Optimizing Accuracy and Eliminating Numerical Oscillations for Transient Power System Simulations

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***Abstract*—In transient power system simulations both high accuracy and elimination of numerical oscillations caused by the integration method are required. Power system engineers, who often are the end users of simulation tools, are not always aware of the latter. In this paper an algorithm which yields optimum accuracy for the nominal frequency of a power system while avoiding numerical oscillations will be presented. This is achieved by pre-warping a damped Trapezoidal rule and appropriately adapting the damping factor of the numerical integrator individually for each element. In particular, but not only, steady-state signals are depicted very accurately, which is essential for applications in the area of power system protection.**

***Keywords*—Transient power system simulation, Trapezoidal integration, backward Euler integration, numerical oscillations, critical damping adjustment (CDA), accuracy, EMTP**

I. Introduction

In EMTP and other transient power system simulation software, Trapezoidal integration with a constant time step Δt has been used successfully for years. To avoid numerical oscillations, the critical damping adjustment (CDA) scheme is often used [1], [2], which switches the integration method to backwards Euler for two time steps of size $\frac{\Delta t}{2}$ at every discontinuity, such as switching or fault events. However, it is not possible to completely eliminate numerical oscillations by using CDA, so that careful modelling of the simulated topology is still necessitated.

While power system engineers, who use power system simulation tools in their daily work (e.g. for testing power system protection) should ideally be aware of the problems, one of the authors' experience in industry has lead to the conclusion that, unfortunately, this is not always the case. Hence appropriate steps (e.g. the inclusion of damping resistors) are often not taken. Therefore, an algorithm which automatically eliminates even more numerical oscillations is to be preferred.

On the other hand, for applications within power system protection, very accurate transient simulations are required. Protection relays measure voltages and currents to detect faulty conditions in the network, where even small deviations in amplitude and phase are evaluated (e.g. within differential protection elements). Any inaccuracies in the test signals can lead to unexpected behaviour of protection relays under test or cause deviations of measurements done with steady-state tests (e.g. exact reaches of distance zones). Power system simulation tools used for protection testing should therefore yield very accurate results particularly for signals at the power system's nominal frequency.

In this paper, a modified damped Trapezoidal rule will be presented. First, for non-ideal inductive and capacitive elements, the accuracy will be increased greatly by correcting the impedance for signals at the nominal frequency. Second, for each individual element, we will choose an appropriate damping factor in order to both accurately depict transient processes and reduce numerical oscillatory effects.

Finally, a novel algorithm to optimize accuracy while eliminating most numerical oscillations will be proposed and compared to standard Trapezoidal integration.

II. Numerical modelling

In EMTP and similar programs, it is necessary to transform continuous integrals into discrete-time approximations, usually using some form of damped Trapezoidal rule [1], [3]

$$\int_t^{t+\Delta t} f(x)\mathrm{d}x \approx \Delta t \cdot \frac{f(t+\Delta t)+\beta\cdot f(t)}{\alpha}, \qquad (1)$$

where $0 \leq \beta \leq 1$ and $\alpha = 1+\beta$. In particular, for $\beta = 0$, we have the Euler backwards method, whereas for $\beta = 1$, we have the Trapezoidal rule. As will become clear from the estimate for the time constant deviation (19) below, it will be useful to adjust the damping not directly using α and β, but instead we will use a third parameter $\Omega := \frac{1+\beta}{1-\beta}$, which can take values between 1 and $+\infty$, where $\Omega = 1$ corresponds to Euler backwards and $\Omega = +\infty$ to the Trapezoidal rule.

It is well known [1], [4] that the Trapezoidal rule yields more accurate results than the Euler backwards method, but is much more vulnerable to numerical oscillations.

