Novel Voltage Source Type Synchronous Machine Model for Nodal Analysis Based Simulations

H. Konara, U.D. Annakkage, C. Karawita

***Abstract*—This paper presents a novel synchronous machine model suitable for nodal analysis based simulations. Guaranteeing numerical stability in existing synchronous machine models requires extra computations which can make a real-time simulation challenging. In this work, the synchronous machine is represented using a voltage source and an impedance by splitting the stator inductance into two components. The time-dependent component is included in the voltage and the rest of the terms are included in the impedance. The impedance part can be directly included in the constant admittance matrix of the network. This increases the simulation efficiency significantly. The derivation of the proposed synchronous machine model is a simple extension to the well-known transient stability model. The proposed model is numerically robust. The robustness of the machine model is evaluated by interfacing the machine into different networks.**

***Keywords*—Synchronous machine, numerical stability, dynamic phasors, electromagnetic transient, nodal analysis.**

I. Introduction

SYNCHRONOUS machine is a fundamental component in a power system. Depending on the study objective the extent of modelling of the synchronous machine is different in simulation programs. The oscillations in the rotor of the machine are slow electromechanical oscillations. The dynamics in the stator winding of the machine has higher oscillatory frequencies and they fall under electromagnetic transients. In Transient Stability (TS) studies that focus only on electromechanical oscillations, the high frequency oscillations from the network are ignored [1]. Therefore, it is not necessary to model dynamics in the stator windings in stability studies. However, it is important to model the dynamics in the stator winding fluxes in Electromagnetic Transient (EMT) simulations [2]. Dynamic Phasor (DP) simulations are similar to TS models. However, it models the high frequency network dynamics [3]. In TS, the network is solved using algebraic equations and in DP the voltages and currents in the network are modelled as state variables. Therefore, the synchronous machine model with stator winding dynamics used in EMT studies is used in the DP model.

Different modelling techniques are available to interface the synchronous machine to an EMT model of the network. Synchronous machine can be interfaced to the network either as a Norton current source or as a Thevenin voltage source. A stability criterion for interfacing a machine into a network is presented in [4]. According to the stability criterion, either one of these methods can be stable at a time depending on the input impedance of the network that it is connected to.

The simplest model available for synchronous machine is in [5], where the machine is represented as a Norton equivalent. The numerical instability problems in this representation are overcome by connecting a shunt resistor with a compensated current source to the model. There has been a significant amount of research done on Voltage Behind Reactance (VBR) type synchronous machine models for EMT type simulations [6–10]. The major concern associated with these methods is that they are not suitable to use in nodal analysis type solvers for following reasons; a) numeral instability; and b) the impedance of the Thevenin equivalent is time-dependant and the system matrix has to be inverted in every time-step. The later reason makes the simulation inefficient.

In this paper, a novel synchronous machine model is introduced. The synchronous machine is modelled as a voltage source and it can be directly interfaced to nodal analysis type simulations. The novel synchronous machine model is demonstrated using a Single Machine to Infinite Bus (SMIB) system. The network is modelled using dynamic phasors. The performance of the system is validated using the machine model available in Real Time Digital Simulator (RTDS). The stability and the efficiency of the model are also analysed.

II. Synchronous machine models for transient analysis

In this section, the synchronous machine models used in EMT simulation programs are briefly reviewed.

Different full order synchronous machine models are used in EMT simulation platforms. In PSCAD-EMTDC program the synchronous machine is modelled as a Norton current source calculated using previous time-step values [5]. The numerical issues due to time-step delay between the machine and the network is addressed by using a resistor with a compensated current source at the interface bus.

A stability criterion for interfacing an external device to a DP simulation with a time-step delay is presented in [4]. It has been pointed out that the device modelled as either Norton equivalent or Thevenin equivalent (not both) will be stable at steady state depending on the input impedance of the network that it is connected to. In DP nodal approach, the state equations of the synchronous machine are solved separately

This work was supported by Natural Sciences and Engineering Research Council (NSERC) and University of Manitoba.

