FDTD Analysis of Distribution Line Voltages Induced by Non-Vertical Lightning

Masashi Natsui, Akihiro Ametani, Jean Mahseredjian, Shozo Sekioka, Kazuo Yamamoto

***Abstract*—This paper investigates lightning induced surges on an overhead line considering "bent" and "computationally-generated" non-vertical lightning channels by using a finite-difference time-domain (FDTD) method. In the bent lightning study, the channel is represented by combinations of vertical and inclined paths with different connecting (bent) heights. It is found that the lightning channel geometry under 100-m altitude is significant for the peak voltage when severe conditions of a 1/200-µs current and a lightning distance of 50 m are assumed. A lightning-like (zig-zag) channel is computationally generated by a probabilistic calculation based on an electric potential distribution in a three-dimensional space, and its induced voltage is compared with that by a simply-inclined channel. When the inclinations are set to the average angles (θ and φ) under 100 m in the computed channel, the induced voltages show good agreement. The difference is less than 10 % and it decreases with higher earth resistivity. These results indicate that realistic non-vertical lightning can be represented by simply-inclined lightning by considering average inclined angles in low altitude.**

***Keywords*: lightning induced voltage, non-vertical lightning, inclined lightning, distribution line, FDTD.**

I. Introduction

A vertical lightning channel is one of general assumptions in lightning-induced overvoltage evaluations in power systems. A number of publications have investigated induced voltages by analytical formulas and transmission-line (TL) based approaches with some assumptions [1]-[10]. The vertical channel and a perfectly conducting earth are quite essential for analytical formulations because the channel can be represented as a dipole antenna or a transmission line with the method of images, which are easily handled by circuit and TL theories. However, actual lightning is not vertical nor straight, and it strikes lossy earth. The influences of these assumptions should be carefully investigated for an accurate evaluation of the lightning induced voltages.

Recent developments of an analytical formula [11] and a transmission-line based approach [12]-[17] reveal some influences of non-vertical lightning on the induced voltages above a perfectly conducting earth. In [15], influences of lightning tortuosity on the voltages are investigated by using a reconstructed lightning channel from a photo, and it is demonstrated that lightning channel geometry in low altitude is more significant for the induced voltage.

The finite-difference time-domain (FDTD) method of numerical electromagnetic analysis, is being widely applied to lightning surge simulations in power systems [18]-[22] because of its capability to deal with three-dimensional non-uniform current paths above a lossy earth. The authors have recently investigated an influence of inclined lightning channel on distribution line voltages above lossy earth by the FDTD [22]. However, in [22], the lightning channel is assumed simply inclined, i.e. the path is straight and tilted with a fixed angle. Considering the knowledge presented in [15], a further study should be performed for the influence of lightning channel geometry especially in low altitude.

This paper investigates the influence of the low-altitude lightning-channel geometry by using "bent" and "computationally-generated" lightning channels. In Chapter II, lightning induced voltages in a distribution line caused by the bent lightning with various bent heights are investigated by the FDTD. Further, effects of earth resistivity, lightning distance to the line, and lightning current waveforms are investigated. In Chapter III, line voltages induced by the computationally-generated channels are compared with those by simply inclined channels, to evaluate the influence of channel non-uniformity on the voltages. Chapter IV summarizes findings and remarks obtained from the investigations in this paper.

II. Induced Voltages by Bent Lightning Channels

A. FDTD Model Circuit

Fig. 1 illustrates an FDTD model circuit. A lightning channel of 800-m height is placed at the center of a working space and in front of a single-conductor overhead line. The channel has various geometries described by channel types (Cases A and B) and bent heights (h_A and h_B) with bent angles (θ and φ) as in Fig. 1 (b) and (c). Case A indicates a combination of a lower-vertical and an upper-inclined lightning channels with various connecting heights of h_A. Case B uses the opposite combinations of Case A with the bent height of h_B. The bent angle θ is for the inclination toward the line, and the angle φ is for the inclination along the line as illustrated in Fig. 1 (b). The angles θ and φ vary between −45˚ and +45˚ and 0˚ and +45˚, respectively. The channels are represented by a series of small current sources known as a traveling-current-source (TCS) model [19], of which the current sources are aligned vertically and horizontally along the lightning paths. Heidler function is used to calculate lightning current waveforms for the sources.

