

Development of a Symmetrical Multi-Phase Synchronous Machine Model for Real-Time Digital Simulation

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Abstract— Multi-phase electric machines have the advantage of fault tolerance, reliability and reduction in the torque ripple. Such properties make them attractive for applications such as electric ship propulsion, traction, aircraft systems, electric vehicles [1]-[2]. Failure of the drive system through a few phases could be tolerated by operating the remaining phases of the machine. Modeling of multi-phase machines for real-time digital simulation is an important step in enhancing these simulators for applications such as electric ship propulsion and new schemes in wind generation.

This paper presents development of a symmetrical multi-phase (three or more phases) synchronous machine model for the Real-Time Digital Simulator (RTDS®). The paper starts with the introduction and analysis of symmetrical multi-phase machines. The extension of two-reaction theory to multi-phase machines is presented. The developed model allows the user to choose the number of stator phases. The model is validated using off-line numerical simulations. Additional control components are developed to enhance the application of this model in multi-phase systems.

Keywords: real-time digital simulation, synchronous machine, symmetrical components, multi-phase, two reaction theory, digital transient network analyzer, wind generation

I. INTRODUCTION

In conventional power system applications, the number of phases is three as it generally results in a lower transmission costs. Historically 3-phase systems are selected for transmission and electric machines in both generation and loads. When it comes to applications where the electric machine is connected to power electronic converters, there is no need to be restricted to a system with 3 phases [1].

Historically, electric machines with higher number of phases (multi-phase machines) were introduced to lower ripples in electric torque, and also for their lower current ratings. In fact, the later was a reason that promoted the development of multi-phase machines in early stages of circuit breaker development. Additionally, lower current rating per phase allows the deployment of power electronic converters with lower ratings.

In addition to lower torque ripple and lower current ratings, multi-phase machines have other advantages such as higher force density, reduction in the noise characteristic and the stator copper losses [2]. Reliability and fault tolerance is another benefit of multi-phase machines. A Few phases can be

lost yet the machine can still be operated utilizing the remaining phases by providing proper excitation through power electronic converters.

For the above reasons, multi-phase machines have applications in electric ship propulsion, traction, aircraft systems, electric vehicles [2]-[3] and new schemes of wind turbines. There are variations of multi-phase machines such as split-phase, dual stator, higher order symmetrical, etc. which can be studied in literature.

A Real Time Digital Simulator (RTDS) is a combination of computer hardware and software designed specifically for the solution of power system and power electronic electromagnetic transients in real time. There are many areas where this technology has been successfully applied. Modeling of multi-phase machines for real-time digital simulation is an important step in enhancing these simulators for the above mentioned applications.

Modeling of multi-phase machines poses a few challenges especially when it comes to real-time digital simulation; one of the main problems is the diversity of these models. Even if the phases are displaced in a symmetric manner, the number of phases may vary between 3 to 18 or even more [4]. Developing and maintaining a large number of machine models in the library of any commercial software is challenging and time-consuming. If the models are developed directly in the phase domain, not only it become a tedious task to form the inductance matrix of the machine, but also it creates a large computational burden for the simulation tool.

A comprehensive analysis in rotor frame of reference seems necessary as it responds to the question of general analysis of multi-phase machines. Note that, in practical applications, these machines are driven by power electronic converters, therefore, models need to be implemented with a smaller simulation time-step of few microseconds. This makes the modeling in rotor frame of reference more attractive.

This paper presents the development of a multi-phase synchronous machine model for real-time digital simulation. The paper starts with the introduction and analysis of symmetrical multi-phase synchronous machines. Differential equations of a symmetrical multi-phase synchronous machine are described and challenges of modeling and analysis in these machines are outlined. Two reaction theory [5]-[6] is reviewed and its extension for multi-phase machines is described. A general equivalent circuit in rotor frame of reference is presented from which a transient multi-phase (three or more phases) synchronous machine model is developed and incorporated into the network solution of the RTDS real-time digital simulator. The model is validated through comparison with an off-line phase domain solution. Capabilities of the

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model such as flexibility in selecting the number of phases are described.

II. ANALYSIS OF SYMMETRICAL MULTI-PHASE SYNCHRONOUS MACHINES

This section briefly describes the analysis and modeling method of symmetrical multi-phase synchronous machines. Since the goal is incorporation of the model into an electromagnetic transient program, the coupled electric circuit approach is used for modeling of this type of machine. This means that the machine is considered as an idealized Park machine assuming sinusoidal distribution for the magnetomotive forces and permeance.

A. General Description of a Symmetrical Multi-Phase Synchronous Machine

Similar to an ordinary 3-phase synchronous machine, a multi-phase synchronous machine consists of stator windings known as armature, mounted in stator slots and a field winding wound on the rotor. Also, similar to a 3-phase synchronous machine, a damper grid or a set of *amortisseur windings* (which consists of copper or brass rods embedded in the pole face) is provided to damp speed deviations. In a symmetrical multi-phase synchronous machine, magnetic axes of stator windings are displaced by the electrical angle of $\delta = 2\pi/N$ where N is the number of stator phases. There are alternative arrangements such as “dual wound synchronous machines” [7][8] with two sets of arbitrarily displaced 3-phase windings which is not the subject of this paper.

