Different approaches on modeling of overhead lines with ground displacement currents

Antonio C. S. Lima, Rodolfo A. M. Moura, Marco A. O. Schroeder, Maria Teresa Correia de Barros

Abstract—**There is a growing concern regarding a more accurate assessment of the transient behavior of overhead lines and underground cables. Besides the well-known behavior of the skin effect in conductor and ground, recent works have shown that there are cases where the ground return admittance must be considered, such as lightning performance assessment, transients in short lines or cables. However, traditionally, such analyses when carried out using time-domain transient programs such as ATP-EMTP, EMTP-RV, or PSCAD do not consider the possibility of the inclusion of the ground return admittance in the so-called line/cable parameters routine. In this paper, we propose to analyze the impact of displacement currents in the time-domain modeling of two distinct overhead lines.**

***Keywords*: Transmission line modeling, Transmission line theory frequency domain synthesis, Time-domain analysis.**

I. Introduction

THE assumption of ground as a good conductor is one of the most common assumption in the development of transient models for overhead lines and underground cables [1]-[3]. Typically, closed-form approximations are used instead of the traditional infinite integrals expressions [5]-[7]. There are some possibilities to derive closed-form approximation and among them, the usage of the so-called image methods provide simpler expressions based on logarithmic functions. As shown in [8][9], even when ground displacement currents are considered, it is possible to derive simple expressions to include the ground return admittance.

The developments of frequency dependent soil parameters shed a new light of interest in the applicability/suitability of closed form for an adequate assessment of the transient performance of overhead lines [10]-[12]. One interesting aspect of the inclusion of ground displacement currents is that there are different formulations of the line parameters which might lead to distinct frequency and time domains behavior. Thus, in this paper, we propose to investigate the impact of these distinct approaches for the inclusion of ground displacement currents in the modeling of two distinct overhead lines. Two test cases are considered to provide a more general overview, involving circuits with "vertical" and "horizontal" configuration. To avoid instability issues in the Method of Characteristic due to inaccurate interpolation of modal travel times [13], the time responses are obtained using the Numerical Laplace Transform [14][15].

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II. Impedance & Admittance Expressions

From a rigorous point of view, a complete characterization of the electromagnetic field associated with overhead lines can be obtained using the so-called full-wave formulation [16][17][18]. Unfortunately, an extension of this formulation to the multi-phase configuration has not been developed and the usual solution is to resort to a quasi-TEM (transverse electromagnetic) approximation. Given that a quasi-TEM approximation must deal with complex infinite integrals is common to use closed-form approximations to approximately represent the ground return impedance and more recently the ground return admittance of overhead lines[8][9].

Consider an overhead line with infinitely long conductors *i* and *j*, both at a constant height, h_i and h_j respectively, and with radius *r* as depicted in Fig. 1. Both air and ground are characterized by a permittivity ϵ_i, conductivity σ_i where *i=1* for air and *i=2* for the ground, permeability $\mu_1 = \mu_2 = \mu_0$, and propagation constant $\gamma_i = \sqrt{j\omega\mu_0(\sigma_i + j\omega\varepsilon_i)}$.

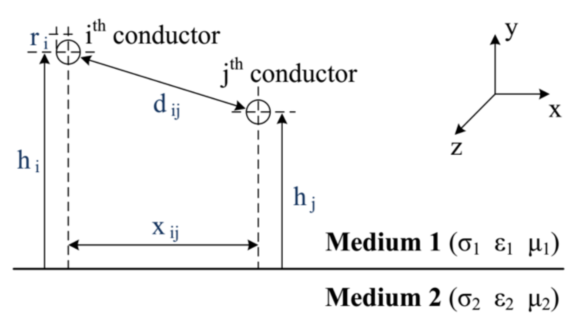


Fig. 1. Configuration of the conductors for an overhead line

The voltage to ground of each conductor U_n is given by [9]