For modelling the current through a non-ideal inductance (an inductance in series with a resistor), transformation of the equation

$$v(t) = R\cdot i(t) + L\cdot \frac{\partial i}{\partial t}(t) \qquad (2)$$

into the z-domain (see e.g. [4], [5], [6]) yields the transfer function

$$\frac{V(z)}{I(z)} = R + L\cdot s, \qquad (3)$$

where

$$s = \frac{1}{\Delta t}\cdot\alpha\cdot\frac{z-1}{z+\beta}. \qquad (4)$$

While such a transformation is necessary to the numerical simulation, it also introduces a phase shift which is caused

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by the approximation of the differential operator s. While, ideally, we would have [5] $z = e^{s\Delta t}$, our approximation is

$$z = \frac{\alpha + \beta s\Delta t}{\alpha - s\Delta t}. \tag{5}$$

Assuming $\left|\frac{\Delta t}{\alpha} s\right| < 1$, this leads to the series expansion

$$z = 1 + \Delta t s + \frac{1}{\alpha}(\Delta t s)^2 + \cdots, \tag{6}$$

which, compared to

$$e^{s\Delta t} = 1 + \Delta t s + \frac{1}{2}(\Delta t s)^2 + \cdots, \tag{7}$$

results in an error of $\left(\frac{1}{\alpha} - \frac{1}{2}\right) \cdot (\Delta t s)^2 + \mathcal{O}\left((\Delta t s)^3\right)$. In particular, we have a second-order error only for the Trapezoidal rule.

However, for linear elements, we can avoid this by pre-warping the z-transformation to the correct impedance [8]. For an RL-element, this means we introduce correcting factors $k_R, k_L \in \mathbb{R}$ such that, given $\omega = 2\pi f$ and $z = e^{j\Delta t\omega}$,

$$R + jL\omega = k_R R + k_L L s. \tag{8}$$

Note that, for the Trapezoidal rule, we have $k_R = 1$ and $k_L = \frac{\frac{\omega\Delta t}{2}}{\tan\frac{\omega\Delta t}{2}}$, which is the inverse of the error already mentioned in [3].

Direct calculation of these factors leads to, substituting $\xi := \frac{1-e^{-j\omega\Delta t}}{1+\beta e^{-j\omega\Delta t}}$,

$$k_L = \frac{\omega\Delta t}{\alpha} \cdot (\mathrm{Im}(\xi))^{-1}, \tag{9}$$

$$k_R = 1 - k_L \cdot \frac{L}{R} \cdot \frac{\alpha}{\Delta t}\mathrm{Re}(\xi). \tag{10}$$

Using this impedance correction, the simulation accuracy can be greatly increased. Due to the duality of inductance and capacitance, the same considerations apply for the latter. It is especially noteworthy that, for steady state calculations, there is no notable difference in accuracy between the Euler backwards method and the Trapezoidal rule anymore. This can be seen for different impedance angles and sampling frequencies in Table I. While for the non-corrected versions, quadratic and linear convergence rates for Trapezoidal rule and Euler backwards are evident as well as their slower convergence for high angles, these differences are negligible for the impedance-corrected versions.

While the above kind of impedance correction is very useful for eliminating both magnitude and phase shift in a steady-state solution, it can be somewhat problematic for transient processes. The two correcting factors k_R, k_L are used to warp both the resistance and conductivity of the element in order to gain the correct frequency response, however, this results in a shift in the time constant $\tau := \frac{L}{R}$.

For transient simulations, it is therefore necessary to investigate if, and if so, when this shift in time constant can turn out to be problematic.

		Trapezoidal	Euler
1°	1 kHz	3.59E-04	3.82E-03
	10 kHz	2.03E-06	3.88E-04
	100 kHz	2.03E-08	3.88E-05
89.99°	1 kHz	2.27E-02	2.80E-01
	10 kHz	2.33E-04	2.28E-02
	100 kHz	2.33E-06	2.22E-03

(a) without impedance correction

		Trapezoidal	Euler
1°	1 kHz	6.62E-15	6.68E-15
	10 kHz	7.68E-15	8.35E-15
	100 kHz	7.70E-15	7.99E-15
89.99°	1 kHz	8.08E-15	4.83E-14
	10 kHz	7.46E-15	1.23E-14
	100 kHz	4.42E-14	2.07E-14

(b) using impedance correction

TABLE I: Relative error to theoretical solution for the current through an inductor with either very high or low impedance angle, for an AC Voltage source with $f = 50Hz$ and different sampling frequencies, using the Trapezoidal rule and Euler backwards method. The first column shows the impedance angle, the second the sampling frequency.

A. Time constant considerations

In order to estimate the error caused by the shift in time constant, it will prove to be useful to look at the relative error

$$\Delta\tau := \left|\frac{\hat{\tau} - \tau}{\tau}\right| = \left|1 - \frac{k_L}{k_R}\right|, \tag{11}$$

where $\tau = \frac{L}{R}, \hat{\tau} = \frac{k_L \cdot L}{k_R \cdot R}$.

