Non-Uniform Lines – Review of the Theory and Measured / Simulation Examples

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Abstract—**This paper reviews existing theories dealing with a non-homogeneous line composed of cascaded homogeneous lines. The theories neglect lead wires between the cascaded lines. This is mathematically permissible, but physically impossible and thus the theories result in an in-accurate or erroneous solution of the non-homogeneous line. The lead wires are non-uniform because the geometrical configuration at the sending end of the lead wires, i.e. the first homogeneous line, differs from that at the receiving end, i.e. the next homogeneous line. Measured and FDTD computed results of the non-uniform line characteristics are demonstrated. Finally approximate calculations by EMTP are compared with the measured and FDTD computed results, and the limit of the EMTP simulations is explained.**

***Keywords*: non-uniform line, homogeneous/non-homogeneous line, measured result, EMTP, FDTD.**

I. INTRODUCTION

When cross-bonded underground cables were installed in 1960s and 1970s, it became a significant problem to find an efficient method for calculating transient voltages on the cables by a computer in that time, because it necessitated a large amount of memories and CPU times. Therefore it was essential to develop an equivalent homogeneous cable so as to make a transient simulation by a circuit-theory based approach possible [1]-[6]. This was the origin of the theory of non-homogeneous lines. The same method is applicable to an overhead transposed line [7]. When an extra-high-voltage (EHV) line was constructed and put in services in 1970s, a lightning surge analysis became important, and modeling of a transmission tower (vertical conductor) involved a theory of a non-uniform line of which the voltage was position-dependent. There are a number of papers investigating and discussing the non-uniform lines [8]-[26]. Related to a lightning surge in a substation, inclined and non-parallel conductors from the gantry to the bus are non-uniform and cause a difficulty in an accurate simulation of the lightning surge by EMTP [18]-[21].

The above non-homogeneous and non-uniform lines require a theory to deal with the lines for steady-state and transient simulations and at the same time the impedance and admittance of the line for EMTP simulations.

A recent publication [7] discusses a negative conductance required for matching of a non-homogeneous line composed of two homogeneous lines. The theories is only mathematical neglecting the physical parameters of the lines.

Considering the above, this paper reviews existing theories of non-homogeneous lines and discusses problems associated with the theories. The characteristics of non- uniform lines are demonstrated based on measured and FDTD (finite-difference time-domain) computed results. Finally, an approach to deal with the non-homogeneous and non-uniform lines by EMTP is explained, and EMTP simulation results are compared with the measured and FDTD computed results.

II. THEORY OF NON-UNIFORM LINE

A. Definition

The definition of a non-uniform line is given in the following form as a function of the impedance and admittance.

$$V(x) = Z(x)\, I(x),\quad I(x) = Y(x)\, V(x) \qquad (1)$$

where x: position, either vertical, inclined or horizontal, to the voltage reference, Z(x) and Y(x): position- (length-) dependent impedance and admittance.

In this paper, a non-homogeneous line is defined as a cascaded homogeneous lines of which the impedance and admittance are not position-dependent but are defined as “per-unit-length (pul)”.

B. Review of existing theories and impedances

B1. Theories of cross-bonded cable / transposed line

References [1] to [6] explains a theory of a non-homogeneous line composed of cascaded homogeneous lines i.e. a minor section of a cross-bonded cable. The theory is applicable to an overhead transposed line [7]. It should be noted that lead wires connecting the next section is only some meters in the cross-bonded cable, but those reach more than some ten meters in the transposed line.

B2. Impedance / admittance formulas

(1) Vertical conductor

References [8] to [17] show the impedance and admittance formulas of vertical conductors, a transmission tower for example.

(2) Inclined and non-parallel conductors

[18] to [21] show the impedance and admittance formulas of inclined and non-parallel conductors, and investigate the frequency and time responses.

C. Problems found in the theories

C1. Non-homogeneous line

Most references [1]-[7] neglect lead wires for cross-bonding and transposing phase conductors. The lead

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wires are almost always non-uniform, i.e. inclined and / or non-parallel. If those are to be considered, all the theories in [1] to [7] become not accurate or impossible to use. For example in [7], the lead wires transposing phases a', b' and c' on the left-hand side of a tower to phases a, b and c on the right of the tower might exceed 35 m in the case of a 500 kV horizontal line where the separation distance between phases a and c is more than 20 m. The tower span is 300 to 500 m. Thus the lead wire length reaches 10 % of the span length.