H. Konara is with Department of Electrical Engineering, University of Manitoba, Canada (e-mail of corresponding author: konarhkk@myumanitoba.ca). U. D. Annakkage is with the Department of Electrical Engineering, University of Manitoba, Canada (e-mail: udaya@ieee.org). C. Karawita is with the TransGrid Solutions Inc., Canada (e-mail: ckarawita@tgs.biz).

Paper submitted to the International Conference on Power Systems Transients (IPST2019) in Perpignan, France June 17-20, 2019.

from the network and therefore, an inherent time-step delay exists in the simulation. Since the network impedance is usually much higher than the sub synchronous reactance of the machine the Norton equivalent-type synchronous machine will be numerically unstable.

VBR models of synchronous machines for EMT simulations are discussed in [6–9], where the machine is modelled using qd coordinates. Numerical instability is one of the main concern with the qd method. In order to interface the qd model to the network it is necessary to predict the machine variables [10]. Any prediction errors may lead to numerical instability . The methodology in [9] is called the Phase-Domain (PD) method where the stator winding flux is modelled as part of the network. The machine is represented using a Thevenin equivalent with a equivalence resistance. This is more numerically stable since the stator is part of the network's admittance matrix. However, this method is computationally demanding for the nodal analysis approach since the equivalent resistance is time-varying.

The VBR method is extended for the EMT nodal analysis method in [11] where the machine model is interfaced to the network by discretizing the stator voltage using the implicit trapezoidal rule. Then the discretized equations are used to come up with a general form given in (1) that can be included into EMT solution as a Thevenin equivalent circuit.

$$v_{abc}(t) = R_{eq}(t) i_{abc}(t) + E_h(t) \tag{1}$$

Here, the terms $R_{eq}(t)$ and $E_h(t)$ are the equivalent resistance and history term respectively. The term R_{eq} is time-dependant and is modelled externally to the network as a Thevenin impedance. We propose an alternative approach where the stator reactance is split into two components: (a) a fixed reactance corresponding to base frequency ($L\omega_0$) and (b) a component representing change of reactance due to change in frequency ($L\Delta\omega_r$). The fixed component is included in the network and the variable component is included in the sub-transient voltage.

The proposed Voltage Source (VS) type synchronous machine model is derived in Section II-C using the same set of steps that are used to derive the synchronous machine model in typical transient stability studies [1]. The synchronous machine model presented in this paper is numerically robust and it has a constant impedance that can be included in the admittance matrix of the DP part of the network. Hence, at each time-step the voltage source value is given as an input to the DP model as a boundary voltage.

A. *Synchronous Machine Model*

The synchronous machine model considered in this paper is the same detailed machine model that is used in EMT simulations with stator flux dynamics. Without loss of generality let us consider a synchronous machine with one field winding, one damper winding on the direct axis (d-axis), two damper windings on the quadrature axis (q axis), one stator winding on d-axis and one stator winding on q-axis for GENROU (round rotor generator) type generators. Hence, there are eight differential equations governing the dynamics of a synchronous machine. Equations (2) and (3) are used to represent the dynamics of the rotor [1].

$$\Delta\dot{\omega_r} = \frac{1}{2H}\left(T_m - T_e - K_D \Delta\omega_r\right) \tag{2}$$

$$\dot{\delta} = \Delta\omega_r \omega_0 \tag{3}$$

Parameters δ and $\Delta\omega_r$ are the rotor position and the angular speed deviation respectively. The mechanical torque and electrical torque of the machine are denoted by T_m and T_e respectively. The dynamics of the field winding and damper winding flux are expressed as in (4) to (7),

$$\dot{\varphi_{fd}} = e_{fd} + \frac{\varphi_{ad} - \varphi_{fd}}{L_{fd}} R_{fd} \tag{4}$$

$$\dot{\varphi_{1d}} = \frac{\varphi_{ad} - \varphi_{1d}}{L_{1d}} R_{1d} \tag{5}$$

$$\dot{\varphi_{1q}} = \frac{\varphi_{aq} - \varphi_{1q}}{L_{1q}} R_{1q} \tag{6}$$

$$\dot{\varphi_{2q}} = \frac{\varphi_{aq} - \varphi_{2q}}{L_{2q}} R_{2q} \tag{7}$$

where the terms φ_{ad} and φ_{aq} are given by:

$$\varphi_{ad} = L_{ad}^{''}\left(-i_d + \frac{\varphi_{fd}}{L_{fd}} + \frac{\varphi_{1d}}{L_{1d}}\right) \tag{8}$$

$$\varphi_{aq} = L_{aq}^{''}\left(-i_q + \frac{\varphi_{1q}}{L_{1q}} + \frac{\varphi_{2q}}{L_{2q}}\right) \tag{9}$$