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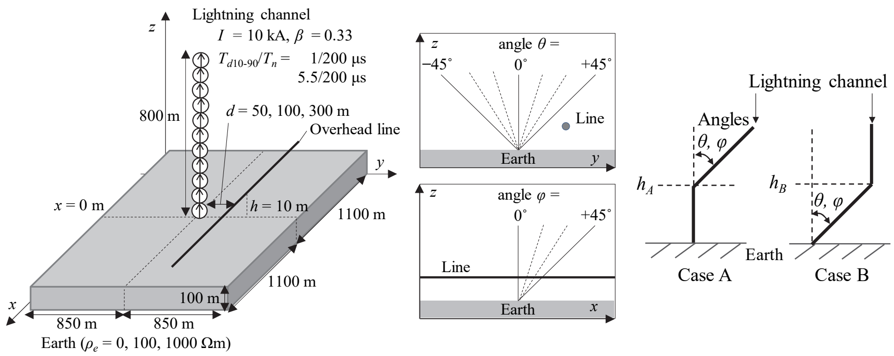


(a) FDTD model. (b) Inclined angles. (c) Lightning channel types.

Fig. 1 Model circuit.

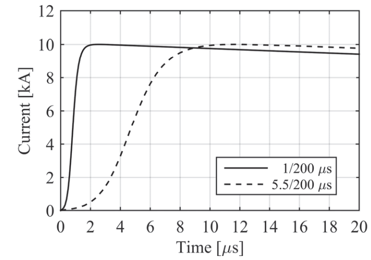


Fig. 2 Return stroke current.

$$I(t) = \frac{I_0}{\eta} \cdot \frac{(t/\tau_1)^n}{1+(t/\tau_1)^n} \cdot e^{(-t/\tau_2)} \tag{1}$$

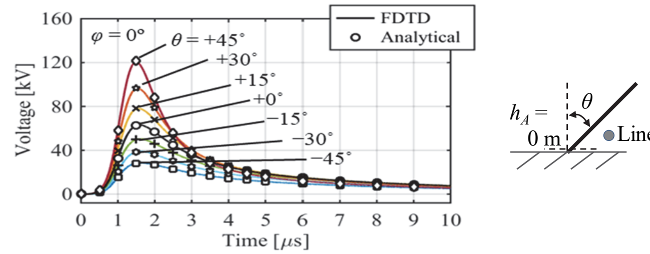
The current amplitude I_0 is assumed to be 10 kA. Two different waveforms of $T_{d10\text{-}90}/T_n$ = 1/200 μs and 5.5/200 μs are investigated in this paper as shown in Fig. 2. Return stroke velocity is taken 1×10^8 m/s. Earth resistivity ρ_e is set to 0 (perfectly conducting), 100, and 1000 Ωm with relative permittivity of ε_r = 10. The overhead line of 10 m-height is placed at distances d = 50 to 300 m away from the lightning channel base. A voltage measuring point is set at the center of the line (in front of the lightning channel base). The FDTD working space of 2200 m × 1700 m × 900 m is composed of 2-, 4-, and 8-m cell lengths. The minimum length is applied to a region including the lightning channel base and voltage measuring point. The working space is covered by Liao's absorbing boundary to suppress unintended reflections from the boundaries [23].

B. Induced Voltages by Bent Lightning

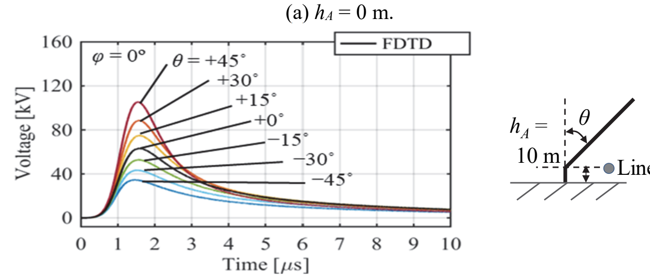
1) Case A above a perfectly conducting earth: angle θ

Fig. 3 shows lightning induced voltages in Case A for the angle θ where the lightning rises vertically at first and then inclined toward the line at height h_A. The lightning channel base is placed 50-m away from the line and the earth resistivity is set to 0 Ωm. The bent height h_A is varied from 0 m to 100 m. In Fig. 3 (a), when h_A = 0 m, the lightning channel is assumed simply inclined on the earth surface with the angle θ. It is clear in the figure that the induced voltages are significantly influenced by the inclined angle, as already reported in [22]. The voltage for θ = +45° reaches about 200 % of that for θ = 0° (vertical lightning). The change is also confirmed in comparison with results calculated by an analytical formula for the inclined lightning [11].