Fig. 1 shows the diagram of an idealized N-phase synchronous machine. Windings $a_s, b_s \dots n_s$ with angular displacement of $\delta = 2\pi/N$ represent stator windings. In an idealized synchronous machine, in the absence of structural asymmetries, the behavior of the damper grid can be represented by windings on rotor d- and q-axes. This is shown by windings k_{dr} and k_{qr} in Fig. 1. The field winding is represented by a winding on the rotor d-axis.

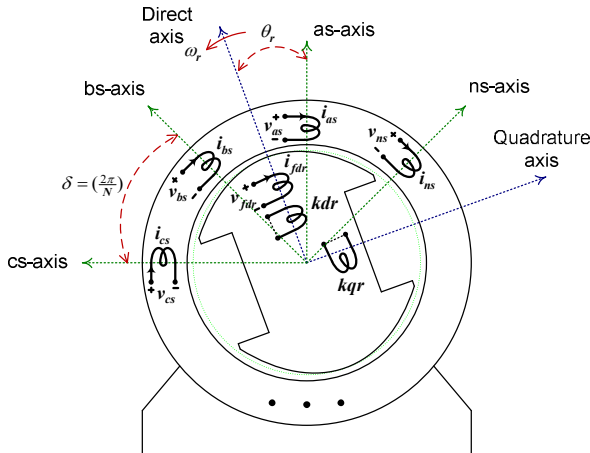


Fig. 1. Diagram of an idealized N-phase synchronous machine.

B. Voltage and Flux Linkage Equations

Based on the above description of a symmetrical multi-phase synchronous machine, voltage and flux linkage

equations of this type of machine are developed. The following assumptions are made in this analysis:

- A multi-pole synchronous machine is modeled as an equivalent two pole machine.
- It is assumed that the machine windings produce a sinusoidal MMF, and permeance is sinusoidally distributed; therefore, space harmonics are ignored.
- By the definition of symmetry, stator windings are displaced with the electrical angle of $\delta = 2\pi/N$. For all stator windings, phase resistances and leakage inductances are equal.

Voltage and flux linkage equations for a N-phase synchronous machine are presented in (1). Equation (1) shows the relation between vectors of voltages, currents and flux linkages of the machine. Here, vectors of stator voltages, currents and flux linkages are shown by $\underline{v}_{ab\dots n_s}$, $\underline{i}_{ab\dots n_s}$ and

$\underline{\psi}_{ab\dots n_s}$ respectively. A similar arrangement is made for rotor quantities.

$$\begin{aligned} \begin{pmatrix} \underline{v}_{ab\dots n_s} \\ \underline{v}_{dq_r} \end{pmatrix} &= \begin{pmatrix} [r_s] & [0] \\ [0] & [r_r] \end{pmatrix} \begin{pmatrix} \underline{i}_{ab\dots n_s} \\ \underline{i}_{dq_r} \end{pmatrix} + \frac{d}{dt} \begin{pmatrix} \underline{\psi}_{ab\dots n_s} \\ \underline{\psi}_{dq_r} \end{pmatrix} \\ \begin{pmatrix} \underline{\psi}_{ab\dots n_s} \\ \underline{\psi}_{dq_r} \end{pmatrix} &= \begin{pmatrix} [L_{ss}] & [L_{sr}] \\ [L_{sr}]^T & [L_{rr}] \end{pmatrix} \begin{pmatrix} \underline{i}_{ab\dots n_s} \\ \underline{i}_{dq_r} \end{pmatrix} \end{aligned} \quad (1)$$

where:

$$\begin{aligned} \begin{pmatrix} f_{ab\dots n_s} \end{pmatrix}^T &= (f_{as} \ f_{bs} \ \dots \ f_{ns}) \\ \begin{pmatrix} f_{dq_r} \end{pmatrix}^T &= (f_{dr_1} \ f_{dr_2} \ \dots \ f_{qr_1} \ f_{qr_2} \ \dots) \end{aligned}$$

Self-inductance matrices of stator and rotor are shown by $[L_{ss}]$ and $[L_{rr}]$. $[L_{sr}]$ is the matrix for the mutual inductances between the stator and rotor windings. The values of these inductances are functions of rotor position θ_r and saturation. As an example, the stator inductance matrix $[L_{ss}]$ is shown in (2). Here, L_{ls} is the stator winding leakage inductance and N is the number of stator windings. Inductance values M_{0s} and L_{2s} are functions of winding distribution, effective number of turns, effective air-gap length and saturation. As mentioned previously, higher order space harmonics of the machine are ignored.