A first-order approximation with respect to $\omega\Delta t$ of the correcting factors is given by

$$k_L \approx 1, \tag{12}$$

$$k_R \approx 1 - \frac{L\omega}{R} \cdot \frac{\omega\Delta t}{2} \cdot \frac{1}{\Omega}. \tag{13}$$

Seeing as Δt can in practical applications be chosen to be fairly small, the accuracy of these approximations will in the following assumed to be decently good.

It follows from (12)-(13) that, while k_L is not an issue, it is advisable to choose Ω as large as possible, in order to keep k_R as close to 1 as possible.

Since most transient processes are closely linked to exponential functions of the form $e^{-t/\tau}$, in many cases the error resulting from the shift in time constant can be estimated by the difference

$$\left|e^{-t/\tau} - e^{-t/\hat{\tau}}\right|. \tag{14}$$

As

$$\left|e^{-t/\tau} - e^{-t/\hat{\tau}}\right| = \left|e^{-t/\tau} - \left(e^{-t/\tau}\right)^{\tau/\hat{\tau}}\right|, \tag{15}$$

substituting $Z := e^{-t/\tau}$, $x := \frac{L\omega}{R} \cdot \frac{\omega \Delta t}{2} \cdot \frac{1}{\Omega}$ and $\rho := \tau/\hat{\tau} \approx 1 - x$ transforms this to

$$|Z - Z^{\rho}|, \tag{16}$$

which has a maximum value of roughly (second-order approximation with respect to x)

$$\frac{1}{e} \cdot \left(x - \frac{x^2}{2}\right) \tag{17}$$

In particular, this means that, in order to reduce the error in transient processes, it is sufficient to reduce the value of x. Seeing as, by (12)-(13) and for small x, we have

$$\Delta\tau = \left|1 - \frac{k_L}{k_R}\right| \approx \left|1 - \frac{1}{1-x}\right| \approx |1 - (1+x)| = x, \tag{18}$$

this is equivalent to the intuitive solution of keeping the time constant shift as small as possible. Looking back at our definition of x, without changing the time step size, the only - but fortunately a rather simple - way to achieve this is by increasing Ω. Given any $\varepsilon > 0$ and otherwise fixed system parameters, we can give an estimate for a minimal Ω such that $\Delta\tau < \varepsilon$ by

$$\Omega_{\varepsilon} = \frac{L\omega}{R} \cdot \frac{\omega \Delta t}{2} \cdot \frac{1}{\varepsilon}. \tag{19}$$

This shows that, considering only the time constant shift, we would like Ω to be as large as possible. As, for small Ω and large impedance angles, it is entirely possible for the time constant to take negative values after impedance correction, it will turn out to be essential that such a bound for $\Delta\tau$ is introduced and Ω is then chosen at least as big as in (19).

B. Oscillatory behaviour

While the Trapezoidal rule has great properties in terms of stability and accuracy, it is prone to suffer from oscillations [3], [10], [5]. Two of the most common solutions to this are the Critical Damping Adjustment (CDA), consisting of a change to the Euler backwards method for a short time after an event, and the use of a damped Trapezoidal rule as in (1) [1], [7]. The latter is, for ideal elements, the mathematical equivalent to including a physical damping resistor [3]. For non-ideal elements, while this simple physical interpretation is not easily applicable anymore, using a dampened Trapezoidal rule is still an excellent tool to reduce oscillations without great losses in terms of accuracy. There are, of course, other options as well, such as using a higher order numerical integration method, as in [11], but these shall not be discussed here.

When calculating the current through an RL branch, the corresponding differential equation

$$v(t) = \quad R \cdot i(t) + L \cdot \tfrac{\partial i}{\partial t}(t) \tag{20}$$

$$\Rightarrow \quad i(t+\Delta t) = \quad i(t) + \textstyle\int_t^{t+\Delta t} v(x) - R \cdot i(x) \,\mathrm{d}\,x \tag{21}$$

leads to recursions of the form

$$i(t+\Delta t) = \quad A_1 \cdot i(t) + B_1 \cdot v(t) + C_1 \cdot v(t+\Delta t), \tag{22}$$

$$v(t+\Delta t) = \quad A_2 \cdot v(t) + B_2 \cdot i(t) + C_2 \cdot i(t+\Delta t). \tag{23}$$