The notations and the current direction conventions that are used in this paper are same with [1]. Using the terms φ_{ad} and φ_{aq} the electric torque of the machine can be expressed as (10) .

$$T_e = \varphi_{ad} i_q - \varphi_{aq} i_d \tag{10}$$

The dynamics of the stator winding flux in the d and q axes are expressed by (11) and (12).

$$\dot{\varphi_d} = \omega_r \varphi_q + R_a i_d + e_d \tag{11}$$

$$\dot{\varphi_q} = -\omega_r \varphi_d + R_a i_q + e_q \tag{12}$$

B. *Voltage-Behind-Reactance Model of Synchronous Machine*

The derivation of the VBR model for 8^{th} order synchronous machine is explained in this section. Let us consider (11), which can be re-written as (13).

$$e_d = \dot{\varphi_d} - \omega_r \varphi_q - R_a i_d \tag{13}$$

Equation 13 can be then written as (14) using (8), (9) and the relationships of $\varphi_d = \varphi_{ad} - L_l i_d$ and $L_d^{''} = L_{ad}^{''} + L_l$,

$$e_d = -L_d^{''}\frac{d}{dt} i_d + \omega_r L_q^{''} i_q - R_a i_d + E_d^{''} \tag{14}$$

where the term $E_d^{''}$ is equals to:

$$E_d^{''} = \frac{L_{ad}^{''}}{L_{fd}}\dot{\varphi_{fd}} + \frac{L_{ad}^{''}}{L_{1d}}\dot{\varphi_{1d}} - \omega_r \frac{L_{aq}^{''}}{L_{1q}}\varphi_{1q} - \omega_r \frac{L_{aq}^{''}}{L_{2q}}\varphi_{2q} \tag{15}$$

The difference between the above model and the commonly used TS model of a synchronous machine is that (14) has the extra term, $L_d^{''}\frac{d}{dt}i_d$, representing the dynamics of the d-axis component of the stator current. The first three terms in (14) are identical to the dynamic equation of an RL branch in DP and thus it can be modelled as part of the network solution. The term ω_r in (14) depends on the rotor position and it is time-varying. Therefore, it does not allow this equation to be directly included into the nodal analysis method. If one tries to solve this equation using the nodal approach, the admittance matrix of the network has to be updated at each time-step. Computing the inverse of the network's admittance matrix at each time-step is computationally demanding.

C. Propopsed Voltage Source Type Synchronous Machine Model

The synchronous machine model suitable for nodal analysis method is derived in this section. Let us go back to (14) and replace the term ω_r using $\omega_r = \Delta\omega_r + 1$. It should be noted that the speed terms considered here are in per unit values.

$$\begin{aligned} e_d = -L_d^{''}\frac{d}{dt}i_d + (\Delta\omega_r + 1)L_q^{''}i_q - R_a i_d + \frac{L_{ad}^{''}}{L_{fd}}\dot{\varphi}_{fd} + \\ \frac{L_{ad}^{''}}{L_{1d}}\dot{\varphi}_{1d} - \omega_r\frac{L_{aq}^{''}}{L_{1q}}\varphi_{1q} - \omega_r\frac{L_{aq}^{''}}{L_{2q}}\varphi_{2q} \end{aligned} \tag{16}$$

Now (16) can be re-arranged as (17) and (18).

$$e_d = -L_d^{''}\frac{d}{dt}i_d + L_q^{''}i_q - R_a i_d + E_{\omega d}^{''} \tag{17}$$

$$\begin{aligned} E_{\omega d}^{''} = \frac{L_{ad}^{''}}{L_{fd}}\dot{\varphi}_{fd} + \frac{L_{ad}^{''}}{L_{1d}}\dot{\varphi}_{1d} - \omega_r\frac{L_{aq}^{''}}{L_{1q}}\varphi_{1q} - \\ \omega_r\frac{L_{aq}^{''}}{L_{2q}}\varphi_{2q} + \Delta\omega_r L_q^{''}i_q \end{aligned} \tag{18}$$