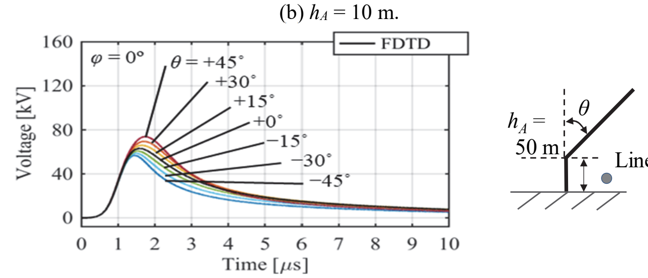
However, as observed in Fig. 3 (b) to (d), the influence of the lightning inclination quickly decreases as h_A increases. The peak voltage for θ = +45° decreases about 15 % even when only 10 m of the vertical part (h_A = 10 m, same as the line height) is considered. The influence of the angle θ on the peak voltages becomes very minor when h_A = 50 m, and the peak voltages converge when h_A = 100 m. These results indicate the peak voltage is mainly influenced by the lightning channel geometry under 100-m altitude, at least when severe conditions of 1/200-μs waveform and a lightning distance d of 50 m are assumed.



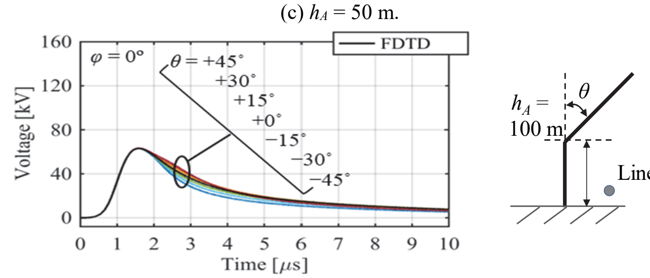
(a) h_A = 0 m.



(b) h_A = 10 m.



(c) h_A = 50 m.

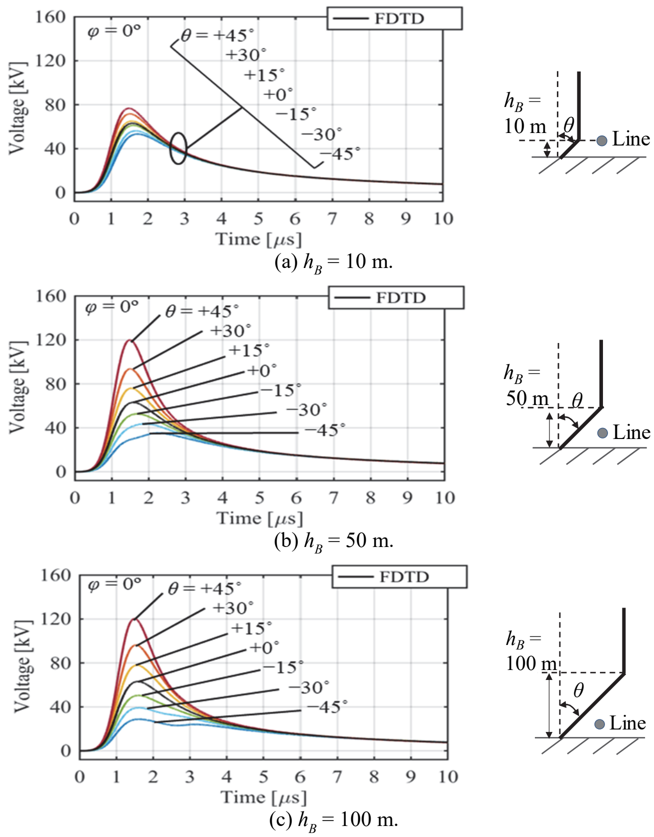


(d) h_A = 100 m.

Fig. 3 Lightning induced voltages on a single-conductor overhead line by bent lightning with various bent heights and inclined angles θ: Case A, 1/200 μs, ρ_e = 0 Ωm, d = 50 m, φ = 0°.