Similarly, equations for rotor inductances and mutual inductances between stator and rotor can be developed which is not mentioned here. Taking into account these time-varying inductance matrices, the set of differential equations in (1) can be solved directly in the phase-domain and incorporated into the network solution of electromagnetic transient programs [9][10]. Although this approach is numerically superior, it has the drawback of being computationally extensive. The phase domain inductance matrix of the machine needs to be inverted every time-step and equivalent conductance matrix of the machine continuously needs to be computed and passed to the network solution [9][10]. Normally these machines are inverter driven and because of high frequency PWM switching, the simulation needs to be performed with the time-step of 1~3 μs . Furthermore, it would be very difficult for

ordinary users of simulation packages to acquire enough design data to calculate constants such as M_{0s} and L_{2s} . Usually, data for rotating machines is available in the form of equivalent circuits for rotor d- and q-axes. Thus, it is necessary to analyze the symmetrical multi-phase synchronous machines in the dq frame of reference.

$$[L_{ss}] = [L_{ss-l}] + [L_{ss-oh}] + [L_{ss-2h}]$$

where:

$$\begin{cases} [L_{ss-l}]_{ik} = \begin{cases} L_{ls} & i = k \\ 0 & i \neq k \end{cases} \\ [L_{ss-oh}]_{ik} = \left(\frac{2}{N}\right) \cdot M_{0s} \cdot \cos[(k-i)\delta] \\ [L_{ss-2h}]_{ik} = \left(\frac{2}{N}\right) \cdot L_{2s} \cdot \cos[2\theta_r - (i+k-2)\delta] \end{cases} \quad (2)$$

$$\delta = \left(\frac{2\pi}{N}\right) \quad i, k \in 1, 2 \dots N$$

The two-reaction theory for the analysis of symmetrical 3-phase synchronous machines [5] was proposed by R. H. Park in 1929. In this work, Park generalized the work of Blondel, Dreyfus, Doherty, and Nickle and provided a complete analysis for 3-phase machines with multiple rotor circuits. Park transformation in its non-orthogonal form is shown in (3).

$$P(\theta_r) = \frac{2}{3} \begin{pmatrix} \cos \theta_r & \cos(\theta_r - \frac{2\pi}{3}) & \cos(\theta_r - \frac{4\pi}{3}) \\ \sin \theta_r & \sin(\theta_r - \frac{2\pi}{3}) & \sin(\theta_r - \frac{4\pi}{3}) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{pmatrix} \quad (3)$$

This transformation has been used by researchers to analyze and model multi-phase machines with triple number of phases (e.g. 3, 6, 9, etc.) [7]-[8]. Alternative transformations have been proposed for 5-phase machines considering the third harmonic of the air-gap MMF [1].

At the same time that R. H. Park presented the two reaction theory on 3-phase machines, Yu H. Ku analyzed 3-phase machines [11] using operational calculus and Fortescue's symmetrical components [12]. The advantage of the two reaction theory over Ku's method is that it uses real transformations whereas Fortescue's symmetrical component transformation and forward-backward transformation proposed by Ku are complex transformations. This could have been one of the reasons that Ku's method is not as popular today. The advantage of his method, however, is the fact that symmetrical component transformation is applicable to a system with any number of phases. This allowed Ku to extend the two reaction theory to symmetrical multi-phase machines [15]. If one applies the symmetrical component transformation to the magnetizing part of the stator self-inductance matrix (i.e. $[L_{ss-oh}]$ and $[L_{ss-2h}]$ shown in (2)), the resulted matrices will be diagonal with non-zero elements only on the positions corresponding to positive and negative sequences. This is true for symmetrical synchronous machines with any number of phases. From the sequence domain, differential equations of machine can be brought up to the general stator two-phase quantities ($\alpha\beta$) and ultimately to the rotor two reaction components (dq). Appendix II provides the necessary transformations. This process of calculations is lengthy and it

is not outlined in this paper. An equivalent circuit is provided in Fig. 2 which describes the differential equations of the synchronous machine in rotor two reaction components (dq).