$$U_n = V_n + j\omega\int_0^{h_n} A_y(r_n,\zeta)d\zeta \tag{1}$$

where r_n is the radius of conductor n, A_y is the vertical component of the magnetic vector potential $\mathbf{A}(x,y)$, and V_n is the difference of the electric scalar potential $\varphi_n(x,y)$ between conductor n and ground given by

$$V_n = \varphi_n(r_n, h_n) - \varphi_n(0,0) \qquad (2)$$

Assuming a thin wire and quasi-TEM approximations, and $\mu_1 = \mu_2 = \mu_0$, the per-unit-length impedance and admittance matrices are then

$$\begin{aligned} \mathbf{Z} &= \mathbf{Z}_i + \frac{j\omega\mu_0}{2\pi}\left[\mathbf{P} + \mathbf{S}_1 - \left(\mathbf{S}_2 + \mathbf{S}_3\right)\right] \\ \mathbf{Y} &= 2\pi\left(j\omega\epsilon_0\right)\left[\mathbf{P} - \mathbf{S}_3\right]^{-1} \end{aligned} \qquad (3)$$

where $\mathbf{Z}_i$ stands for the internal impedance using Bessel functions, and the elements in $\mathbf{P}$ are given by

$$\mathbf{P}_{ii} = \ln\frac{2h_i}{r_i} \qquad \mathbf{P}_{ij} = \ln\frac{D_{ij}}{d_{ij}} \qquad (4)$$

with $D_{ij} = \sqrt{\ell_{ij}^2 + x_{ij}^2}$, $\ell_{ij} = h_i + h_j$ and d_{ij} and x_{ij} are as shown in Fig. 1. The elements of $\mathbf{S}_1$, $\mathbf{S}_2$ and $\mathbf{S}_3$ are given by

$$S_{1ij} = \int_{-\infty}^{\infty} \frac{\exp\left(-\ell_{ij}\lambda\right)}{\lambda + \bar{u}} \exp(j\lambda x_{ij})\, d\lambda \qquad (5)$$

$$S_{2ij} = \int_{-\infty}^{\infty} \frac{\exp\left(-\ell_{ij}\lambda\right)}{n^2\lambda + \bar{u}} \exp(j\lambda x_{ij})\, d\lambda \qquad (6)$$

$$S_{3_{ij}} = \int_{-\infty}^{\infty} \frac{\bar{u}}{\lambda}\left(\frac{\exp\left(-\ell_{ij}\frac{\lambda}{2}\right) - \exp\left(-\ell_{ij}\lambda\right)}{n^2\lambda + \bar{u}}\right)\exp(jx_{ij}\lambda)\, d\lambda \qquad (7)$$

where $\bar{u} = \sqrt{\lambda^2 + \gamma_2^2 - \gamma_1^2}$ and n is the refractive index of the ground. The expression of $\mathbf{S}_1$ are a simple extension of the ground return model of Pollaczek [1] and Carson [2] and $\mathbf{S}_2$ was proposed by Wise [19]. For the inclusion of the conductor losses we must assume that its propagation constant γ_c is such that $\gamma_c \gg \gamma$, where γ is the overall propagation constant of a given overhead line. For power transmission circuit is condition is true for a wide frequency range, thus one can easily include the skin effect in the conductor´s internal impedance.

Alternatively, one may use (2) instead of (1), i.e., use the potential difference V_n instead of the conductor voltage U_n to define the line parameters. This is equivalent to disregard the effect of A_y in the line parameters. In this case, the following expressions are obtained

$$\begin{aligned} \mathbf{Z} &= \mathbf{Z}_i + \frac{j\omega\mu_0}{2\pi}\left[\mathbf{P} + \mathbf{S}_1 - \mathbf{S}_{20}\right] \\ \mathbf{Y} &= 2\pi\left(j\omega\epsilon_0\right)\left[\mathbf{P} + \mathbf{S}_2 - \mathbf{S}_{20}\right]^{-1} \end{aligned} \qquad (8)$$