In the time of [1] to [6] for a cross-bonded cable up to 125 kV, a cross-bonding lead is less than 3 m while the length of one minor section is more than 200 m. Thus it was quite possible to neglect the cross-bonding lead.

It should be noted that the cross-bonding lead is almost always considered in an EMTP simulation of a transient on the cross-bonded cable, either as an inductance or a distributed line [6], [27], [28]. On the contrary, lead wires for transposing an overhead line are almost always neglected as in [7].

C2. Non-uniform line

It should be noted that all the impedance and admittance formulas of non-uniform lines in [15]-[21] are derived based on a circuit theory. Transient phenomena on the non-uniform lines are often associated with a non-TEM mode of wave propagation which cannot be handled by the circuit theory as discussed in [29]-[36]. Only a possible approach to deal with phenomena associated with the non-TEM propagation is a numerical electromagnetic analysis (NEA) method [32]-[36]. The non-TEM wave propagation is observed even in an infinite-length conductor with the pul impedance and admittance in both frequency and time domains as mode transition [37]-[40].

III. Transients on non-uniform lines

A. *Measured results*

A1. Vertical conductor

Fig. 1 shows measured results of transient responses at various positions (height) of a vertical conductor with height h = 25 m and radius r = 25 mm [16]. It is observed that the voltage waveforms at height $x_1 = h = 25$ m, $x_2 = 12$ m and $x_3 =$ 9 m are noticeably different from each other before the first reflection from the bottom (x = 0) to the position of x_3 appears at around $t = 2x_3 / c_0 = 18 / 0.3 = 60$ ns. Remind that the measurement starts considering a time delay, but not from time t = 0 when the source is applied. Because the conductor length (height) is not large, the wave deformation observed in the figure is not due to the frequency-dependent effect of the propagation constant. But it is dominantly caused by negative reflection at every instance due to the position-dependent impedance as defined in (1) in Section II-A, for Z_0 (x_1) greater than $Z_0(x_2)$. When the initial traveling wave reaches the conductor bottom, large negative reflection is produced and the reflected wave decreases the voltages as in Fig. 1 (b) to (d). This is a typical characteristic of a transient on a vertical conductor when a source is applied at the conductor top.

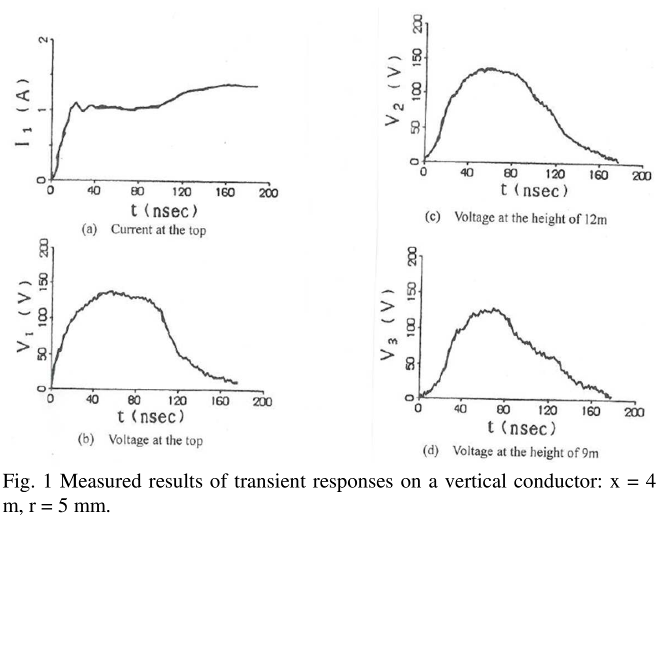


Fig. 1 Measured results of transient responses on a vertical conductor: x = 4 m, r = 5 mm.

A2. Horizontal single conductor

Table 1 shows the surge impedance Z_0 of a horizontal single conductor with length x = 4 m and radius r = 5 mm as a function of the conductor height "h". It is clear that Z_0 calculated by the finite-length impedance [20] agrees better than Z_0 calculated by the pul impedance implemented into EMTP. The difference decreases as the ratio (x / h) increases.