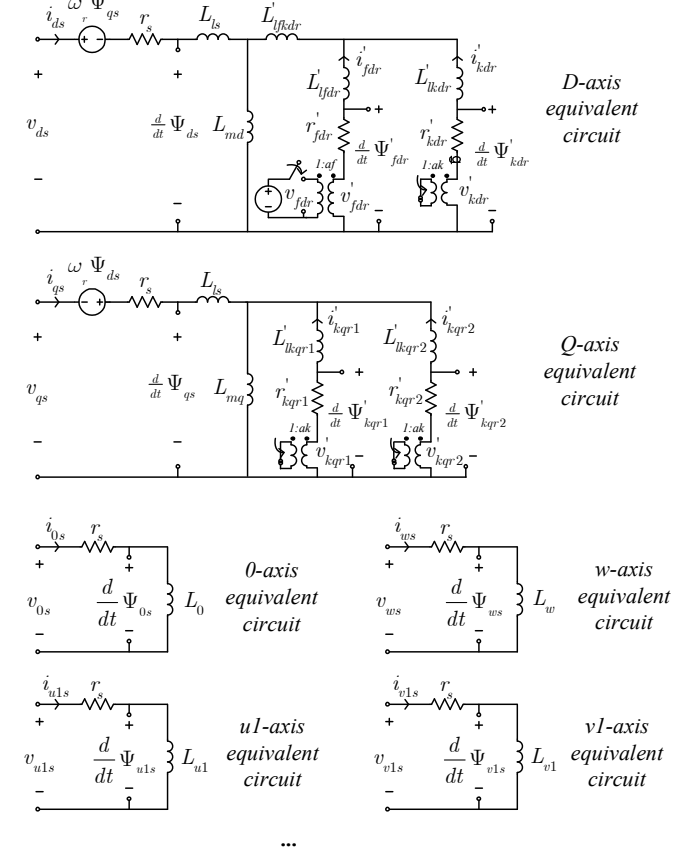


Fig. 2. Equivalent circuit of an idealized N-phase synchronous machine.

In Fig. 2, in addition to the well-known d - q - and 0 -axes equivalent circuits, there exist other zero sequence circuits shown by w , u and v . D - q - and 0 -axes components correspond to (+), (-) and (0) sequences in symmetrical component frame. W , u and v components correspond to other sequences in symmetrical component frame when the number of phases is more than 3. Note that w -axis only exists when the number of phases is even. Depending on the number of phases, additional u - and v - zero sequence circuits may exist. U and v components always appear in pairs.

Existence of this equivalent circuit is a great advantage in modeling symmetrical multi-phase machines with arbitrary number of phases as the differential equations of the zero sequence circuits can be solved independently. D - and q - axes equivalent circuits are also identical to the ones for 3-phase machines. Measurement of machine parameters is also possible through well-known tests [13]. Additional tests is needed for the measurement of zero sequences proposed here. Such procedures and possible experimental validation are subjects of future papers.

III. IMPLEMENTATION OF THE MULTI-PHASE SYNCHRONOUS MACHINE MODEL INTO THE ENVIRONMENT OF THE REAL-TIME DIGITAL SIMULATOR

The RTDS simulator is a combination of computer hardware and software designed specifically for the solution

of power system electromagnetic transients in real time. Each unit of RTDS is called a rack. Every rack consists of processors dedicated to the solution of the power system network, power system devices, and control components [14]. Each simulation time-step is divided into computation intervals and communication intervals. Similar to other electromagnetic transient programs such as EMTP and EMTDC, Dommel algorithm [16] is used for discretization of the system's differential equation. Subsequently, the nodal solution is used for calculation of node voltages. Differential equations of power system components are discretized to a Norton equivalent of conductances and current sources which are passed to the network solution during a communication interval (T_0) every time-step [14]. This is how the network solution is solved for the "normal time-step module" used for general power system studies.

Small Time-Step Module:

One of the simulation tools in the RTDS simulator is called the "small time-step module". This module which was introduced in 2005 is suitable for modeling circuits containing converters with high frequency switching such as PWM [19]. This module contains a library of its own including power electronic components, electric machines, transformers, transmission lines, non-linear elements, control components, etc. The small time-step network solution as well as components in the circuit are executed with a time-step of ($1.4\mu s \sim 2.5\mu s$).

Due to the small size of the simulation time-step and the relative convenience of modeling electric machines in rotor dq0 frame of reference, the *interfaced-based approach* [17], [18] is used to model the multi-phase synchronous machine. In this approach, a machine is considered as an *external* sub-network to the main system; it receives the terminal voltages at every time-step, computes winding currents and injects them back to the network solution. Special projection and integration techniques are used to make the model mathematically precise.

Torque and Mechanical Swing Equations:

Per-unit electromagnetic torque is calculated using (4). As expected, zero sequences do not contribute to electromagnetic torque. If the mechanical swing equations are solved inside the model, then the following equation as shown in (4) can be used to calculate the rotor speed. Alternatively, the mechanical swing equations can be calculated using a multi-mass model as this option is available in other machine models in the library [14].

$$T_e = \frac{1}{2} (\Psi_d \cdot i_q - \Psi_q \cdot i_d) \quad (4)$$

$$T_m - T_e = 2H \cdot \frac{d}{dt} \omega + D \cdot \omega$$

Features and Capabilities of the Model:

With the above description, the multi-phase synchronous machine model is implemented in the environment of the RTDS simulator. This section describes the capabilities of this model. The basic icon for this component appears as shown in Fig. 3. The number of phases for the machine can be selected

by the user from a menu. Based on the selected number of phases, stator terminals and neutral will be available for external connection. Additionally, an option is available to access both ends of each stator winding as shown in Fig. 3. This option allows users to connect both ends of stator windings to power electronic converters as desired in particular configurations [2].