2) Case B above a perfectly conducting earth: angle θ

Fig. 4 shows lightning induced voltages in Case B for the angle θ where the lightning is firstly inclined toward the line and then bent back to the vertical position at a height h_B. The rest parameters are the same as those in the previous section. Note that results for h_B = 0 m are not presented in Fig. 4 because they are the same as that by the vertical lightning. When h_B =

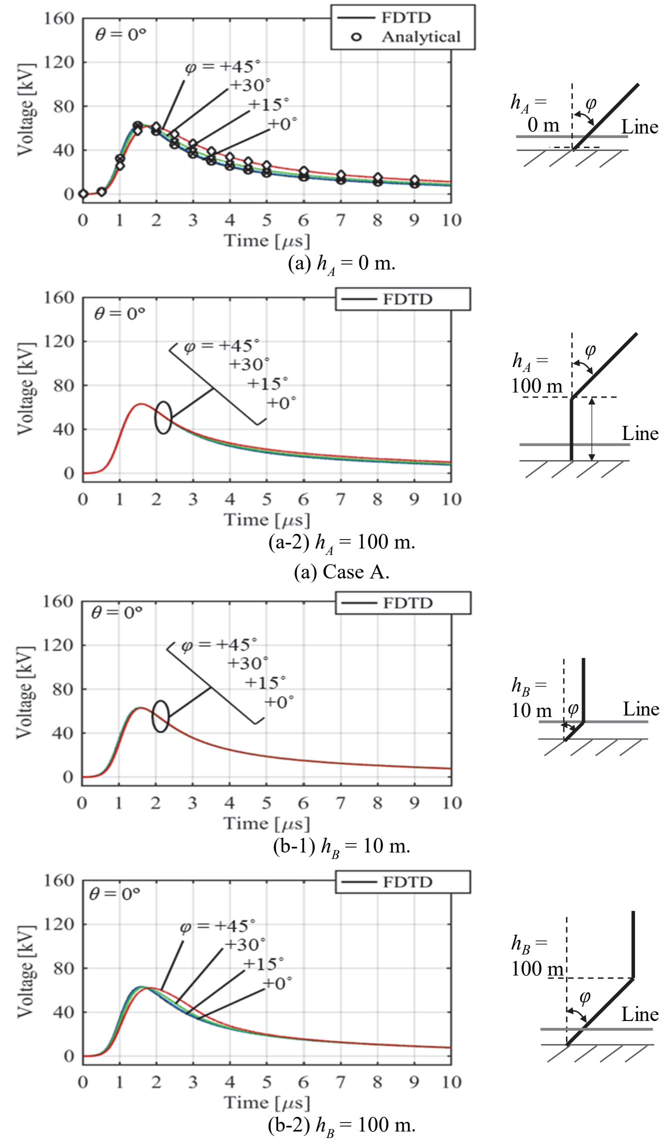


(a) h_B = 10 m.

(b) h_B = 50 m.

(c) h_B = 100 m.

Fig. 4 Lightning induced voltages on a single-conductor overhead line by bent lightning with various bent heights and inclined angles θ: Case B, 1/200 μs, ρ_e = 0 Ωm, d = 50 m, φ = 0º.



(a) h_A = 0 m.

(a-2) h_A = 100 m.

(a) Case A.

(b-1) h_B = 10 m.

(b-2) h_B = 100 m.

(b) Case B.

Fig. 5 Lightning induced voltages on a single-conductor overhead line by bent lightning with various bent heights and inclined angles φ: Case A and B, 1/200 μs, ρ_e = 0 Ωm, d = 50 m, θ = 0º.

10 m, only the inclination of 10-m height is considered, the induced voltage starts to vary according to the inclined angle. The voltages for h_B = 50 and 100 m become almost the same as those by the inclined lightning in Fig. 3 (a). These results also support the importance of the lightning channel geometry under 100-m altitude.

The influence of h_B is more noticeable than that of h_A because the lightning channels in Case B become closer to the line than those in Case A when $h_A = h_B$. However, both Case A and B can clearly demonstrate the importance of the lightning channel geometry in low altitude. Thus, only Case A is investigated after section II-B 4).