TABLE I
Surge impedances of an overhead conductor with length *x* = 4 m

h [cm]	*x/h*	Surge impedance [Ω] Measured	Finite	Infinite
21.0	19.1	201	218	224
39.5	10.2	233	251	262
61.0	6.57	256	272	287
83.5	4.80	278	285	307
102	3.93	295	293	319
Error [%]		——	5.1	10.9

A3. Non-parallel horizontal conductor

Fig. 2 shows measured results of induced voltages at the receiving end in a non-parallel two-conductor system when a step-like voltage is applied at the sending end of conductor 1.

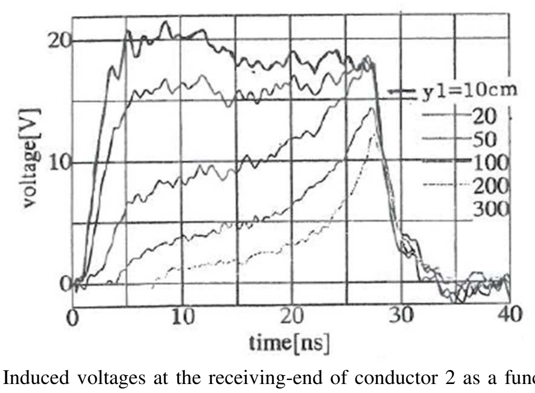


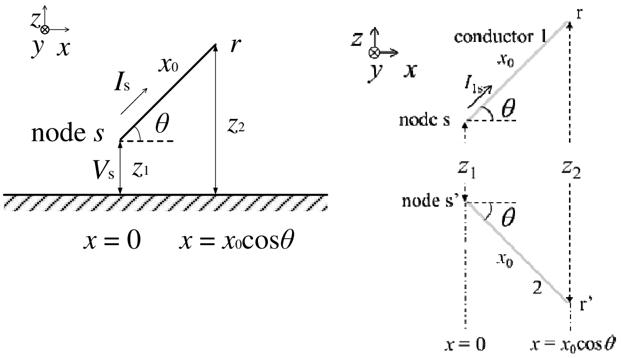
Fig. 2 Induced voltages at the receiving-end of conductor 2 as a function of separation distance y_1 at the sending end: $h_1 = h_2 = 0.4$ m, $r_1 = r_2 = 5$ mm, $x_1 = x_2 = 4$m, $y_2 = 10$ cm.

It is observed that the voltage increases as separation y_1 becomes smaller. When $y_1 = y_2 = 10$ cm, the transient voltage shows a typical waveform on a parallel conductor system.

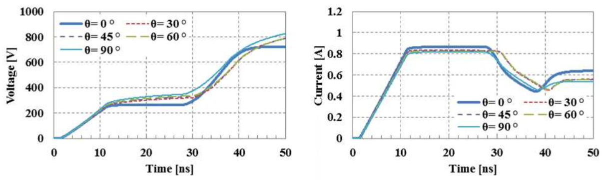
B. FDTD computed results

B1. Inclined conductor

Fig. 4 (a) shows a model circuit of an inclined conductor to the earth surface and FDTD simulation results of transient voltage and current for various angle θ when a current source with rise time T_f = 10 ns is applied at node "s"[31].



(a) Model circuit: a=5mm, x_0=4m, z_1=0.4m.(b) Equivalent circuit in free space



(c) Sending-end voltage v_s (d) Sending-end current i_s

Fig. 4 Transient voltage and current waveforms on an inclined conductor

The voltage and current waveforms in Fig. 4 (c) and (d) are nearly the same until t = 10 ns corresponding to the rise time T_f of the applied current. Then, the waveforms deviate from each other. The voltage is the smallest in the case of $\theta = 0°$, i.e. a horizontal conductor, and is the highest when $\theta = 90°$, i.e. a vertical conductor. Correspondingly, the current is the largest in $\theta = 0°$, and the smallest in $\theta = 90°$. The apparent (surge) impedance Z_0 at the sending end is estimated by the ratio of voltage v_s and current i_s. Fig. 5 shows the surge impedance Z_0 as a function of inclined angle θ. It is observed that Z_0 increases as angle θ increases. In general, the surge impedance is proportional to the inclined angle θ. The surge impedance can be theoretically evaluated by approximate formulas in [15] and [21].