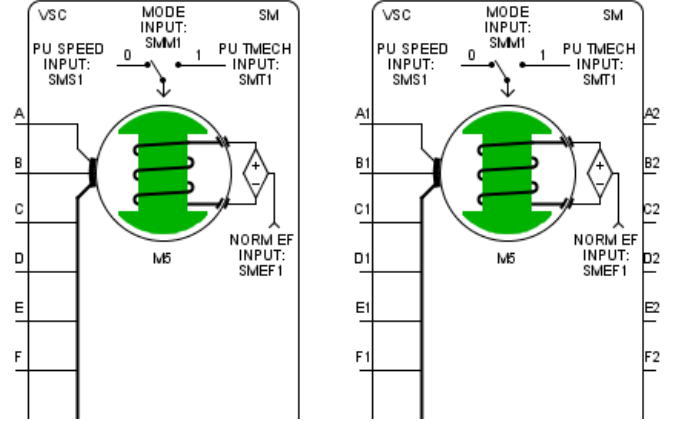


Fig. 3. The icon for multi-phase synchronous machine model.

The modeling technique can be applied to any number of phases, and in fact the model is tested for up to 12 phases. However, for practical reasons, the number of phases will be limited to 6 for the small time-step version of the model.

Similar to other machine models in the library, mechanical swing equations can be solved either inside the model or outside using a multi-mass model. Field excitation voltage can be received in normal units from the excitation system [14]. Magnetic saturation is also incorporated into this model [18]. The machine accepts d- and q-axis data both in the form of reactances and time constants. Another menu receives machine zero sequence reactances and resistances in pu.

TABLE I shows the approximate execution time (on a PB5 processor [14]) for the machine with different number of phases.

TABLE I
EXECUTION TIME VS NUMBER OF PHASES

Number of Phase	3	5	6	9	12
Execution Time (μs)	1.2	1.4	1.6	1.9	2.4

IV. VALIDATION OF THE MODEL AND TIME-DOMAIN SIMULATION RESULTS

This section provides validations for the multi-phase synchronous machine model presented in this paper. The focus of this section is to provide validations for the capability of the proposed model in accurately solving the differential equations of a symmetrical multi-phase synchronous machine stated in (1) and (2).

To validate the performance of the proposed model in transient conditions, a small off-line stand-alone electromagnetic transient program including a stand-alone multi-phase synchronous machine model is developed. In order to provide a validation to the analysis which is provided

in Section II and the d- q- and additional zero sequence equivalent circuits provided in Fig. 2, the differential equations of the multi-phase synchronous machine are directly solved in *phase domain*.

The data for the synchronous machine under the test is shown in Appendix I. Although multi-phase synchronous machines rating are generally in the range of maximum few MVAs, the data for the machine presented in this paper belongs to a larger 100MVA salient-pole synchronous machine. The reason for this selection is that transient and sub-transient time constants of larger machines are more distinct. The higher MVA ratings provides a better platform for thorough validation of the model under transient condition especially short circuit faults.

To check the validity of the model for a conventional 3-phase synchronous machine, a single phase fault is simulated in both the real time model and the stand-alone program.

To check the validity of the model for a synchronous machine with higher order phases, a 5-phase synchronous machine is modeled. Transient behavior of the model is studied under both symmetrical 5-phase and asymmetrical single phase faults.

A. Asymmetrical Fault on a 3-Phase Synchronous Machine

In this section, the proposed multi-phase synchronous machine model is configured to be a conventional 3-phase synchronous machine model. During the real-time simulation, while the rotor speed is set to synchronous, the machine is operating in open-circuit and its field voltage is adjusted such that the terminal voltage of 1 pu is achieved (1 normal field voltage).

After reaching steady state conditions, a solid phase-neutral short-circuit is applied to the terminal A of the machine. The fault is applied at the time that phase A voltage crosses zero. Variations of the field current and the stator phase A current of the machine for the first 3.5 seconds of the short circuit is shown in Figures 4a and 4b respectively. In each figure there are three curves, one corresponding to the real-time model labeled as “Proposed Model”, one corresponding to the PSCAD standard synchronous machine model labeled as “Standard” and the other corresponding to the off-line phase domain solution labeled as “Off-Line Solution”.

As shown in Fig 4, the three curves are indistinguishable, establishing the fact that the proposed model is equivalent to the off-line phase domain machine model. Consequently the integrity of transformations used in this paper is validated. Note that applying a single-phase fault excites the zero sequence circuit in addition to the d- and q-axes circuits.

B. Asymmetrical Fault on a 5-Phase Synchronous Machine

In this section, the above experiment in Section IV-A is repeated when the number of machine phases is 5. The proposed multi-phase synchronous machine model is configured to be a symmetrical 5-phase synchronous machine model. During the simulation, the machine is operating in open-circuit and its field voltage is adjusted such that the

terminal voltage of 1 pu is achieved (1 normal field voltage).