For oscillatory behaviour in case of a sudden drop in voltage or current, the important terms in the above equations are $A_{1,2}$, in particular, numerical oscillations will occur if one of these terms is negative [1], [10]. Substituting the dampened Trapezoidal rule, we arrive at

$$A_1 \quad = \quad \frac{\alpha L - \beta R \Delta t}{\alpha L + R \Delta t}, \tag{24}$$

$$A_2 \quad = \quad -\beta. \tag{25}$$

From this we can conclude that, firstly, calculating the current through an RL element will not be problematic provided that $R\Delta t < L$. If this were not the case, it would translate to a very small impedance angle, which is not relevant for most practical applications. If one wanted to still implement a safeguard for this case, it would be possible to substitute α, β by $\frac{2\Omega}{\Omega+1}, \frac{\Omega-1}{\Omega+1}$ and then calculate a maximum Ω such that $A_1 \geq 0$. The time-constant shift caused by such an upper bound for Ω can be shown to be less than $\left(\frac{\omega \Delta t}{2}\right)^2$, and is negligible. However, as the modelled inductances tend to have rather high impedance angles, this shall not be discussed in detail here.

The second conclusion we can draw from (24)-(25) is that, for calculating the voltage across an RL branch, the only way to rule out numerical oscillations is by using the Euler backwards method, otherwise we have oscillations decaying with β^n, where n is the number of samples after the event.

Therefore, in order to reduce oscillations, we would very much like to keep our Ω as small as possible.

C. Proposed Algorithms

From the last two sections, we have two conflicting requirements for our Ω - firstly, it should be large enough that the time constant shift $\Delta\tau$ does not matter too much, and secondly, it should be rather small such that numerical oscillations can be eliminated. It is also obvious that the exact requirements for Ω differ from element to element, depending on the material parameters. Therefore, it would be ideal to use different integration methods for different elements. By substitution into the corresponding matrices in [3] and some rearranging, it can be shown that, if each element is properly approximated by the corresponding method, then, for uncoupled elements, this is indeed possible. It may be worth pointing out that something similar is already implemented in ATP, where one can select a damping value K_p for different elements separately [9], [10]. However, this reasoning cannot be immediately adopted, seeing as in ATP, the same process is explained not by selecting a different integration method, but by choosing an element with slightly altered physical parameters (that is, the additional damping resistor).

In order to give an algorithm and choose an appropriate Ω for each element, the one thing that seems to be mandatory is to implement a lower bound Ω_{min} for Ω, restricting $\Delta\tau$ as in (19). The condition $\Omega \geq \Omega_{min}$ ensures that $\Delta\tau \leq \varepsilon$. .

The first algorithm aims to keep Ω as large as possible, so as to achieve great transient accuracy. The resultant oscillations can then be reduced using some other method, for instance the previously mentioned CDA. We will still not allow for

$\Omega = \infty$, however, so that randomly occurring numerical errors do not propagate and are reduced within a reasonable time, and instead aim to set Ω to 500. Taking into account the above, this is only admissible if $\Omega_{min} \leq 500$. Hence, the first algorithm looks as follows:

Algorithm I:

1) Select a maximum allowed $\Delta\tau$, and calculate the corresponding Ω_{min}.
2) If $(\Omega_{min} \leq 500)$, then $\Omega = 500$.
3) Else $\Omega = \Omega_{min}$.

The second algorithm is supposed to eliminate oscillations as much as possible. It will select the minimum Ω such that the time constant constraints are not broken:

Algorithm II:

1) Select a maximum allowed $\Delta\tau$, and calculate the corresponding Ω_{min}.
2) $\Omega = \Omega_{min}$.

III. Simulation results

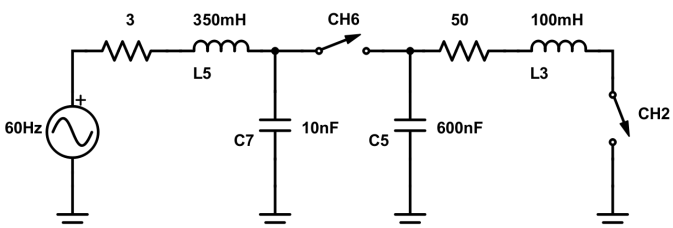

Fig. 1: Testing circuit. The first switch CH6 is closed at $t = 0.005$s, while the second simulates a fault event at $t = 0.05$s.