where $\mathbf{S}_{20} = \mathbf{S}_2 + \mathbf{S}_4$. The elements in $\mathbf{S}_4$ are given by

$$S_{4_{ij}} = \int_{-\infty}^{\infty} \left(\frac{\exp\left(-\ell_{ij}\frac{\lambda}{2}\right) - \exp\left(-\ell_{ij}\lambda\right)}{n^2\lambda + \bar{u}}\right)\exp(jx_{ij}\lambda)\, d\lambda \qquad (9)$$

Even simpler expressions are obtained if one considers only $\varphi(r_n, h_n)$ to define the parameters which then leads to the following

$$\begin{aligned} \mathbf{Z} &= \mathbf{Z}_i + \frac{j\omega\mu_0}{2\pi}\left[\mathbf{P} + \mathbf{S}_1\right] \\ \mathbf{Y} &= 2\pi\left(j\omega\epsilon_0\right)\left[\mathbf{P} + \mathbf{S}_2\right]^{-1} \end{aligned} \qquad (10)$$

For the sake of clarity, the following is adopted in the remainder of this paper. If (3) is used to define the line parameters, we call this *voltage formulation*. If (8) is used then it is called *potential difference formulation* and if (10) is used then it is termed *potential formulation*. It is worth mentioning that if the ground displacement currents are neglected both $\mathbf{S}_2$, $\mathbf{S}_3$ and $\mathbf{S}_4$ tend to zero and all formulations lead to the well-known results with the ground assumed as a good conductor.

III. Closed-Form Approximations

Regardless of the formulation used to define the line parameters, the main issue lies in the need to deal with infinite integrals. One may resort to Gaussian quadrature methods as in [23][24] although it might be preferable to use an approximated solution to improve the overall process of evaluating the per unit length parameters. A possible alternative consists in using an approximated integrand to allow a closed-form based on logarithms. This process is summarized in the appendix and leads to the following

$$S_{1_{ij}} \approx \overline{S_{1_{ij}}} = \ln\left(1 + \frac{2}{\eta\sqrt{\ell_{ij}^2 + x_{ij}^2}}\right) \qquad (11)$$

$$S_{2_{ij}} \approx \overline{S_{2_{ij}}} = \frac{2}{n^2+1}\ln\left(1 + \frac{n^2+1}{\eta\sqrt{\ell_{ij}^2 + x_{ij}^2}}\right) \qquad (12)$$

$$S_{3_{ij}} \approx \bar{S}_{3_{ij}} = 2\ln 2 + \frac{2n^2}{n^2+1}\left(\ln\frac{1 + \frac{n^2+1}{\eta\sqrt{\ell_{ij}^2 + x_{ij}^2}}}{1 + 2\frac{n^2+1}{\eta\sqrt{\ell_{ij}^2 + x_{ij}^2}}}\right) \qquad (13)$$

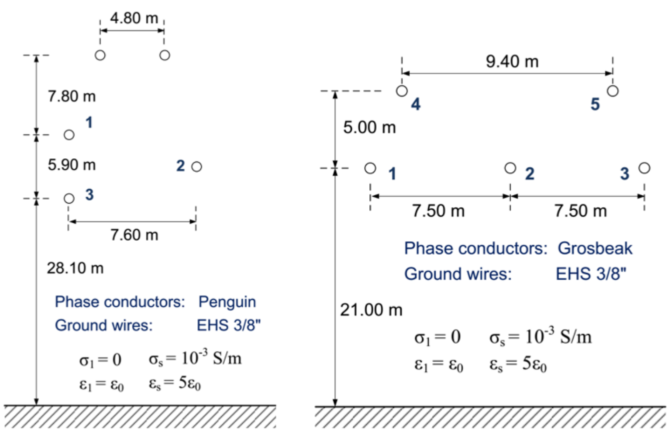
$$S_{4_{ij}} \approx \overline{S_{4_{ij}}} = \frac{2}{n^2+1}\left(\ln\frac{1 + \frac{n^2+1}{\eta\sqrt{\ell_{ij}^2 + x_{ij}^2}}}{1 + 2\frac{n^2+1}{\eta\sqrt{\ell_{ij}^2 + x_{ij}^2}}}\right) . \qquad (14)$$

