{2\pi f k}{N f_0} + \phi\right) \quad (1)$$

where X_m is the signal amplitude, f is the signal frequency, ϕ is the initial phase angle, f_0 is the nominal system frequency (50 or 60 Hz), and N is the sampling rate in samples/cycle. The signal can be expressed as,

$$\begin{aligned} x(k) &= \frac{1}{2}\left[X_m e^{j\left(\frac{2\pi f k}{N f_0} + \phi\right)} + X_m e^{-j\left(\frac{2\pi f k}{N f_0} + \phi\right)}\right] \\ &= \frac{1}{2}\left[\bar{x} e^{j\frac{2\pi f k}{N f_0}} + \bar{x}^* e^{-j\frac{2\pi f k}{N f_0}}\right] \end{aligned} \quad (2)$$

where $\bar{x} = X_m e^{j\phi}$ and $\bar{x}^*$ denotes the complex conjugate of $\bar{x}$. The fundamental frequency component within the DFT of $x(k)$, evaluated at the r^{th} sample, is given by,

$$\hat{x}_r = \frac{2}{N}\sum_{k=0}^{N-1} x(k+r)\, e^{-j\frac{2\pi k}{N}} \quad (3)$$

Let consider the system frequency deviation is Δf; thus,

$$f = f_0 + \Delta f \quad (4)$$

By performing some algebraic manipulations after substituting (2) into (3), it is possible to show that [4]-[6]:

$$\hat{x}_r = A_r + B_r \quad (5)$$

where,

$$A_r = \frac{\bar{x}}{N} e^{j\frac{2\pi(f_0+\Delta f)r}{N f_0}} \cdot e^{j\frac{\pi(N-1)\Delta f}{N f_0}} \frac{\sin\left(\frac{\pi \Delta f}{f_0}\right)}{\sin\left(\frac{\pi \Delta f}{N f_0}\right)} \quad (6)$$

$$B_r = \frac{\bar{x}^*}{N} e^{-j\frac{2\pi(f_0+\Delta f)r}{N f_0}} \cdot e^{j\frac{\pi(N-1)(f_0+\Delta f)}{N f_0}} \frac{\sin\left(\frac{\pi(f_0+\Delta f)}{f_0}\right)}{\sin\left(\frac{\pi(f_0+\Delta f)}{N f_0}\right)} \quad (7)$$

Define the exponential kernel in (6) and (7) as,

$$a = e^{j\frac{2\pi(f_0+\Delta f)}{N f_0}} \quad (8)$$

Let,

$$w = a + a^{-1} = 2\cos\left(\frac{2\pi(f_0+\Delta f)}{N f_0}\right) \quad (9)$$

Then, the frequency deviation, Δf is given by,

$$\Delta f = \frac{N f_0}{2\pi}\cos^{-1}\left(\Re\left(\frac{w}{2}\right)\right) - f_0 \quad (10)$$

where $\Re(\,)$ denotes the real part of a complex number. If w is known, then the system frequency can be estimated. From (6)-(8), we can find the following relations.

$$\begin{aligned} A_r &= a A_{r-1} \\ B_r &= a^{-1} B_{r-1} \end{aligned} \quad (11)$$

Consider three consecutive DFT fundamental components based on (5),

$$\begin{aligned} \hat{x}_r &= A_r + B_r = a A_{r-1} + a^{-1} B_{r-1} \\ \hat{x}_{r-1} &= A_{r-1} + B_{r-1} \\ \hat{x}_{r-2} &= A_{r-2} + B_{r-2} = a^{-1} A_{r-1} + a B_{r-1} \end{aligned} \quad (12)$$

Then, w can be estimated as,

$$w = \frac{\hat{x}_r + \hat{x}_{r-2}}{\hat{x}_{r-1}} \quad (13)$$

The phasor can be estimated by rearranging (6) as,

$$A_r = \frac{X_m}{N} \frac{\sin\left(\frac{\pi \Delta f}{f_0}\right)}{\sin\left(\frac{\pi \Delta f}{N f_0}\right)} \cdot e^{j\left(\frac{2\pi(f_0+\Delta f)r}{N f_0} + \frac{\pi(N-1)\Delta f}{N f_0} + \phi\right)} \quad (14)$$