In (18), suffix ω is used with the sub-transient voltage $E_{\omega d}^{''}$ to differentiate it from the commonly used term $E_d^{''}$. With this re-arrangement, $(\Delta\omega_r + 1)L_q^{''}i_q$ is separated into $L_q^{''}i_q$ and $\Delta\omega_r L_q^{''}i_q$. The former is included in (17) and the latter is included in (18). The terms $\dot{\varphi}_{fd}$ and $\dot{\varphi}_{1d}$ in (18) can be found from (4), (5) and (8).

Similarly, we can write the q-axis voltage equation using (12) as shown in (19)

$$e_q = -L_q^{''}\frac{d}{dt}i_q - L_d^{''}i_d - R_a i_q + E_{\omega q}^{''} \tag{19}$$

where $E_{\omega q}^{''}$ is equals to,

$$\begin{aligned} E_{\omega q}^{''} = \frac{L_{aq}^{''}}{L_{1q}}\dot{\varphi}_{1q} + \frac{L_{aq}^{''}}{L_{2q}}\dot{\varphi}_{2q} + \omega_r\frac{L_{ad}^{''}}{L_{fd}}\varphi_{fd} + \\ \omega_r\frac{L_{ad}^{''}}{L_{1d}}\varphi_{1d} - \Delta\omega_r L_d^{''}i_d \end{aligned} \tag{20}$$

and the terms $\dot{\varphi}_{1q}$ and $\dot{\varphi}_{2q}$ can be found from (6), (7) and (9).

Equation (21) can be derived by adding (17) and (19) as follows:

$$\begin{aligned} e_d + je_q = -L_d^{''}\frac{d}{dt}i_d + L_q^{''}i_q - R_a i_d + E_{\omega d}^{''} + \\ j\left(-L_q^{''}\frac{d}{dt}i_q - L_d^{''}i_d - R_a i_q + E_{\omega q}^{''}\right) \end{aligned} \tag{21}$$

With sub-transient saliency neglected $L_d^{''} = L_q^{''} = L^{''}$, (21) can be re-written as,

$$E_t = -L^{''}\frac{d}{dt}I_t - jL^{''}I_t - R_a I_t + E_t^{''} \tag{22}$$

where $E_t = e_d + je_q$, $E_t^{''} = E_{\omega d}^{''} + jE_{\omega q}^{''}$ and $I_t = i_d + ji_q$. All the parameters in these equations are in per unit. However, to integrate these equations to the network, the per-unit time needs to be converted into seconds. The resulting equation can be written as (23).

$$E_t = -\frac{L^{''}}{\omega_0}\frac{d}{dt}I_t - jL^{''}I_t - R_a I_t + E_t^{''} \tag{23}$$

The derived model of the synchronous machine can then be interfaced to the network as a Thevenin voltage source (Fig.1).

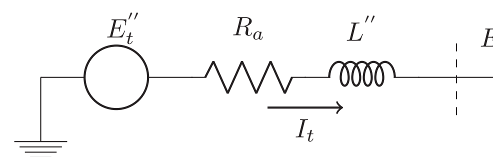

Fig. 1. Synchronous machine represented by its internal voltage behind the sub-transient reactance

It is straightforward to interface the equivalent model of the synchronous machine shown in Fig. 1 to nodal analysis method. The term $R_a + jL^{''}$ can be added to the network's admittance matrix and the voltage $E_t^{''}$ can be considered as a time-varying boundary voltage updated at each integration time-step.