The expressions in (11) to (14) are multi-phase generalization of the single-phase expressions presented in [9], the exception being (13) which is slightly different from the one proposed in [9].

Naturally, the main option for the analysis of electromagnetic transients is the voltage formulation. However as initially shown in [20] and later discussed in [21][22], the usage of closed-form leads to numerical instability in the nodal admittance matrix for short lines as passivity violations appear in high frequency range. Thus, there is a need to identify whether the alternative formulation might lead to accurate responses when compared with the voltage formulation.

IV. Test Cases

Two test cases were considered for the evaluation of the impact of the distinct approaches presented in the previous section. The first one consists of a vertical circuit and the second deals with a horizontal one as depicted in Fig. 2. to ensure that high frequency components are presented in the transient voltages/current, it is assumed here a line length of 500 m. To avoid possible passivity violations associated with the voltage formulation when image-type approximations are used [20][21][22], only the Numerical Laplace Transform was used to obtain the time responses.



(a) 138 kV vertical circuit (b) 230 kV horizontal circuit

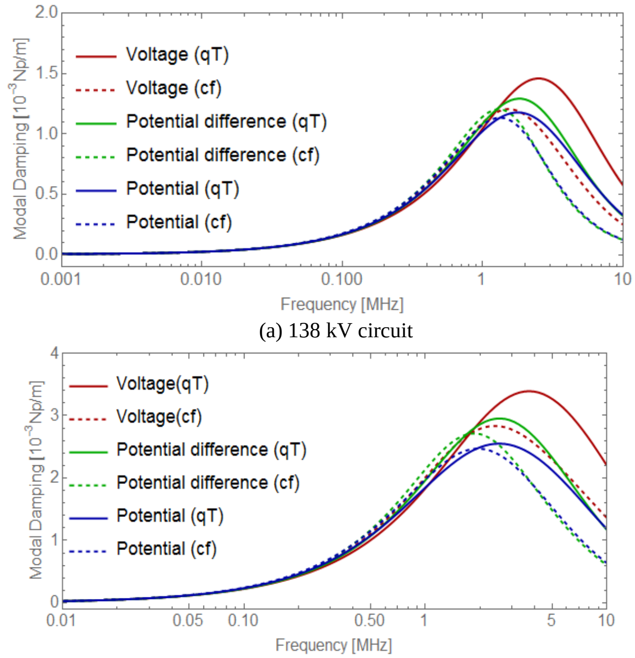
Fig. 2. Overhead line configurations considered.

A. *Ground mode propagation constant*

The three distinct formulations presented in the previous section lead to essentially the same natural modes with the exception being the ground mode. The natural propagation modes are obtained as the eigenvalues of $\sqrt{\mathbf{Z}\cdot\mathbf{Y}}$.

Figure 3 depicts the behavior of the ground mode damping for both circuits, considering the quasi-TEM (qT) and closed-from (cf) approximations. All approximations are close to their quasi-TEM counterpart close to 1 MHz. The voltage formulation is the one with the larger mismatch. The potential difference and the potential formulation are very close to each other in all bandwidth and if we consider only their closed-form approximations, they are essentially identical apart from frequency of the maxima. It is worth mentioning that the closed-form approximation of the ground mode damping is quite close to the one obtained using the potential formulation.