Then,

$$X_m = |A_r| \frac{N \cdot \sin\left(\frac{\pi \Delta f}{N f_0}\right)}{\sin\left(\frac{\pi \Delta f}{f_0}\right)} \quad (15)$$

$$\phi = \text{angle}\,(A_r) - \frac{\pi(N-1)\Delta f}{N f_0} \quad (16)$$

In (16), the phasor rotates when the data window advances by one sample. Therefore, the phasor angle should be corrected to obtain a stationary phasor.

$$\phi = \text{angle}\,(A_r) - \frac{\pi(N-1)\Delta f}{N f_0} - \frac{2\pi}{N} m \quad (17)$$

where m is a counter varies from 0 to $(N-1)$.

Then, solve (9) to obtain,

$$a = \frac{w}{2} \pm \frac{\sqrt{4 - w^2}}{2} \quad (18)$$

From (12),

$$A_r = \frac{a^2 \hat{x}_r - a \hat{x}_{r-1}}{a^2 - 1} \quad (19)$$

B. Extension of SDFT for Synchrophasor Estimation

A new phasor is calculated at every sampling point; however, all of them are not reported since the PMU reporting rate is always less than the sampling rate. The number of samples between two consecutive reportings is known as decimation factor, M and is given by,

$$M = \frac{N f_0}{F_s} \quad (20)$$

where F_s is the PMU reporting in frames/s. Then, every M^{th} phasor is reported as a PMU measurement.

Investigations revealed that estimated phasors show a numerical oscillation especially when the nominal system frequency is 60 Hz where the sampling interval is an irregular value. Therefore, it is proposed to filter the estimate w obtained from (13) using a *mean filter* with an order of $1.5N$. Thus, $(2.5N + 1)$ data samples are required to estimate a phasor. The time tag was set at the middle of this data window, resulting in a measurement delay of only $1.25N$ samples. When compared to the delay of 164 samples ($N = 16$

and $F_s =$ 60 frames/s as an example) of the reference M-class algorithm [1], [2], this is a significant improvement achieved with a less amount of memory and computational resources.

III. SDFT Model Validation

The new SDFT PMU model was implemented in the RTDS simulator. First, the simulated waveforms (70 V nominal) are internally fed to the PMU model, which has fixed sampling rate of 16 samples/cycle. Then, phasors are estimated based on the SDFT algorithm discussed in Section II. Performances of the proposed PMU model were evaluated under a variety of conditions that are specified in [1], [2]. In this paper, signal frequency range, linear frequency ramp, measurement bandwidth, and step response tests are presented to demonstrate compliance of the PMU model in a real-time simulation environment. Steady-state compliance tests such as signal magnitude (voltage/current), harmonic distortion and out-of-band interference tests were also performed; the SDFT PMU passed all stead-state tests but detailed results are omitted due to space limitation. Power system frequency was selected as 60 Hz for the demonstration purpose, but the PMU model proves similar performances for 50 Hz power system as well. PMU reporting rate was selected as 60 frames/s.

Errors of the PMU model were expressed in terms of total vector error (TVE) [1] and compared with the reference M-class model in the RTDS simulator. The automated PMU test setup proposed in [9] was used to test the proposed PMU model. The test setup has been developed around the RTDS simulator and capable to execute a series of tests, collects measurements, calculates errors, and checks conformity (pass/fail assessment) as per the synchrophasor standard [1], [2] and the IEEE synchrophasor measurement test suite specification guidelines [10] with minimal user interaction.

A. Signal Frequency Range

The signal frequency range test demonstrates a deviation of frequency in the power system. In this test, the signal frequency is varied from 55 Hz to 65 Hz. When the frequency is deviating from the nominal frequency of 60 Hz the magnitude measured in the SDFT model shows slightly better performance. For example, if the signal frequency is set to 55 Hz the magnitude measured in the SDFT model is 69.998 V whereas measured value in the M-class model is 69.947 V as shown in Fig 1(a). Phase angle measurements of both models very closely follow the reference value as shown in Fig 1(b). The corresponding TVEs are 0.003% and 0.174% respectively and well below to the error limits specified in the standards [1], [2] as shown in Fig 1(c). Therefore, both PMU models show satisfactory performances under frequency range test.