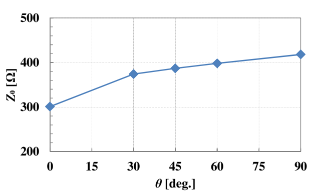
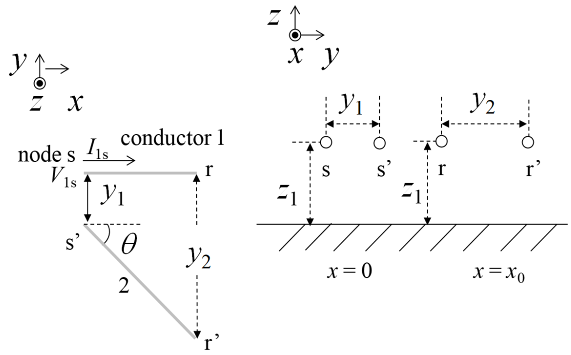


Fig. 5 Surge impedance Z_o as a function of inclined angle θ

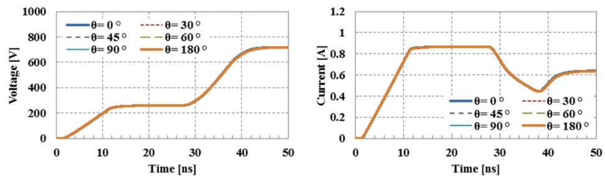
If we assume that the image theory is applicable to the inclined conductor in Fig. 4 (a), then Fig. 4 (b) is obtained as an equivalent circuit of Fig. 4 (a). EMTP simulation results of transients in Fig. 4 (b) are identical to those in Fig. 4 (c) and (d). Thus, it is concluded that the image theory can be applied to the inclined conductor including a vertical one, i.e. a transmission tower. This is the reason why a number of EMTP simulations have been performed to investigate lightning surges in a transmission system containing many towers [10]-[17].

B2. Non-parallel conductor

Fig. 6 (b) and (c) show voltage and current waveforms at the sending end (node s) of conductor 1 in Fig. 6 (a). Fig. 7 shows induced voltage waveforms on conductor 2. The applied-phase voltage and current in Fig. 6 are nearly independent of angle θ of the conductor 2, and are the same as those in Fig. 4 (c) and (d) for $\theta = 0°$, i.e. a horizontal conductor. It is clear in Fig. 7 that the induced voltages for $\theta \geqq 30°$ are far smaller than that for $\theta = 0°$. The reason for this is higher attenuation and lower surge impedances in the non-parallel conductors. Fig. 8 shows the mutual impedance as a function of angle θ. As θ increases, the induced voltage (mutual impedance) decreases for θ smaller than 90°. Then, the induced voltage becomes negative for θ equal and greater than 90°. The negative voltage for θ = 180° is greater than that

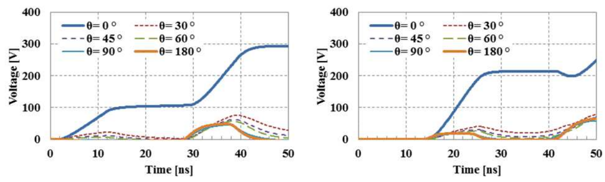


(a) Non-parallel conductor: a=5mm, , x_0=4m, z_1=0.4m, y_1=0.1m.

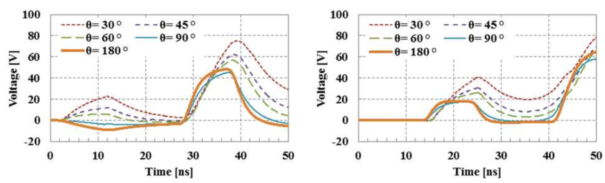


(b) Voltage v_{1s} (c) Current i_{1s}

Fig. 6 Applied phase (conductor 1) voltage and current waveforms at the sending-end in Fig. 6 (a).



(a) Voltage $v_{2s'}$ at node s' (b) Voltage $v_{2r'}$ at node r'



(c) Expanded waveform of $v_{2s'}$ (d) Expanded waveform of $v_{2r'}$

Fig. 7 Induced voltage waveforms on conductor 2.