The algorithms were tested for an RLC-circuit as illustrated in Fig. 1, with impedance angles for the capacities set to $85°$. They were compared with the normal Trapezoidal rule, as well as a constant Ω of 5.4 and 9.8, which is roughly equivalent to the upper and lower bound of damping suggested in [3]. The images shown correspond to the voltage and current that would be measured by the protection relays that is, the voltage potential to the left of CH6 and the current across, but also present a good representation of the behaviour of other branch currents/voltages.

The frequency f was chosen to be 60 Hz, $\Delta t = 10^{-4}$s. To serve as a reference, the simulation was also done using a Trapezoidal rule, with $\Delta t = 10^{-6}$s, using CDA. The maximum allowed time constant deviation was set to $\Delta\tau = 0.01$.

As can be seen in Fig. 2 (for algorithms I, II) and Fig. 3 (for $\Omega = 5.4, 9.8$), the transients are in general depicted more accurately by the proposed algorithms I and II than by a normal damped Trapezoidal rule with $\Omega = 5.4, 9.8$, the sole exception being the second switching event (after $t = 0.05$) for algorithm II. This is, in the authors' opinions, easily compensated for by the more precise results for the first transient ($t = 0.005$ until $t = 0.05$). As expected, there is a slight loss in the second algorithm compared to the pure

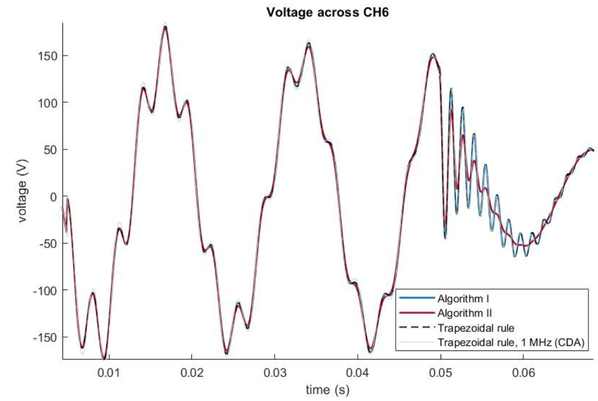

Fig. 2: Voltage as measured by protection relays, using proposed algorithms.

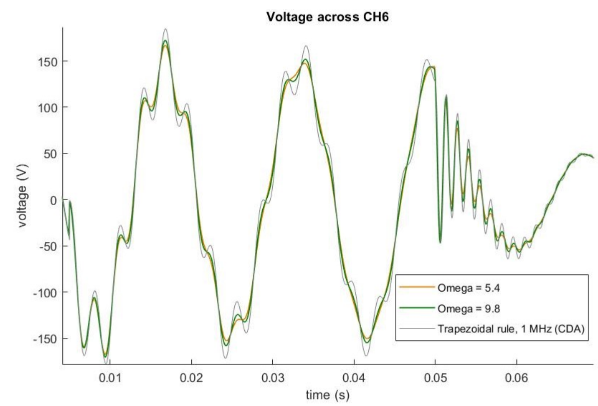

Fig. 3: Voltage as measured by protection relays, using a damped Trapezoidal rule.

Trapezoidal rule or the proposed first algorithm, all of which is to be expected due to its higher emphasis on damping. To see whether the resulting reduction of oscillation can offset this disadvantage, we will next consider the current as measured by the protection relays, which is naturally prone to oscillatory effects.

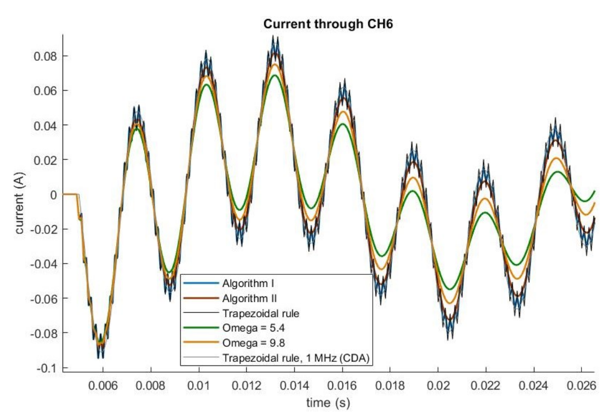

Fig. 4: Current as measured by protection relays.