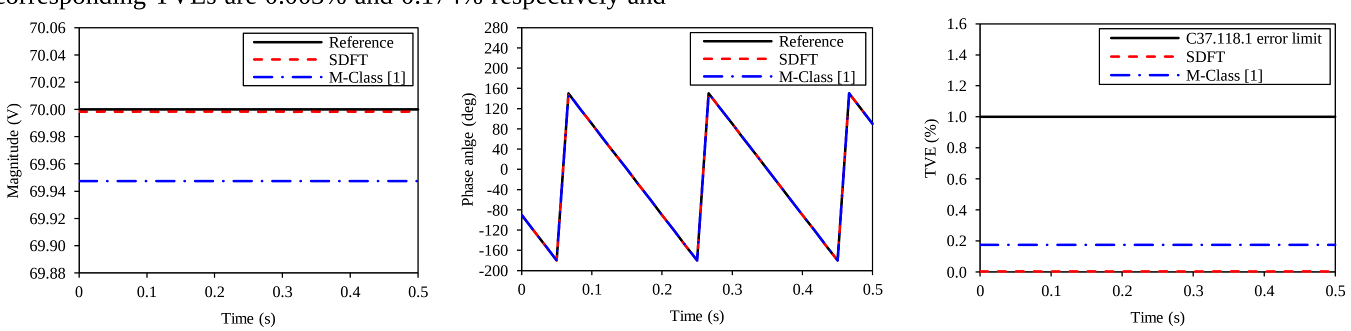
B. Ramp of Signal Frequency

A sudden loss of large generation or load results a power system imbalance and then, system frequency ramps from its nominal value. The ramp can either be positive or negative. The ramping rates specified in [1], [2] are +1 Hz/s and –1 Hz/s and the ramping range is 55 Hz to 65 Hz. As shown in Fig. 2 both PMU models show satisfactory performances and again the SDFT model shows slightly better accuracy.

The reference M-class algorithm is design to operate within the ramping range of 55 to 65 Hz. Occasionally, extreme imbalance situations may result to deviate the system frequency even beyond that range and the M-class PMU model shows poor performances if the system frequency deviates beyond 55-65 Hz. As shown in Fig. 3 if the ramping range is expanded to 60-70 Hz, the magnitude measured in the M-class model shows significant deviation from the reference value. Thus, the TVE exceeds the specified limit of 1% around 67.3 Hz. It was observed that the TVE of the M-class model reaches to 6% at 70 Hz. However, the SDFT model shows better performances even at 70 Hz with a TVE of 0.035%. Similar results were observed if the ramping range is expanded to 50-60 Hz. Therefore, the SDFT model is appropriate to measure phasor quantities over a wide range of frequencies.