III. Test System and Simulation Results

SMIB system [1] is used as the simulation case. The equivalent circuit of the SMIB system is shown in Fig. 2, where E_t is the infinite bus voltage. The synchronous machine is modelled using the novel voltage source model. The transmission line connected to the machine is modelled using a DP model. Equations (24) and (25) are used to model RL and RC ($G = 1/R$) branches in dynamic phasors [12].

$$\hat{V} = (R + j\omega L)\hat{I} + L\frac{d}{dt}\hat{I} \tag{24}$$

$$\hat{I} = (G + j\omega C)\hat{V} + C\frac{d}{dt}\hat{V} \tag{25}$$

The transmission line of the SMIB system considered in this paper is represented using an RL branch. Equation (24) is used to model the transmission line and it is re-arranged as a state equation as shown in (26). The terms R and L are the resistance and the inductance of the transmission line respectively.

$$\frac{d}{dt}\hat{I} = \frac{1}{L}\left(\hat{V} - (R + j\omega L)\,\hat{I}\right) \tag{26}$$

The novel synchronous machine model is validated using an EMT simulation done in RTDS.

Figures 3-6 show the comparison of machine variables (speed, electric torque, generator terminal voltage and d-axis

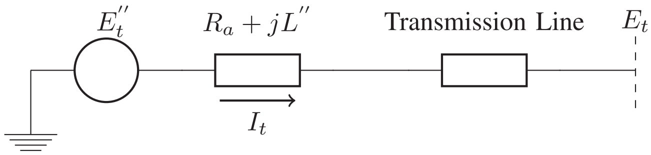

Fig. 2. Synchronous machine connected to the infinite bus through a transmission line

current respectively) with the EMT simulation when the infinite bus voltage is dropped to 2% for a period of $100ms$. It can be seen from the figures that the novel voltage source type synchronous machine model closely follow the EMT simulation results. It accurately captures the high frequency component in electric torque and d-axis current.

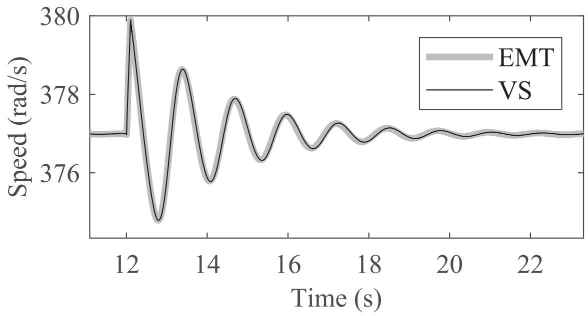

Fig. 3. Comparison of generator speed

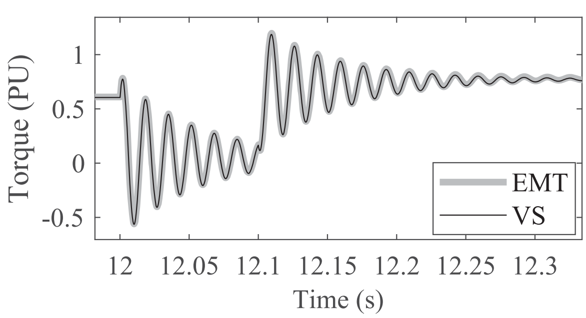

Fig. 4. Comparison of generator electrical torque

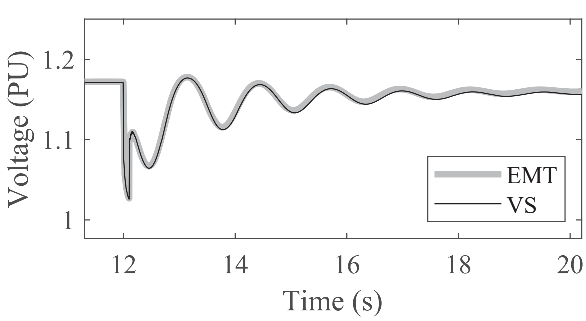

Fig. 5. Comparison of generator terminal voltage

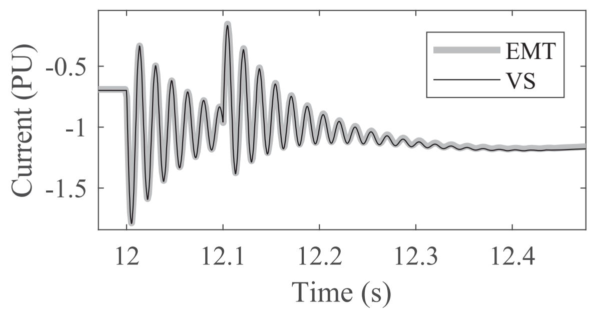

Fig. 6. Comparison of generator d-axis current

IV. Discussion

One advantage of the proposed method compared to existing synchronous machine models is that the proposed formation leads to a network bus admittance matrix that is independent of time. This leads to a significant reduction in computation time. The other advantage is that the proposed model is numerically stable regardless of the network that it connected to. These points will be further explained in Section IV-A and IV-B below.