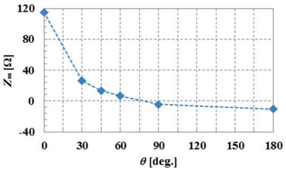


Fig. 8 Mutual surge impedance Z_m as a function of angle θ.

for $\theta = 90°$. This fact, i.e. the polarity change of the induced voltage for $\theta \geqq 90°$, has not been well realized. For $\theta \geqq 90°$, the inducing current direction looks negative from the induced conductor, or the mutual impedance can be said negative as is observed in Fig. 8.

IV. EMTP Simulations of Non-uniform Lines

A. *Non-uniform line representation*

The impedance and admittance of a non-uniform line are easily calculated as a function of frequency by adopting the formulas explained in [15]-[21]. A transient response on the non-uniform line can be calculated by representing the non-uniform line with staircase horizontal lines [18] except a vertical conductor. For example, an inclined single conductor in Fig. 4 (a) is approximated by the staircase horizontal conductors as illustrated in Fig. 9 (a), where the following condition is to be satisfied.

$$x_1 + x_2 + - - - + x_n = x_0 \qquad (2)$$

height of the first conductor (x_1): $h_1 = z_1$

height of last conductor (x_n): $h_n = z_2$

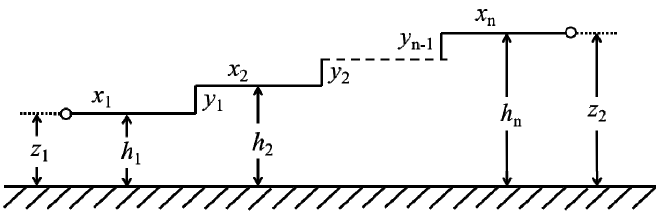
height of i-th conductor (x_i): $h_i = z_1 + i\,(z_1 - z_2)\,/n$

The length of a lead wire connecting the (i-1)th and the i-th conductors is assumed to be "0" ($y_1 = y_2 = - - - = y_{n-1} = 0$), i.e. short-circuited in EMTP in most cases. If necessary, (2) can be rewritten as

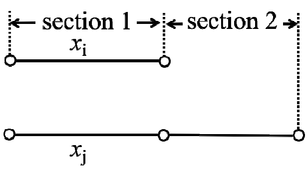
$$x_1 + x_2 + - - - + x_n + (y_{1\,+}\, y_{2+}{=}+ - - - + y_{n-1}) = x_0 \qquad (3)$$

In this case, the surge impedance of the vertical part (y_i) can be given as an average of those of the adjacent conductors. A similar approach can be used for a non-parallel conductor.

In the case of a parallel conductor with different length x_i and x_j as illustrated in Fig. 9 (b), the mutual impedance between conductors i and j calculated by the formulas given in [19] to [21] is to be used as the mutual impedance of section 1. Then, the system in Fig. 9 (b) is represented as a cascaded circuit composed of the section 1 of a two-conductor circuit and section 2 of a single conductor in an EMTP simulation.



(a) Inclined conductor with length x_0.



(b) Parallel conductor with different conductor length x_i and x_j.

Fig. 9 Approximate representation of non-uniform lines.

B. *Simulation examples on vertical conductors*

B1. Surge response on a transmission tower

Fig. 10 shows (a) measured result and (b) EMTP result of a tower top voltage on a 500 kV transmission tower [10], [17]. In EMTP simulation, a frequency-dependent tower model [41] is adopted. The simulation result shows a satisfactory agreement with the measured result. It should be pointed out that the wave deformation observed in the voltage is dominantly caused by the position-dependent impedance of a vertical conductor rather than the frequency-dependent effect as explained in Section III-A1. The positon dependence can be handled in the same manner as the frequency dependence because the tower response is obtained as the ratio of the frequency-responses of the applied current and tower top voltage [41].