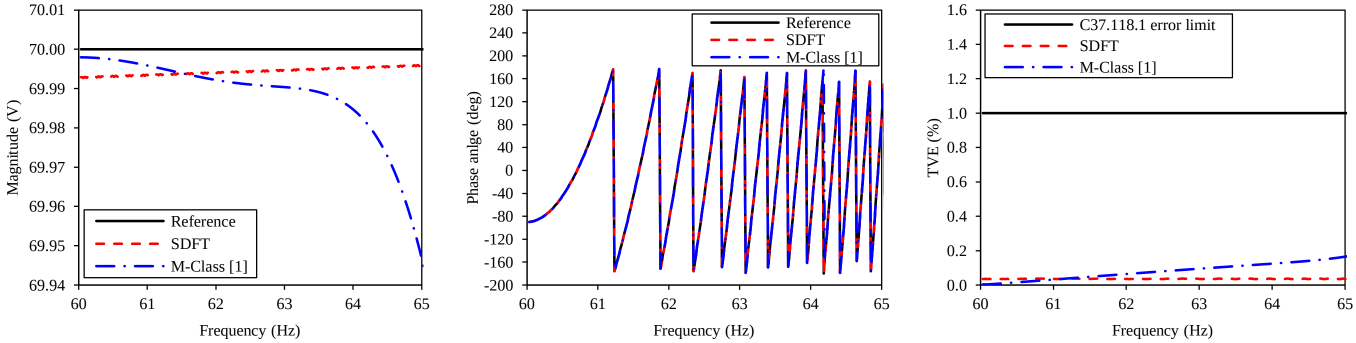
C. Measurement Bandwidth

The measurement bandwidth test demonstrates oscillations in the power system. This test includes two tests; magnitude modulation and phase angle modulation [1], [2]. In the magnitude modulation test, 10% modulation signal is added to the signal magnitude and modulation frequency is varied over the range of 0.1 to 5 Hz. The phase angle modulation test is similar, but 10% modulation signal is applied to the phase angle. Fig. 4 and Fig. 5 compare the performances of two PMU models when the modulation frequency is set to 5 Hz. As seen in the figures, both PMU models satisfy the specified error limits of the standards [1], [2]. The SDFT model shows a slight oscillation in phase angle when magnitude is modulated. If phase angle is modulated, the SDFT model shows a slight oscillation in magnitude. These oscillations cause maximum TVE to reach 0.54%, but it is well within specified limit of 3% [1], [2]. The M-class model shows better performances compared to the SDFT model in this particular test.