3) Case A/B above a perfectly conducting earth: angle φ

Fig. 5 shows lightning induced voltages in Case A and B for the angle φ where the lightning is inclined along the line. In contrast to the angle θ, the influence of φ on the induced voltages is very minor even when h_A = 0 m and h_B = 100 m. These results indicate that considering only the inclination toward the line (angle θ) is enough to investigate the induced voltage by non-vertical lightning, at least for the voltage measured at the line center in this study.

4) Effect of earth resistivity

It is well-known that the induced voltage waveform is dependent on the earth resistivity [24], [25]. The peak voltage becomes higher and the waveform becomes a little wider as the resistivity increases. It would affect the dominant lightning-channel height to induce the peak voltage.

Fig. 6 shows induced voltages in Case A for the angle θ when the earth resistivity ρ_e is 1000 Ωm. The voltages increase because of the high resistivity. However, the influence of the bent height h_A is similar to the results of ρ_e = 0 Ωm in Fig. 3. The lightning channel geometry under 100-m altitude is still dominant for the peak voltage when the lossy earth is assumed.

5) Effect of lightning distance to the line

Fig. 7 shows induced voltages in Case A for the angle θ when the lightning becomes farther to the line (d = 100 and 300 m). The voltages become lower and wider compared with those for d = 50 m as is well-known. In addition, the voltages still differ from each other when h_A = 100 m. This means that the lightning channel geometry of over 100-m altitude is still significant for the peak voltage. The dominant height is estimated about 200 m for d = 100 m and about 500 m for d = 300 m because of their peak voltage position. The delay time from the voltage rise to the peak is about 2 μs for d = 100 m, and the time is equal to 200 m when the return stroke velocity is $1{\times}10^8$ m/s. The delay time of 5 μs for d = 300 m corresponds to 500 m. The estimation

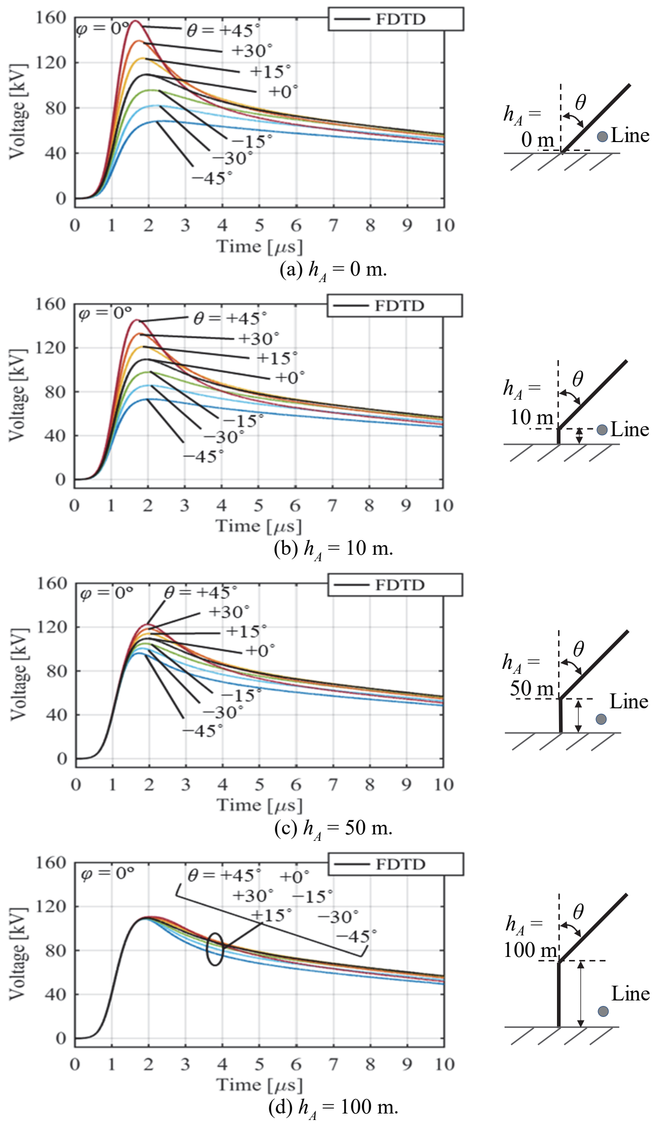


Fig. 6 Lightning induced voltages on a single-conductor overhead line by bent lightning with various bent heights and inclined angles θ: Case A, 1/200 µs, ρ_e = 1000 Ωm, d = 50 m, φ = 0º.

can be confirmed in Fig. 7 (a-3) and (b-3).