Similarly, in steady-state, a solid phase-neutral short-circuit is applied to terminal A of the machine at the zero-crossing of the phase A voltage. Other phases were left open. Variations of the field current and the stator phase A current of the machine for the first 3.5 seconds of the short circuit is shown in Figures 5a and 5b respectively. As can be seen again, the results are matching. As expected, stator currents have lower magnitude compared to the 3-phase machine with the same MVA rating shown in Fig. 4.

Note that applying a single-phase fault on a five phase machine excites multiple zero sequences in addition to the conventional zero sequence circuits known for 3-phase machines. Researchers have developed approximate analytical solutions for the single phase short circuit current in a 3-phase machine [20]. Developing an analytical solution for asymmetrical faults on multi-phase synchronous machines remains a subject of further research.

C. A Symmetrical 5-Phase Fault on a 5-Phase Synchronous Machine

The experiment is repeated with a symmetrical 5-phase short circuit on a 5-phase synchronous machine. The simulation results for the field current and the stator phase A current are shown in Figures 6a and 6b respectively. As can be seen again, the results are identical to the off-line phase domain solution.

Note that in the case of a symmetrical 5-phase fault on a five phase machine, only d- and q-axes circuits are excited and the results will be similar to the ones for a 3-phase machine in per-unit quantities.

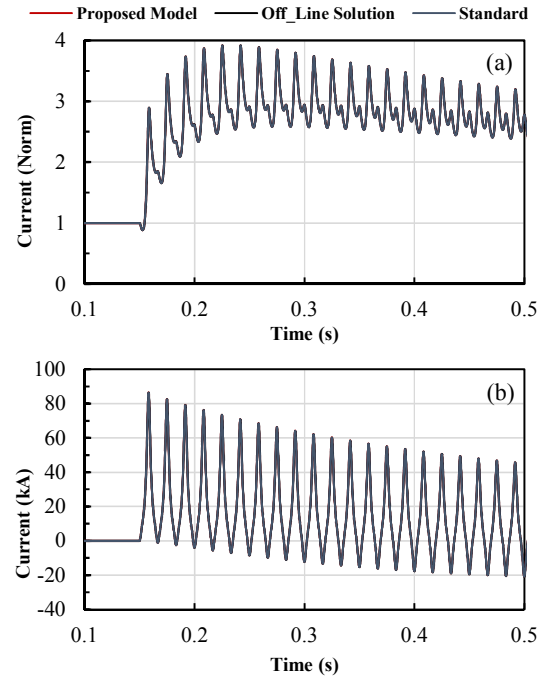


Fig. 4. Single phase fault on a 3-phase synchronous machine. (a) Field winding current. (b) Stator phase A current. Results overlap.

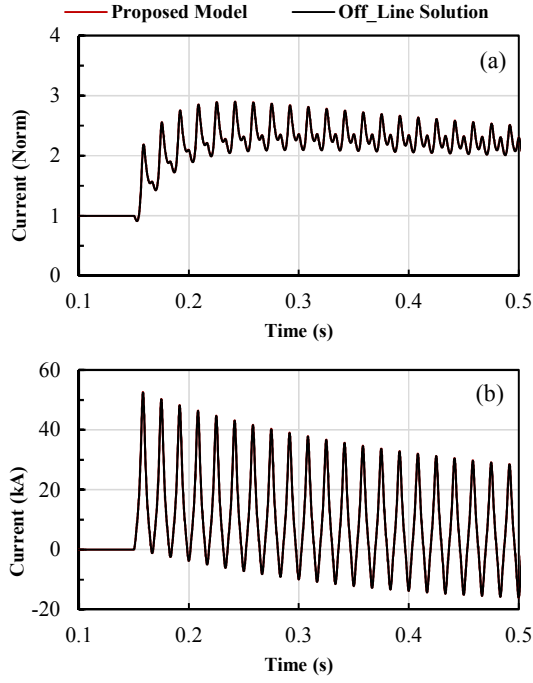


Fig. 5. Single phase fault on a 5-phase synchronous machine. (a) Field winding current. (b) Stator phase A current. Results overlap.