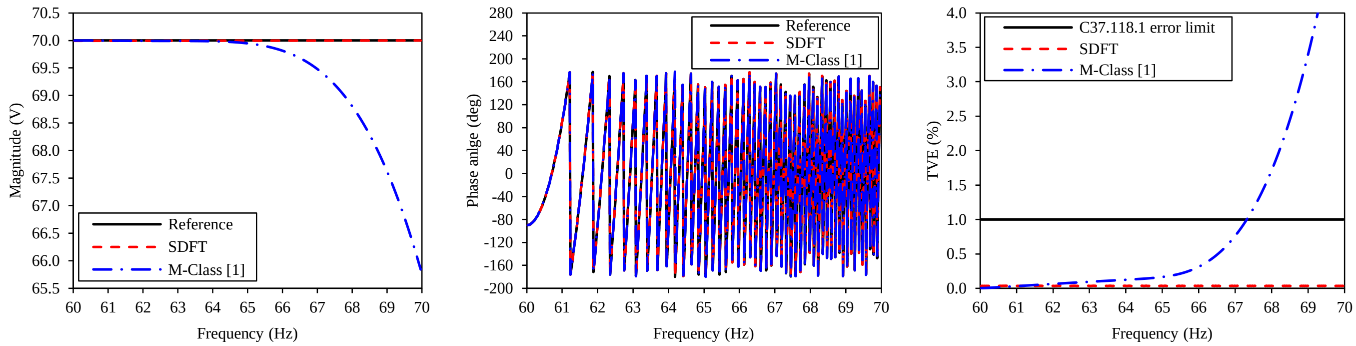
(a) Magnitude (b) Phase angle (c) TVE

Fig. 1. Signal frequency at 55 Hz



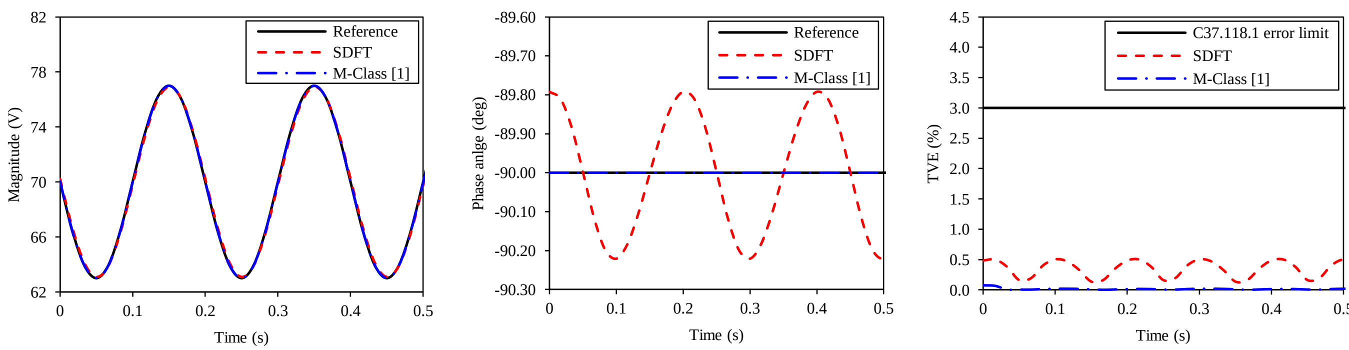
(a) Magnitude (b) Phase angle (c) TVE

Fig. 2. Ramp of signal frequency from 60 Hz to 65 Hz



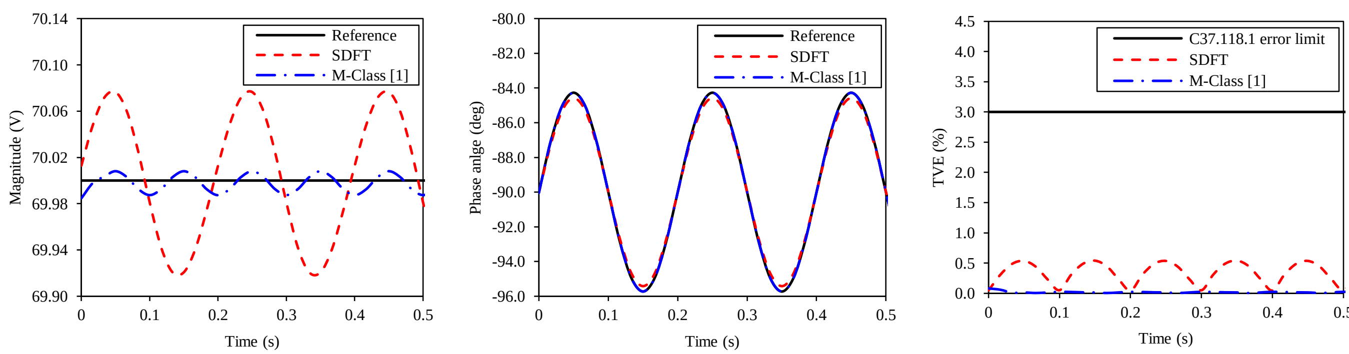
(a) Magnitude (b) Phase angle (c) TVE

Fig. 3. Ramp of signal frequency from 60 Hz to 70 Hz



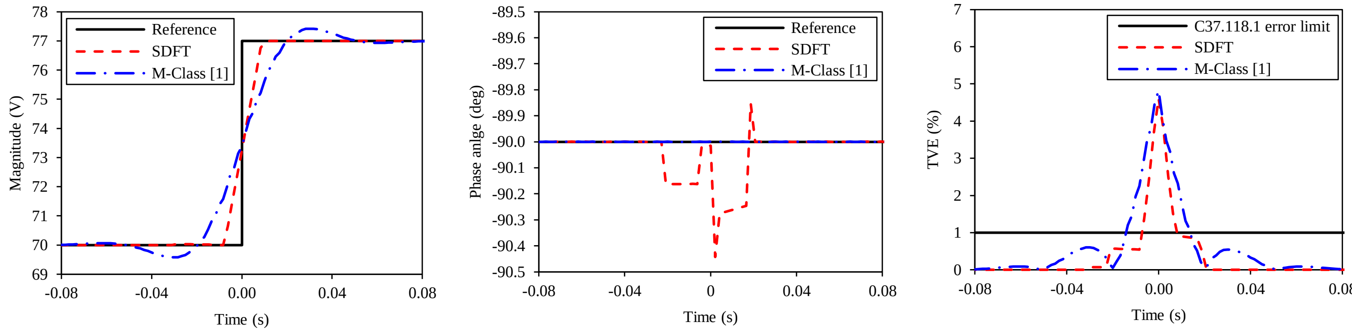
(a) Magnitude (b) Phase angle (c) TVE

Fig. 4. Magnitude modulation at 5 Hz



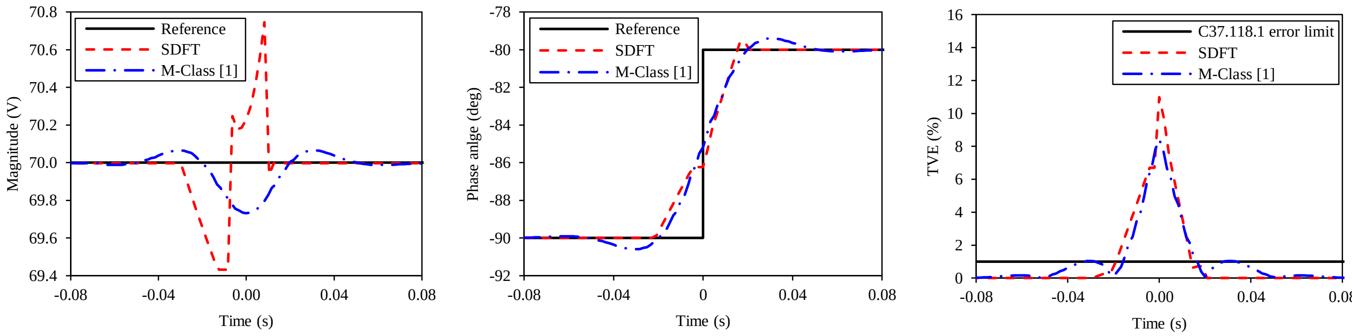
(a) Magnitude (b) Phase angle (c) TVE

Fig. 5. Phase angle modulation at 5 Hz



(a) Magnitude (b) Phase angle (c) TVE

Fig. 6. Magnitude step response



(a) Magnitude (b) Phase angle (c) TVE

Fig. 7. Phase angle step response

D. *Step Response*

The step response test demonstrates the power system switching events. This test also includes two tests; magnitude step and phase angle step responses [1], [2]. In the magnitude step, ±10% step is applied to the signal magnitude whereas $\pm 10^0$ step is applied to the signal phase angle under the phase angle step. The step is initiated by a signal at a precise time, which allows determining response time, delay time, and maximum overshoot/undershoot [10]. Since PMU response time and delay time are small compared to the PMU reporting interval it is difficult to characterize the response of a single step. Therefore, the equivalent sampling approach explained in [11] should be used to achieve the required resolution.