In Fig. 4 we can observe numerical oscillations typical of the Trapezoidal rule. As before, the transient accuracy increases with Ω. On the other hand, we can now see how the choice

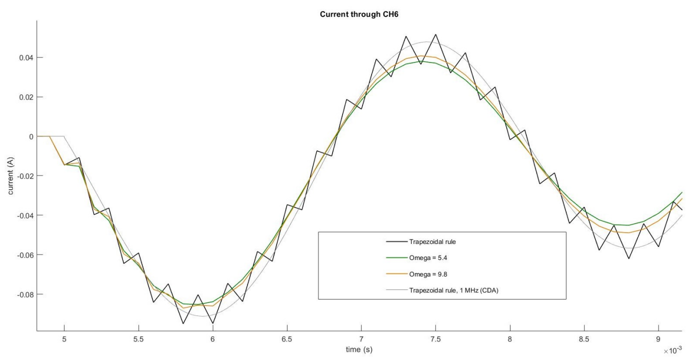

Fig. 5: Current as measured by protection relays, calculated using a damped Trapezoidal rule with $\Omega = 5.4, 9.8$.

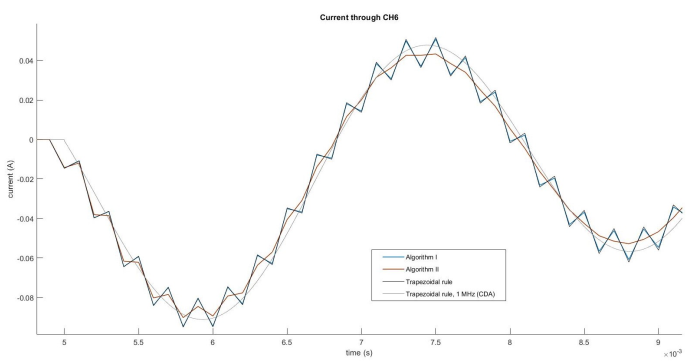

Fig. 6: Current as measured by protection relays, calculated using the proposed algorithms.

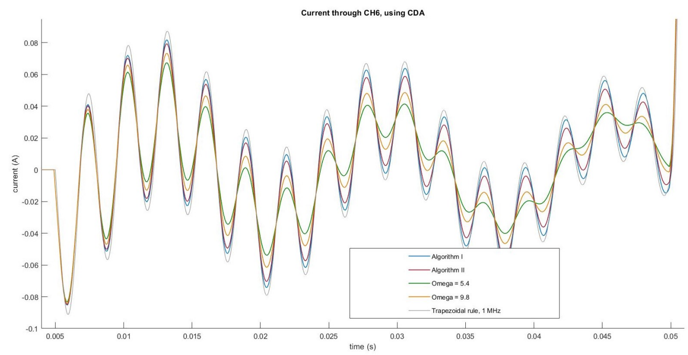

Fig. 7: Current as measured by protection relays, calculated using the proposed algorithms and CDA.

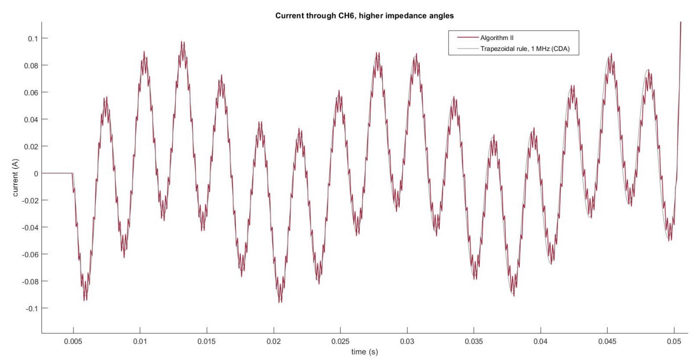

Fig. 8: Current as measured by protection relays, calculated using algorithm II and higher impedance angles.

of Ω is reflected in how the different algorithms handle oscillations.

For fixed Ω of $5.4, 9.8$ we have an almost complete reduction of visible oscillations before the first minimum, though this comes with a notable loss in accuracy, as depicted in Fig. 5. For the second algorithm, while the numerical oscillations are still visible at the beginning, they seem to disappear within roughly a period, while still retaining much better accuracy than the previous versions, as can be seen in Fig. 6. For the first algorithm, the oscillations are only marginally reduced over the whole duration.