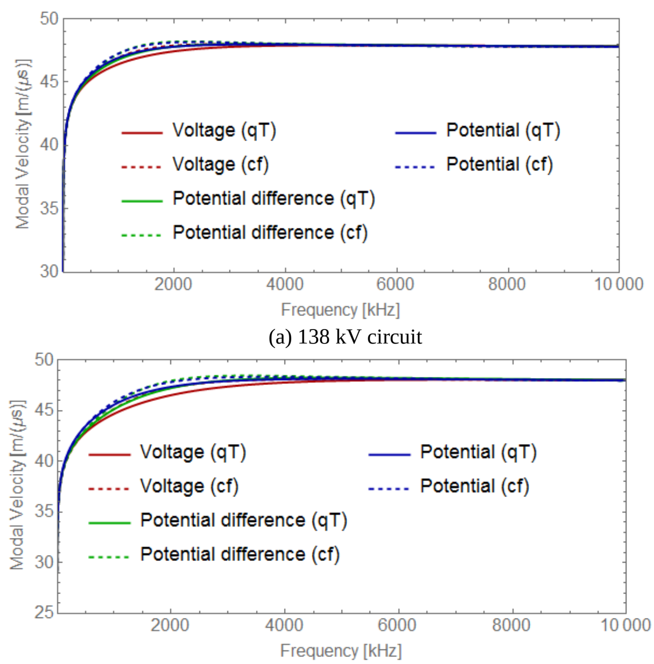
Figure 4 shows the behavior of the ground mode velocity. In this case, again the voltage formulation presents a behavior slightly different from the other two approaches and the potential and potential difference formulations are quite similar in all frequency range.



(a) 138 kV circuit

(b) 230 kV circuit

Fig. 3. Ground mode damping for the two test cases considering the three distinct formulations.



(a) 138 kV circuit

(b) 230 kV circuit

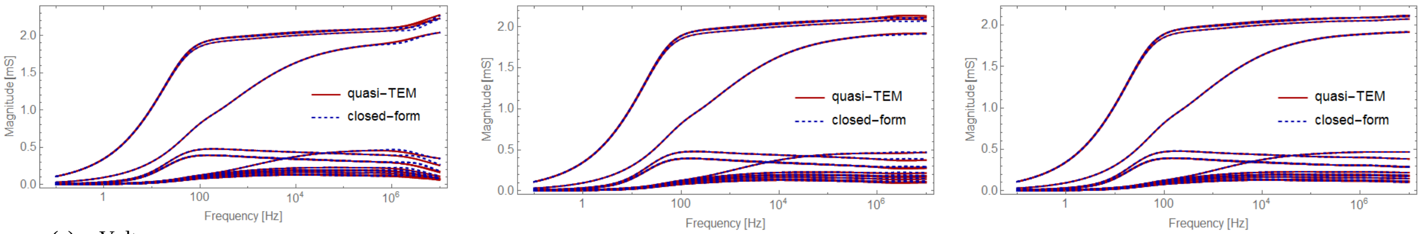
Fig. 4. Ground mode velocity for the two test cases line considering the three distinct formulations.

A. *Characteristic admittance and propagation function*

The propagation function, **H**, and characteristic admittance, $\mathbf{Y}_c$, are calculate as

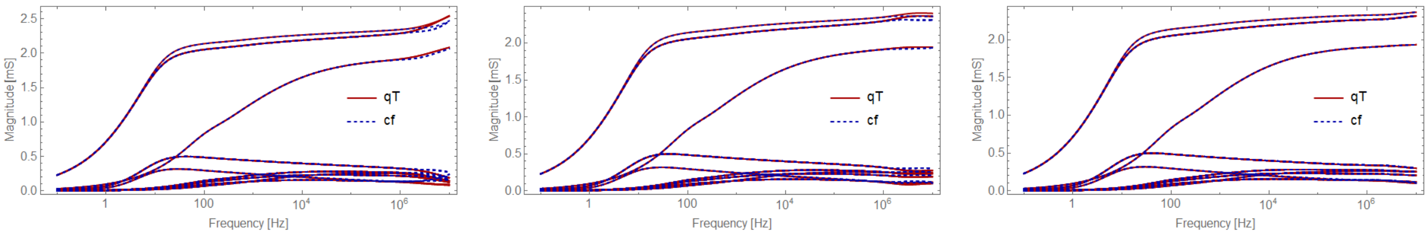
$$\mathbf{Y}_c = \mathbf{Z}^{-1}\sqrt{\mathbf{Z}\cdot\mathbf{Y}} \qquad \mathbf{H} = \exp\left(-\ell\sqrt{\mathbf{Y}\cdot\mathbf{Z}}\right) \tag{15}$$