Fig. 6 and Fig. 7 show the magnitude and phase angle step responses respectively. Table I provides the response time, delay time, and maximum overshoot/undershoot of both PMU models under magnitude and phase angle step responses. The test results show that both PMU models satisfy the standards [1], [2], however, the SDFT model shows better performances compared to the M-class model.

TABLE I
STEP RESPONSE PERFORMANCE

Influence Quantity	PMU Model	Response Time (ms)	|Delay Time| (ms)	Max. Over /Undershoot (% of step)
Magnitude step	C37.118.1 limit	116.67	4.17	10.0
	SDFT Model	15.20	0.58	0.0
	M-class Model	27.85	0.42	6.0
Phase angle step	C37.118.1 limit	116.67	4.17	10.0
	SDFT Model	32.95	2.55	4.7
	M-class Model	66.56	0.42	8.5

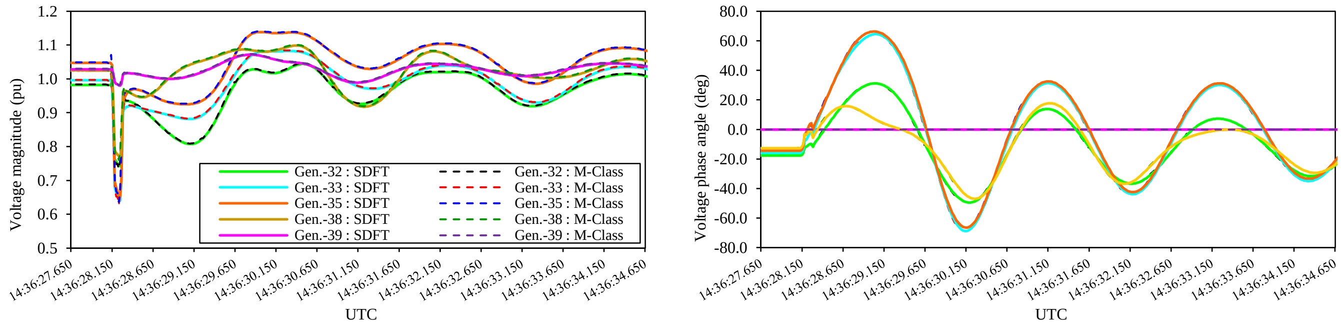
IV. POWER SYSTEM APPLICATION

The IEEE 39-bus test system (New England power system) [12] was used to demonstrate the dynamic performances of the SDFT model. It was assumed that PMUs are installed at the generator terminals. The test system was simulated in the RTDS, which is equipped with a GTNETx2 hardware board to emulate both SDFT and reference M-class PMU modules. A SEL-2407 GPS clock [13] was used to provide inter-range instrumentation group time code format B (IRIG-B) time signal to the RTDS simulator via a GTSYNC card. PMUs in the RTDS were configured to report synchrophasors through a laboratory scale synchrophasor network at 60 frame/s rate. The synchrophasor data were collected by the openPDC v2.0 [14] phasor data concentrator (PDC).