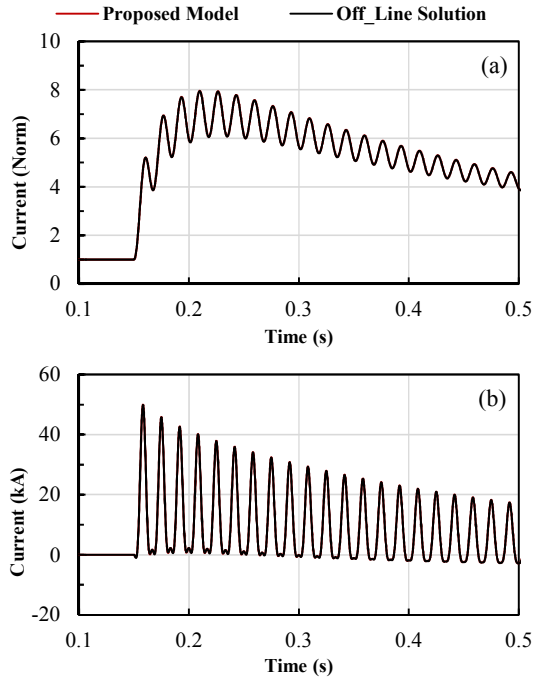


Fig. 6. 5-phase fault on a 5-phase synchronous machine. (a) Field winding current. (b) Stator phase A current. Results overlap.

V. CONCLUSIONS AND CONTRIBUTIONS

Analysis of symmetrical multi-phase synchronous machines is presented through extension of the two reaction theory to symmetrical multi-phase machines.

A detailed transient multi-phase synchronous machine model was developed for real-time digital simulation. The method is applicable to off-line electromagnetic transient programs as well.

The model presented in this paper was validated using stand-alone off-line simulations. Examples for applications of this model in simulating new schemes of wind farms and propulsion systems for naval application is planned to be presented in future publications.

In addition to the multi-phase machine model, multi-phase control components for transforming phase quantities ($ab\dots n$) to general stationary two-phase quantities ($\alpha\beta$) and general rotating two reaction components (dq) are developed and added to RSCAD library. This set of models not only has industrial applications in the above technical fields, but also provides a tool to study and analyze multi-phase systems.

VI. APPENDIX I

TABLE II

PARAMETERS OF THE SYNCHRONOUS MACHINE [21]

Per-Unit Base	Value
Line-neutral rated voltage	7.97 kV
Rated MVA	100 MVA
Rated Frequency	60 Hz
D- and Q-axes Parameters	Per-Unit value
Stator Leakage Reactance	0.13
D-axis Unsaturated Magnetizing Reactance	1.66
Field Winding Leakage Reactance	0.0618
D-axis Damper Leakage Reactance	0.00546
Q-axis Magnetizing Reactance	1.58
Q-axis Damper Leakage Reactance	0.3293
Stator Resistance	0.002
Field Resistance	0.001407
D-axis Damper Resistance	0.00407
Q-axis Damper Resistance	0.01415
Zero Sequence Parameters	Per-Unit value
0 Zero Sequence Reactance	0.13
0 Zero Sequence Resistance	0.002
U1 Zero Sequence Reactance	0.13
U1 Zero Sequence Resistance	0.002
V1 Zero Sequence Reactance	0.13
V1 Zero Sequence Resistance	0.002

VII. APPENDIX II

This appendix presents the summary of transformation matrices used in this paper. Note that transformation matrices listed in this appendix are in the non-orthogonal form. Here, the dimension of matrices is the number of phases.

Transformation matrices from phase quantities ($ab\dots n$) to symmetrical components and the general two-phase quantities ($\alpha\beta$) are shown below. In these equations:

$$\begin{aligned}
 (f_{ab\dots n})^T &= (f_a \ f_b \ \dots \ f_n) \\
 (f_{012\dots N})^T &= (f_0 \ f_1 \ \dots \ f_{N-1}) \\
 f_1 &= f_+ \quad f_{N-1} = f_- \\
 (f_{0\alpha\dots\beta})^T &= (f_0 \ f_\alpha \ f_{\alpha+1} \ \dots \ f_w \ \dots \ f_{v1} \ f_\beta) \\
 i, k &\in 1, 2 \dots N \quad \text{where: } N = \text{number of phases}
 \end{aligned} \tag{5}$$

Transformation matrix from the phase quantities ($ab\dots n$) to

symmetrical components is shown in (6):

$$[S^{-1}]_{ik} = \frac{1}{N} \alpha^{(i-1)(k-1)} \quad \text{where: } \alpha = e^{j\delta} \quad (6)$$

Transformation matrix from the phase quantities to the general two-phase quantities ($\alpha\beta$) is shown in (7):

$$[J]_{ik} = \frac{1}{N} \begin{cases} 1 & i = 1 \\ 2 \cdot \cos[(i-1)(k-1)\delta] & i < \frac{N}{2} + 1 \\ \cos[(\pi)(k-1)] & i = \frac{N}{2} + 1 \\ 2 \cdot \sin[(N+1-i)(k-1)\delta] & i > \frac{N}{2} + 1 \end{cases} \quad (7)$$

Transformation matrix from the general two-phase quantities to general two reaction components (dq) is shown in (8). Note that in the conventions used in this paper, d-axis leads q-axis.

$$\begin{pmatrix} f_d \\ f_q \end{pmatrix} = \begin{pmatrix} \cos(\theta_r) & \sin(\theta_r) \\ \sin(\theta_r) & -\cos(\theta_r) \end{pmatrix} \begin{pmatrix} f_\alpha \\ f_\beta \end{pmatrix} \quad (8)$$

VIII. ACKNOWLEDGMENT

The author is grateful to the valuable recommendations provided by Professor Hamid Toliyat from Texas A&M University. The author sincerely appreciates the high valued technical support from the colleagues at RTDS Technologies Inc.