where **Z** and **Y** are calculated with the three possible formulations and ℓ is the length of the cable system, i.e., 500 m.



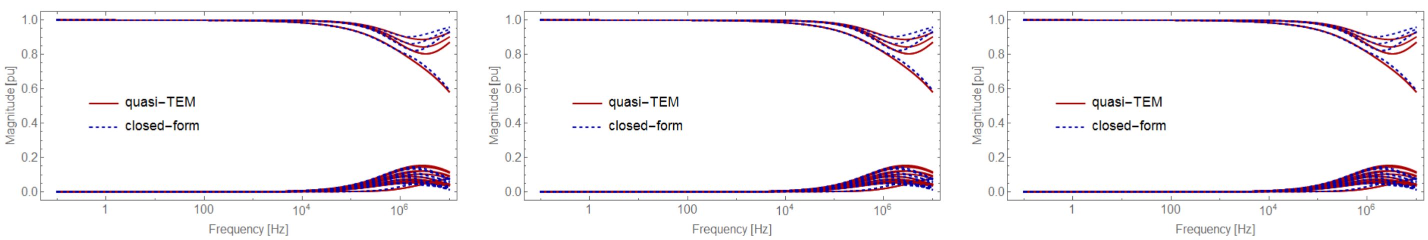
(a) Voltage (b) Potential difference (c) Potential

Fig. 5. Characteristic admittance for the 138 kV line considering the three distinct formulations.



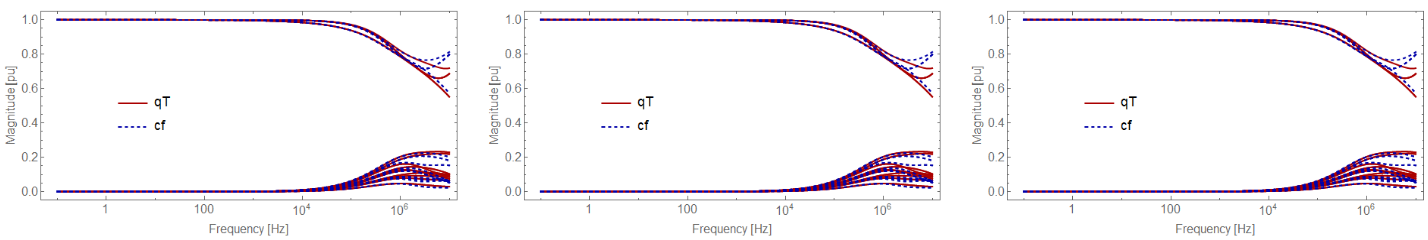
(a) Voltage (b) Potential difference (c) Potential

Fig.6. Characteristic admittance for 230 kV line considering the three distinct formulations.



(a) Voltage (b) Potential difference (c) Potential

Fig. 7. Propagation function for the 138 kV line considering the three distinct formulations.



(a) Voltage (b) Potential difference (c) Potential

Fig. 8. Propagation function for 230 kV line considering the three distinct formulations.

The results for the characteristic admittance are shown in Fig. 5 for the 138kV line and in Fig. 6 for the 230 kV line. In both circuits, the closed-form for the potential formulation is the one that presented the smallest deviation. For the voltage and potential difference formulations, we note that the closed-form start to deviate from the quasi-TEM approximation after 1 MHz.