In order to examine the dynamic performances, two cases were considered. In the first case, a three-phase to ground fault applied on Line 16-17 (at 25% of the length from bus 16) when universal coordinated time (UTC) is 14:36:28.150. The fault was cleared by removing the line after 6 cycles. The variations of the voltage magnitudes and the phase angles obtained from both SDFT and M-class models are shown in Fig. 8. Note that a few generator terminals were selected for the demonstration.

It was observed that the power system is stable following the fault clearance and both PMU models showed similar performances where the trajectories obtained from two PMU models are coincided. The M-class model displayed slight overshoot before and after the fault due to the step change in voltage magnitudes.

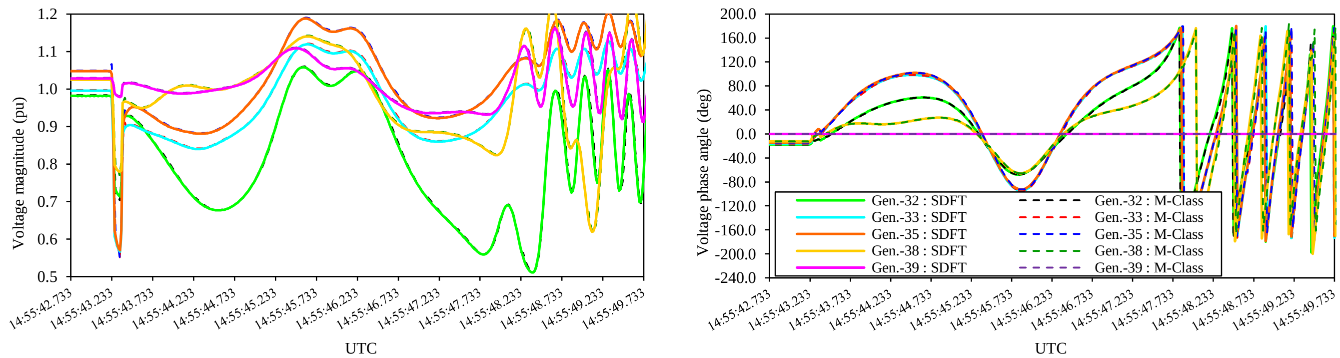
In the second example, a three-phase to ground fault was applied on same Line 16-17 (at 75% of the length from bus



(a) Voltage magnitude

(b) Voltage phase angle

Fig. 8. Voltage phasors at generator terminal : Stable case



(a) Voltage magnitude

(b) Voltage phase angle

Fig. 9. Voltage phasors at generator terminal : Unstable case

16) when UTC is 14:55:43.233 and cleared by removing the line after 6 cycles. Fig. 9 shows the variations of the voltage magnitudes and the phase angles obtained from both SDFT and M-class models. This is a multi-swing instability and both PMU models showed similar performances.

Therefore, the SDFT model implemented in the RTDS simulator precisely captures the dynamic behavior of the simulated power system and realizes the same accuracy level of the reference M-class algorithm [1], with less computational resources.

V. Conclusions

The SDFT algorithm demands a less amount of memory and computational resources compared to the reference M-class algorithm [1] and therefore, it offers a significant advantage in a real-time computing environment. A PMU model based on the SDFT was implemented in a real-time simulator. The standard tests showed that SDFT based PMU is well within the error limits specified in the current synchrophasor standards [1], [2] for M-class PMUs (which are more stringent than P-class) with minimal additional filtering. Comparisons with the reference M-class PMU model [1] showed that SDFT based PMU has improved performance compared to the reference M-class model, except in the measurement bandwidth test. Finally, the developed SDFT PMU model in the real-time environment is applied to analyze the dynamic behavior of the IEEE 39-bus test system and displayed promising performances. Availability of realistic PMU models in electromagnetic transient simulators, especially in real-time simulators, allows verification of critical synchrophasor applications before they are deployed in actual power systems.

VI. References

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