IX. REFERENCES

- [1] Toliyat, H. A, "Analysis and Simulation of Five-Phase Synchronous Reluctance Machines Including Third Harmonic of Airgap MMF", *IEEE Trans. On Industry Application*, vol. 34, no.2, pp. 332-339, March/April 1998.
- [2] L. Parsa, "On advantages of multi-phase machines", Industrial Electronics Society, 2005. IECON 2005. (IECON 2005), Raleigh, Nov 2005.
- [3] E. Levi, "Multiphase Electric Machines for Variable-Speed Applications," *IEEE Trans. on Industrial Electronics*, Vol. 55 (5), May 2008, pp. 1893-1909.
- [4] E.A. Klingshim, "High Phase Order Induction Motors - Part I-Description and Theoretical Considerations", *IEEE Trans. On Power Apparatus and Systems*, vol. PAS-102, no.1, pp. 47-53, Jan. 1983.
- [5] R. H. Park, "Two reaction theory of synchronous machines, Part 1," *AIEE Transactions*, vol. 48, pp. 716-730, 1929.
- [6] R. H. Park, "Two reaction theory of synchronous machines, Part 2," *AIEE Transactions*, vol. 52, p 352, 1933.
- [7] R. F. Schiferl, C. M. Ong, "Six Phase Synchronous Machine with AC and DC Stator Connections, Part I: Equivalent Circuit Representation and Steady-State Analysis", *IEEE Trans. On Power Apparatus and Systems*, vol. PAS-102, no.8, pp. 2685 - 2693, Aug. 1983.
- [8] M.R. Aghaebrahimi, and R.W. Menzies, "A transient model for the dual wound synchronous machine", International Power System Transient Canadian Conference on Electrical and Computer Engineering (CCECE 1997), St John's, May, 1997.
- [9] A. B. Dehkordi, P. Neti, A.M Gole, and T.L. Maguire, "Development and Validation of a Comprehensive Synchronous Machine Model for a Real-Time Environment", *IEEE Trans. On Energy Conversion*, vol. 25, no.1, pp. 34-48, Mar 2010.
- [10] A.B. Dehkordi, A.M Gole, and T.L. Maguire, "Permanent magnet synchronous machine model for real-time simulation", International Power System Transient Conference (IPST 2005), Montreal, June, 2005.
- [11] Y. H. Ku, "Transient Analysis of AC Machinery," *AIEE Transactions*, vol. 48, p. 707, 1929.
- [12] Charles L. Fortescue, "Method of Symmetrical Co-Ordinates Applied to the Solution of Polyphase Networks," *AIEE Transactions*, vol. 37, part II, pp. 1027-1140, 1918.

- [13] P. Vas, Parameter Estimation, Condition Monitoring, Diagnosis of Electrical Machines, Oxford Science. Publications, 1993.
- [14] *RTDS User's Manual*, RTDS Technologies Inc., Winnipeg, Canada, 2016.
- [15] National Taiwan University, Collected Scientific Papers of Professor Yu Hsiu Ku, 1971.
- [16] H. W. Dommel, "Digital computer solution of electromagnetic transients in single and multiphase networks" ,*IEEE Trans. Power Apparatus and Systems*, vol.PAS-88, No.4, pp. 388-399, Apr. 1969.
- [17] A.M. Gole, R.W. Menzies, D.A. Woodford and H. Turanli, "Improved interfacing of electrical machine models in electromagnetic transient programs," *IEEE Trans. Power Apparatus and Systems*, vol. PAS-103, no. 9, pp. 2446-2451, Sept. 1984.
- [18] T.L. Maguire, "An Efficient Saturation Algorithm for Real Time Synchronous Machine Models using Flux Linkages as State Variables," *Electrimacs 2002*, Montreal, Canada, June 2002. *IEEE Trans. Power Apparatus and Systems*, vol. PAS-103, no. 9, pp. 2446-2451, Sept. 1984.
- [19] T. Maguire and J. Giesbrecht, "Small timestep (<2 μ s) VSC Model for the Real Time Digital Simulator", International Conference on Power System Transients (IPST'05) in Montreal, Canada, June 19-23, 2005, paper No. IPST05-168
- [20] C. Concordia, Synchronous machines theory and performance, New York, John Wiley & Sons, Inc., 1951, p. 28.
- [21] IEEE Committee Report, "First benchmark model for computer simulation of subsynchronous resonance" , *IEEE Trans. on Power Apparatus and Systems* , vol. PAS-96, No.5, pp. 1565-1572, Sept./Oct. 1977.



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