For the propagation function the scenario is slightly different, all three approaches presented small deviation after 100 kHz and are more noticeable after 1 MHz. These results are depicted in Figs. 7 and 8. Unlike the conventional approach, when ground displacement currents are neglected, the magnitude of **H** is not a monotonic function in the frequency. This behavior is more noticeable for the diagonal elements in **H**. This is an interesting aspect as this type of behavior might affect the identification of the modal travelling times need for time-domain modeling of overhead lines. This aspect is left for future research and not addressed here as the main focus has been with frequency domain modeling.

B. *Time responses*

For the assessment of the time responses, we consider a very simple configuration as depicted in Fig. 9. For the input, we considered a very narrow current impulse as shown in Fig. 10. A total simulation time of 30 μs was considered and 4096 samples were used. For both overhead lines, the results considered the circuit modeled using quasi-TEM approximation with the voltage formulation. All the other three formulation were calculated using the image-type approximation.

Figures 11 and 12 depicts the voltage at nodes 4 and 6, respectively, for the 138kV line and Figures 13 and 14 show the results for both nodes for the 230kV line. The results are quite similar although there are some small but noticeable discrepancies between them.

It is worth mentioning that in case of a single line span, the assuming of an infinite line to determine the pul parameters is not valid in the high frequency range, typically above a few MHz. The influence of this limitation in the transient response of an overhead line as addressed in [25].

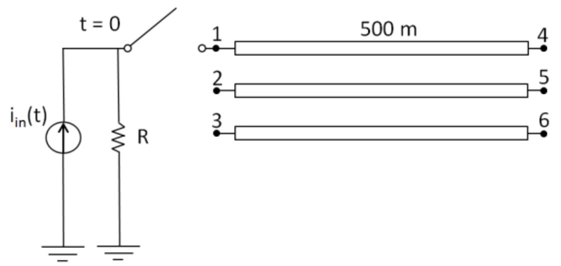


Fig. 9. Circuit for evaluation of time responses

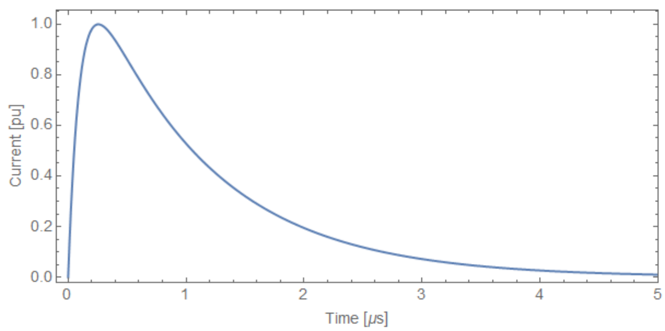


Fig. 10. Impulse current

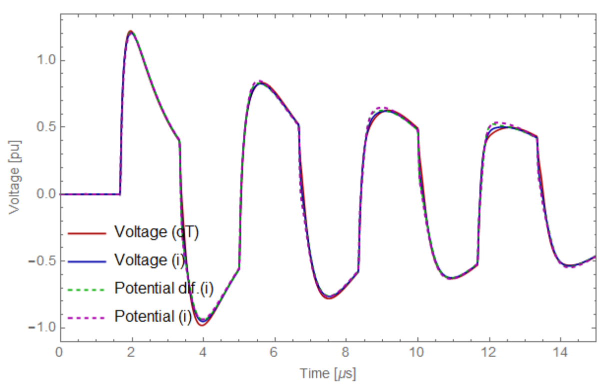


Fig. 11. Voltage response at node #4 for the 138 kV line.