A. Stability Considerations for Interfacing Voltage Injection Type Synchronous Machine Model

Let us consider the circuit given in Fig. 7, where E_1 and R_1 belong to sub-system 1 and E_2 and R_2 belong to sub-system 2. According to the stability criterion given in [4] if the left hand-side reads current from the right hand side and updates the voltage V at the boundary bus, for numerical stability, the following relationship between the impedances should stand.

$$R_1 \leq R_2 \tag{27}$$

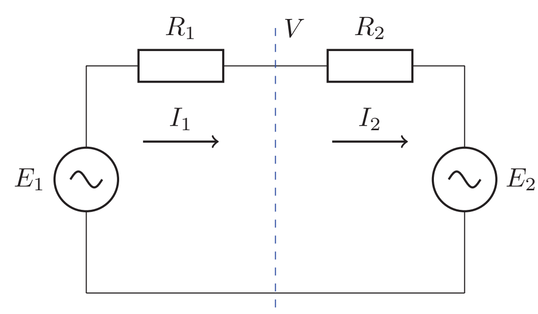

Fig. 7. Simple network to analyse numerical stability

If we model the machine by including the impedance of the machine's Thevenin equivalent as part of the network impedance, R_1 becomes zero. Hence, the stability criteria given is satisfied with any value of R_2. This results in the simulation to be stable with any network impedance. The theory is validated by using different network impedances in the SMIB system as shown in figures 8, 9 and 10. The network side impedance is further reduced in each case. As expected, the results validate that the simulation is always stable regardless of the network impedance.

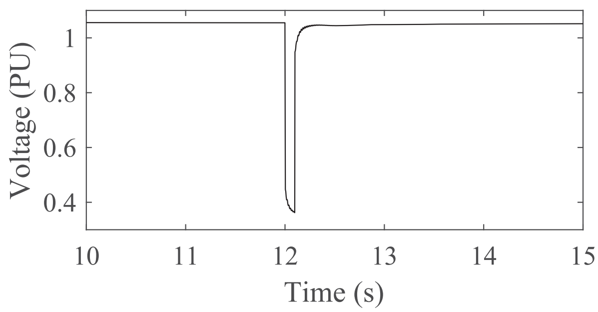

Fig. 8. Generator terminal voltage when the network impedance is reduced to 10%

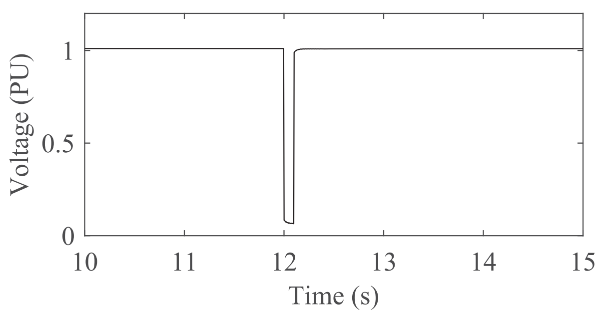

Fig. 9. Generator terminal voltage when the network impedance is reduced to 1%

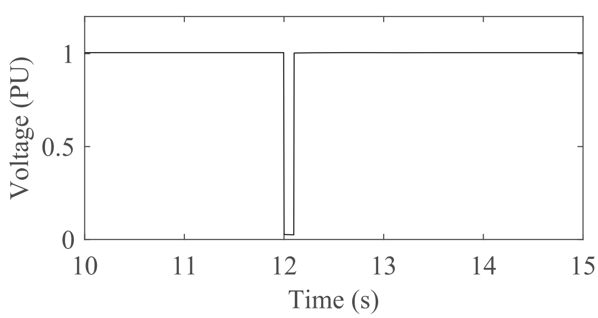

Fig. 10. Generator terminal voltage when the network impedance is reduced to 0.1%