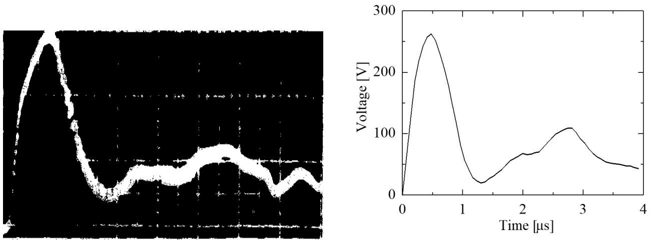
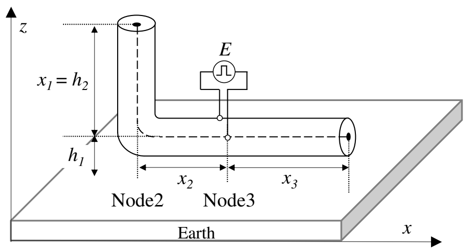


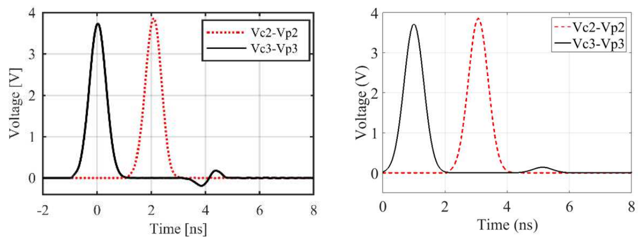
Fig. 10 Measured (left) and EMTP simulation (right) results of the tower top voltage on a 500 kV transmission line.

B2. Surge response on a gas-insulated bus with an elbow part

Fig. 11 (a) shows a gas-insulated bus with an elbow part composed of a horizontal and vertical buses, (b) shows FDTD simulation results with Gaussian pulse voltage and (c) EMTP simulation results with the same voltage starting from t = 0.



(a) Model circuit: x_1=x_3=1m, x_2=0.62m, h_1=0.23m, E=3.72V, ρ_e = 80 Ωm.



(b) FDTD simulation results (c) EMTP simulation results

Fig. 11 Model circuit of a gas-insulated bus with an elbow part and core-to-pipe voltages at Node 3 (bold line) and Node 2 (dotted line).

As discussed in [29] and [42], there exists mutual coupling between the horizontal and vertical parts which cannot be handled by a circuit theory. The mutual coupling is represented in EMTP as the surge impedance different from that of the horizontal part evaluated from the refraction coefficient at Node 2 based on the FDTD result. The EMTP results qualitatively agree with the FDTD results. Thus, a transient involving mutual coupling between horizontal and vertical conductors can be approximately simulated by EMTP.

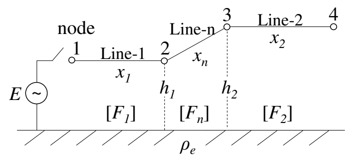
C. Horizontal conductors

C1. Surge impedance of a single conductor

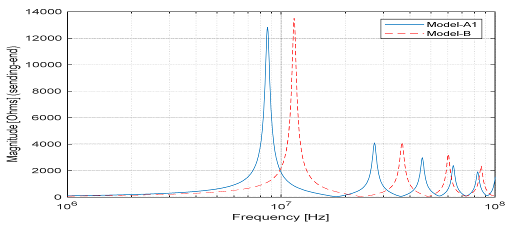
As already explained in Table 1 of Section III-A2, the pul approach of a circuit theory can be adopted when x/h is large. Otherwise, the existing pul impedance such as the formulas by Pollaczek [43], Carson [44] and Sunde [45] cannot be adopted.

C2. Input impedance of a non-homogeneous line

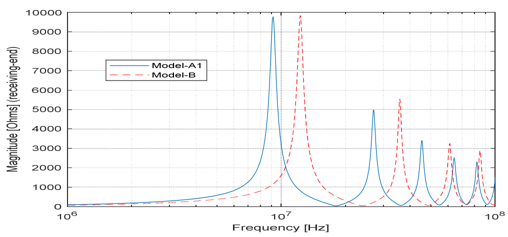
Fig. 12 (a) illustrates a non-homogeneous line composed of two cascaded homogeneous lines (Line-1 and Line-2) and a lead wire (Line-n) connecting Line-1 and Line-2. Note that Line-n is non-uniform because the height at node 2 is h_1 being different from h_2 at node 3. Fig. 12 (b) shows the input impedance seen from node 1, and Fig. 12 (c) the input impedance seen from node 4. In Model-A1, the lead wire (x_n = 50 m) is included, while it is neglected, i.e. node 2 is short-circuited to node 3 in Model-B. It is clear that the input impedance seen from the sending end (node 1) differs from that seen from the receiving end as is well-known. It should be noted that the resonant frequency is significantly different between Model-A1 (lead wire) and Model-B (lead wire neglected). This result clearly indicates the inaccuracy of the theories of the non-homogeneous lines described in [1] to [7].