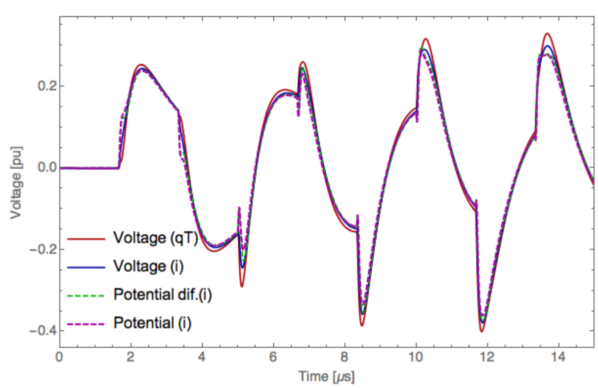


Fig. 12. Voltage response at node #6 for the 138 kV line.

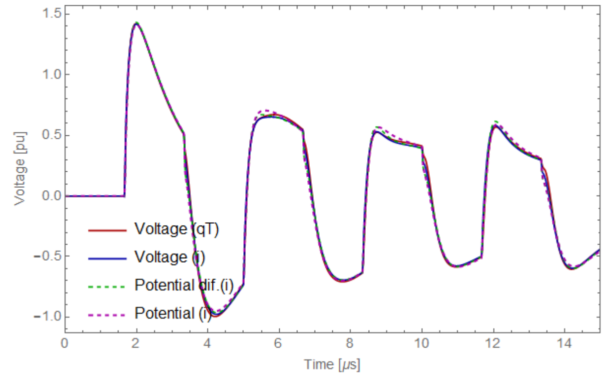


Fig. 13. Voltage response at node #6 for the 230 kV line.

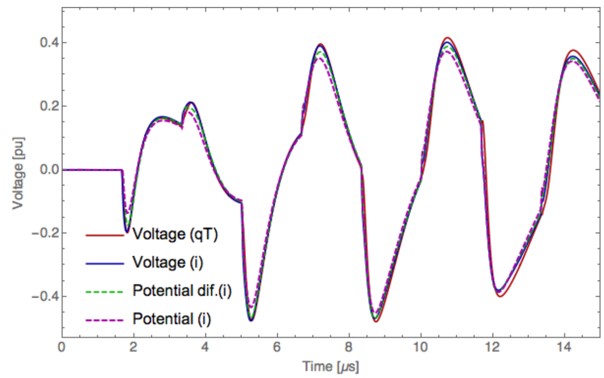


Fig. 14. Voltage response at node #6 for the 230 kV line.

V. CONCLUSIONS

This work has focused on the possible formulation to include the ground admittance for the evaluation of electromagnetic transients. Three distinct formulations are possible and the results indicate that for frequencies below a few MHz almost identical $\mathbf{Y}_c$ and $\mathbf{H}$ are obtained.

The evaluation of the time-response to an impulse test indicate that only small deviations are obtained using the three formulation when compared with the results obtained using quasi-TEM approximation and the voltage formulation. Thus given it simplicity it seems that the potential formulation might be more suitable to be used whenever closed-form expressions are considered.

Future work will deal with the rational modeling of $\mathbf{Y}_c$ and $\mathbf{H}$ and an assessment of this approach for actual lightning performance cases.

APPENDIX A

The process of deriving an image-type approximation consist in further simplifying the infinite integrals found in the quasi-TEM approximation of impedance and admittance to allow a closed-form solution. These integrals have the following structure, see [9] for further details,

$$\int_0^\infty \frac{\exp(-b\lambda)}{a\lambda+\sqrt{\lambda^2+\eta^2}}\cos(c\lambda)\,d\lambda \tag{16}$$

A closed form to (16) can be found if the denominator is approximated by

$$\left(a\lambda + \sqrt{\lambda^2 + \eta^2}\right)^{-1} \approx \frac{1}{\lambda}\frac{1}{1+a}\left[1 - \exp\left(-\lambda\frac{1+a}{\eta}\right)\right] \quad (17)$$

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