These results indicate that to evaluate the induced voltage with long d, the lightning channel geometry of higher altitude increases in its influence and should be modeled in a voltage evaluation. However, the peak voltage is decreased since d is long, and the evaluation itself becomes less significant.

6) Effect of return-stroke current waveform

Fig. 8 shows induced voltages in Case A for the angle θ when the return stroke current of 5.5/200 µs is applied. Note that the time duration in the figure is extended to 20 µs. The peak voltages still differ from each other when h_A = 100 m, indicating the dominant height becomes higher than 100 m similarly to long d. The peak voltages finally converge when h_A = 500 m.

III. Voltages by Non-Vertical Lightning-Like Channel

A. Computationally-generated lightning-like channel

Many lightning photos have been taken and investigated in published papers (e.g. [26]-[28]). Further, some papers investigate lightning induced voltages by using real or rocket-

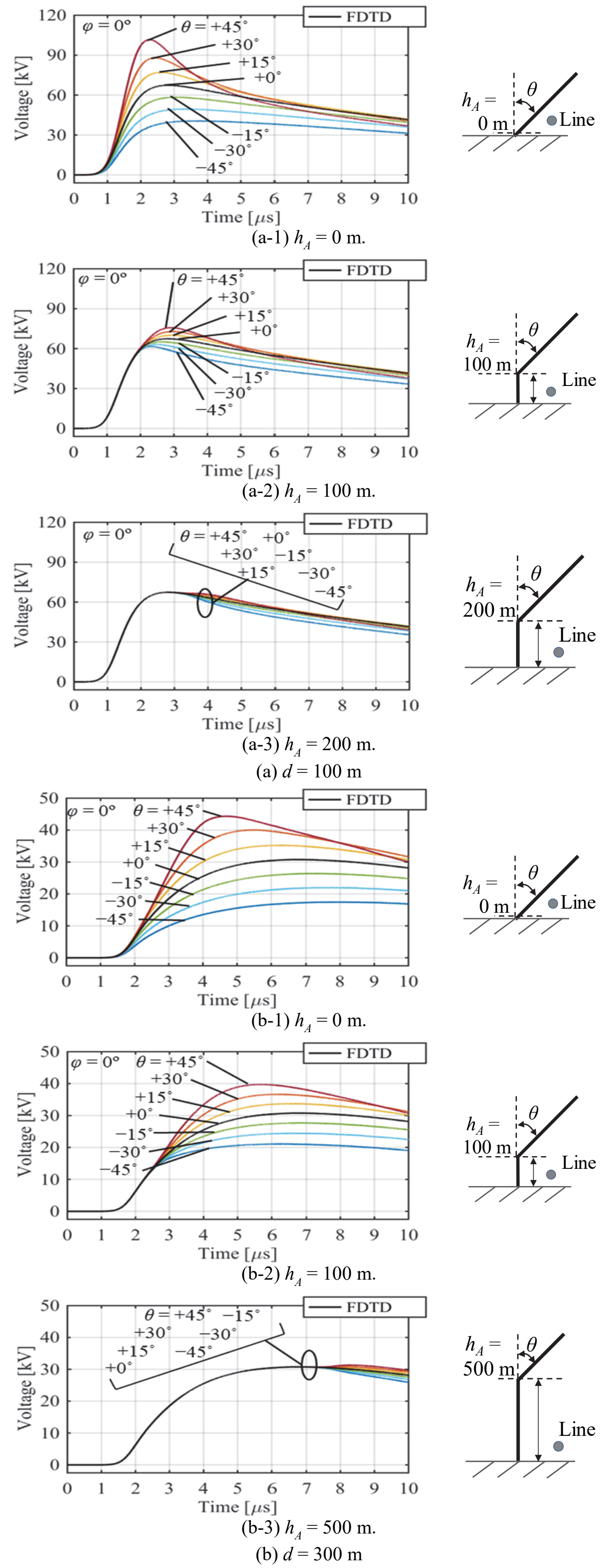
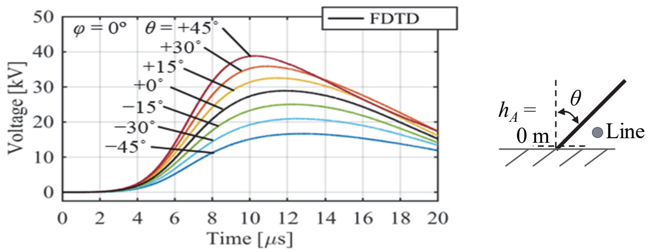
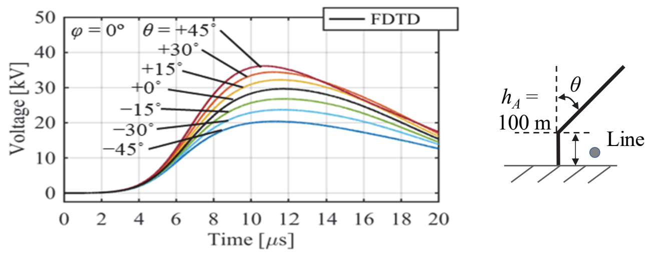