B. Computational Efficiency

In order to use most of the synchronous machine models available in the literature in a nodal analysis type simulation, the admittance matrix of the network has to be inverted in every time-step. The admittance matrix changes every time-step since the equivalent resistance of the Thevenin equivalent is time-dependant. In the proposed method, the impedance of the Thevenin impedance of the synchronous machine is not time-dependent and therefore the network admittance matrix stays as a constant. Hence, the inversion does not have to carry out in every time-step and it could be done only at the beginning of the simulation. A significant computational advantage can be gained by the novel voltage source type synchronous machine model due to this reason.

C. Applicability of the Synchronous Machine Model for an EMT Type Simulation

In this paper the novel synchronous machine model is interfaced directly to DP type network model. However, it can also be interfaced to any EMT type network model. It can be modelled using modified nodal analysis approach [13].

V. Conclusions

A novel, numerically robust voltage source type synchronous machine model was presented. The proposed model is an EMT compatible model where the dynamics of flux in the stator winding are included. The robustness of the model was achieved by splitting the stator inductance into two components. Constant part was included in the network and the remaining part was updated at each time-step and included in the transient boundary voltage. The novel synchronous machine model was validated by using the machine model in the RTDS simulator. The computational efficiency and the numerical robustness of the model were discussed.

VI. Acknowledgment

The authors would like to thank RTDS Technologies Inc. for the software support.

VII. References

[1] P. Kundur, *Power system stability and control*. EPRI power system engineering series, McGraw-Hill, 1994.

[2] J. A. Neville Watson, “Power systems electromagnetic transients simulation,” vol. 39, 2003.

[3] S. Henschel, *Analysis of electromagnetic and electromechanical power system transients with dynamic phasors*. PhD thesis, University of British Columbia, February 1999.

[4] H. Konara, U. D. Annakkage, and C. Karawita, “Stability criterion for interfacing a transient stability model to a dynamic phasor model,” in *2018 IEEE/PES Transmission and Distribution Conference and Exposition (T&D)*, pp. 1–5, April 2018.

[5] A. M. Gole, R. W. Menzies, H. M. Turanli, and D. A. Woodford, “Improved interfacing of electrical machine models to electromagnetic transients programs,” *IEEE Power Engineering Review*, vol. PER-4, pp. 56–57, Sept 1984.

[6] V. Brandwajn, *Synchronous generator models for the analysis of electromagnetic transients*. PhD thesis, University of British Columbia, Canada, April 1977.

[7] H. K. Lauw and W. S. Meyer, “Universal machine modeling for the representation of rotating electric machinery in an electromagnetic transients program,” *IEEE Transactions on Power Apparatus and Systems*, vol. PAS-101, pp. 1342–1351, June 1982.

[8] P. Subramaniam and O. P. Malik, “Digital simulation of a synchronous generator in direct-phase quantities,” *Electrical Engineers, Proceedings of the Institution of*, vol. 118, pp. 153–160, January 1971.

[9] M. Rafian and M. A. Laughton, “Determination of synchronous-machine phase-co-ordinate parameters,”

Electrical Engineers, Proceedings of the Institution of, vol. 123, pp. 818–824, August 1976.

[10] X. Cao, A. Kurita, H. Mitsuma, Y. Tada, and H. Okamoto, "Improvements of Numerical Stability of Electromagnetic Transient Simulation by Use of Phase-Domain Synchronous Machine Models," *Electrical Engineering in Japan 1999*, vol. 128, no. 3, 1999.

[11] L. Wang and J. Jatskevich, "A voltage-behind-reactance synchronous machine model for the emtp-type solution," in *2007 IEEE Power Engineering Society General Meeting*, pp. 1–1, June 2007.

[12] C. Karawita, *HVDC Interaction Studies Using Small Signal Stability Assessment*. PhD thesis, University of Manitoba, Canada, April 2009.

[13] C. W. Ho, A. Ruehli, and P. Brennan, "The modified nodal approach to network analysis," *IEEE Transactions on Circuits and Systems*, vol. 22, pp. 504–509, June 1975.