(a) Non-homogeneous line (500 kV vertical line): r = 0.1785 m, x_1 = x_2 = 300 m, h_1 = 43 m, h_2 = 72 m, x_n = 50 m, ρ_e = 200 Ωm.



(b) Input impedance seen from the sending end (node 1)

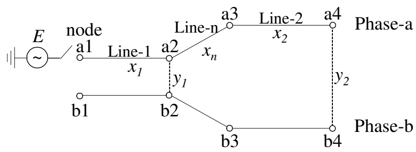


(c) Input impedance seen from the receiving end (node 4)

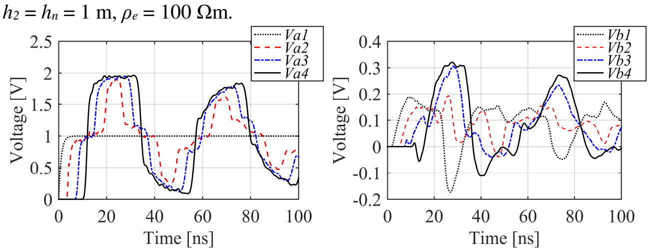
Fig. 12 Input impedance of a non-homogeneous line [46].

C3. Two-conductor system involving a non-uniform line

Fig. 13 (a) illustrates a model circuit of cascaded homogeneous lines and a lead wire which is non-uniform for separation-distance y_1 at node 2 differs from y_2 at node 3. Fig. 13 (b) and (c) show FDTD and EMTP simulation results of a transient when a step-like voltage (E = 1V) is applied to node "a1" to ground. In the figure, (1) is the energized phase and (2) is the induced phase.

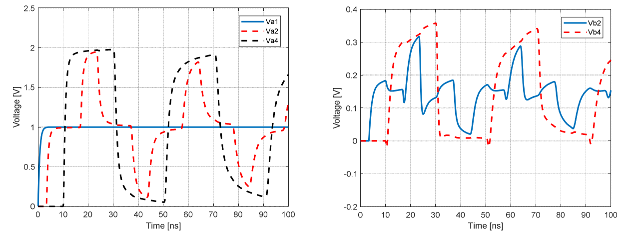


(a) Non-homogeneous two-conductor system: x_1 = x_2 = 1 m, x_n = 1.031m, h_1= h_2 = h_n = 1 m, ρ_e = 100 Ωm.



(1) Energized phase (2) Induced phase

(b) FDTD simulation results



(1) Energized phase (2) Induced phase

(c) EMTP simulation results

Fig. 13 Transient on a non-homogeneous two-conductor system [46].

The EMTP simulation results on phase-a in Fig. 13 (c) qualitatively agree with FDTD results in Figs. 13 (b). Thus, transient responses on the energized phase of a non-homogeneous line can be simulated by EMTP. However, it is rather hard to simulate accurately the responses on the induced phase by the EMTP. The time delay at the sending end of the induced phase cannot be represented by the EMTP, because the delay is due to a direct electromagnetic wave in the air from node "a1" to node "b1".

V. CONCLUSIONS

(1) This paper has reviewed existing theories of non-homogeneous lines and made clear the problems involved in the theories.

(2) The impedance of a non-uniform line differs significantly from the "pul" impedance implemented into EMTP.

(3) Measured and FDTD computed results of transients on non-uniform lines show the position-dependent effect of an

inclined (including vertical) conductor, and non-TEM wave propagation on a non-parallel conductor.

(4) EMTP simulation examples are demonstrated in comparison with measured and FDTD computed results. Because the image theory is applicable to an inclined (to earth) conductor, a transient on the inclined conductor can be analyzed by EMTP, if the parameters are available. When the non-TEM component is dominant in a transient, EMTP cannot be adopted to analyze the transient.

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