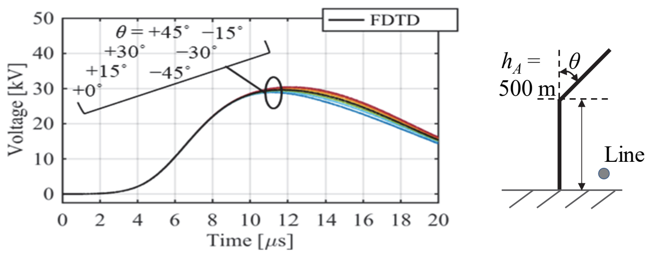
Fig. 7 Lightning induced voltages on a single-conductor overhead line by bent lightning with various bent heights and inclined angles θ: Case A, 1/200 µs, ρ_e = 1000 Ωm, d = 100 and 300 m, φ = 0º.



(a) h_A = 0 m.

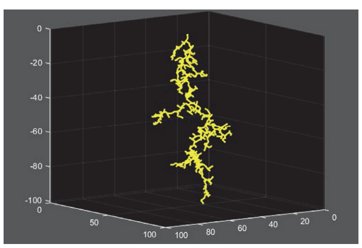


(b) h_A = 100 m.

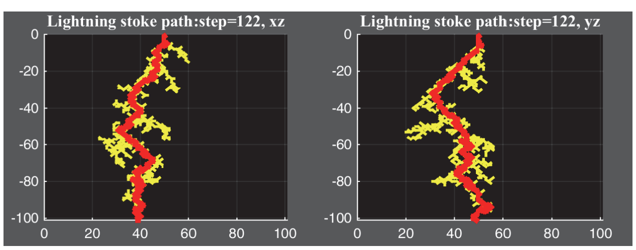


(c) h_A = 500 m.

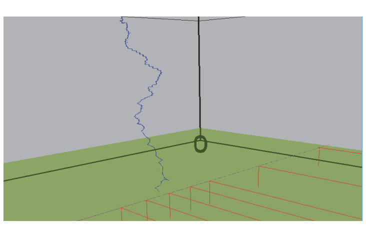
Fig. 8 Lightning induced voltages on a single-conductor overhead line by bent lightning with various bent heights and inclined angles θ: Case A, 5.5/200 µs, ρ_e = 1000 Ωm, d = 300 m, φ = 0º.



(a) Three-dimensional image (height = 1000 m).

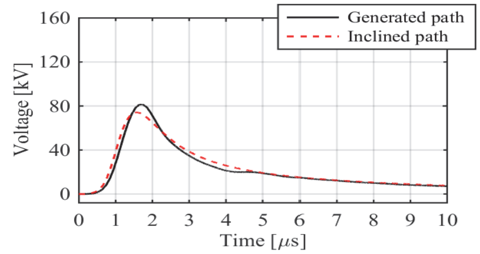


(b) xz and yz plane images (height = 1000 m).

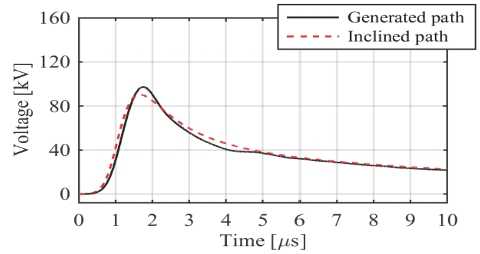


(c) Implementation in FDTD computation.

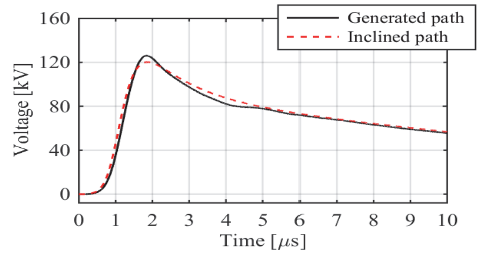
Fig. 9 A lightning path generated by a probabilistic calculation based on the electric potential distribution [29], [30]. Yellow lines show lightning leaders which progress from cloud to ground, and a red line indicates a return stroke path after the leader attaches to the ground.



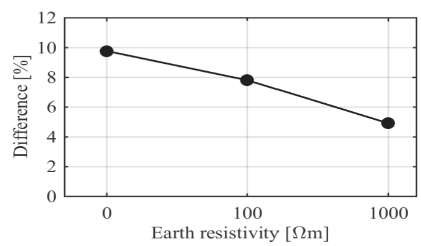
(a) ρ_e = 0 Ωm



(b) ρ_e = 100 Ωm



(c) ρ_e = 1000 Ωm



(d) Difference.

Fig. 10 Lightning induced voltages on a single-conductor overhead line by 1) the computationally-generated and 2) simply-inclined lightning channels: 1/200 µs, d = 50 m, inclined angles θ = 11.5º, and φ = 0.5º.

triggered lightning geometries in three-dimensional space which are reconstructed from the photos [15]-[17]. However, one difficulty is that lightning observations and experiments are expensive, and thus the information of the 3D lightning geometry is quite limited.

Another method to obtain the 3D lightning geometry is a probabilistic calculation based on the electric potential distribution [29], [30]. The similarity of the computed lightning channel to real lightning one is evaluated by fractal dimension. Even though the computed channel is not "lightning" but "lightning-like" channel, it would represent the non-uniformity of the lightning channel geometry.

In this section, lightning leader paths (cloud to ground) are computationally generated by probabilistic calculation, and the return stroke paths are exploited after the leader attaches to the ground. Then, the return stroke paths are imported to FDTD computation to investigate the induced voltage. Fig. 9 shows an example. Details of the probabilistic calculation are presented in [29], [30].

B. Induced voltage

Fig. 10 shows comparisons of voltages induced by 1) the computationally-generated and 2) simply-inclined channels with different earth resistivities. The latter channel is same as h_A = 0 m in Case A. Average angles of 1) in low altitude (between 0- and 100-m heights) are set to the inclined angles θ and φ of 2), i.e. $\theta = 11.5^\circ$ and $\varphi = 0.5^\circ$ for the channel in Fig. 9. The current waveform of 1/200 μs and distance of d = 50 m are adopted.

In Fig. 10, the voltages by the inclined channel agree well with those by the generated channels. The zig-zag channel geometry in the generated path does not significantly affect the entire voltage waveform in all the earth resistivity. The difference of the peak voltages between the generated and inclined channels is less than 10%, and it decreases as the earth resistivity increases. These results indicate that realistic non-vertical lightning channels can be represented by simply-inclined lightning paths. The approximation becomes more reasonable with higher earth resistivity.

IV. Conclusions

Influence of non-vertical lightning on induced voltages have been investigated by FDTD computations when "bent" and "computationally-generated" lightning channels are assumed. The following conclusions are made:

1. The lightning channel geometry under 100-m altitude is dominant for the peak voltage evaluation when a severe condition set of $T_{d10\text{-}90}/T_n$ = 1/200 μs, d = 50 m, and ρ_e = 1000 Ωm is assumed.
2. When the distance d becomes longer and rise time $T_{d10\text{-}90}$ becomes slower, the induced voltage waveform becomes wider and the dominant channel height becomes higher.
3. For a condition of 1/200 μs and d = 50 m, a realistic non-vertical lightning channel can be represented by a simply-inclined lightning path with average inclined angles under 100-m altitude.
4. The difference of the peak voltages between the generated and inclined channels is less than 10%, and it decreases as the earth resistivity increases.

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