It must be noted, however, that for fault events of this kind, numerical oscillations could also be avoided using CDA or a similar technique [6],[10], leading to an image as in Fig. 7. In this case, the main problems of the first algorithm are avoided, leading, compared to the others, to a more accurate result.

If the impedance angles are selected sufficiently high such that (19) leads to an extremely large Ω_{min}, the damping of the proposed algorithms will be reduced accordingly, as dictated by our requirement to the time constant shift, i.e., to the transient accuracy. Fig. 8 shows the almost non-existent damping properties of the second algorithm at $\Delta t = 10^{-4}$s for impedance angles of $89°$. It should be pointed out that, while the proposed algorithm's issue are oscillations, a fixed Ω of 5.4 or 9.8 would lead to a negative time constant and therefore nonsensical transient behaviour, illustrating that, when using impedance correction, time constant considerations are in no way optional but rather a necessity. (19) implies that this problem can be remedied by choosing a smaller Δt is chosen smaller, as this leads to a decrease of the corresponding Ω_{min}. This is illustrated in Fig. 9, where $\Delta t = 10^{-6}$, allowing even for $\Omega = 1$, that is, we can use the Euler backwards method for our calculations while remaining in the bounds given for $\Delta\tau$. If we selected Δt even smaller, this would lead to an increase in accuracy while still retaining our damping properties.

IV. Discussion

As was seen in the previous sections, the damped Trapezoidal rule with impedance correction implemented as above yields a notable increase in accuracy especially for steady-state solutions, while still being similarly, if not somewhat more accurate as previously used methods for transient simulations. Some of this accuracy can be sacrificed to gain improved damping properties, greatly reducing numerical oscillations in many cases without use of a higher-order integration method or introduction of an artificial damping resistance. This may be of particular interest in cases where measures such as CDA are not entirely effective, e.g. if there are nonlinear elements causing discontinuities which, depending on the exact implementation, may not always trigger additional damping mechanisms. However, there are some other points that also need to be addressed.

Firstly, the test results shown were given for single-phase elements. The algorithm can be used for three-phase or

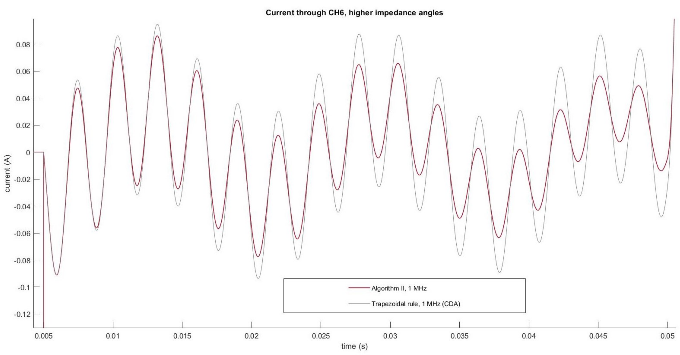

Fig. 9: Current as measured by protection relays, calculated using algorithm II and higher impedance angles, $\Delta t = 10^{-6}$s.

coupled elements as well, but it is important to note that, for any coupled elements, the integration method must stay the same. This means that the parameter Ω needs to be chosen for all elements together, and should be sufficiently large that the time constant shift is within the allowed range for each single element. In tests, the simplistic solution of calculating Ω_{min} for each element of the circuit, and selecting the maximum out of those yielded satisfactory results. This method would, however, still be worth examining in more detail, as it seems likely that improvements are still possible.

Secondly, the algorithm, while very efficient, is also subject to some limitations. The main reason for the increase in accuracy lies in the impedance correction, which eliminates the error naturally occurring by using the Trapezoidal rule. It is therefore necessary that the frequency of the network be known, else this phase shift cannot be properly corrected. In particular, this means that the algorithm may suffer from losses in accuracy for DC sources (which are of little importance to most end users of AC power system protection), but also for nonlinear elements, which introduce signal components with different frequencies. Seeing as there is not one chosen integration method that is used globally, but (uncoupled) elements can be integrated differently, this can sometimes be remedied.

It should also be noted that this algorithm does not allow for ideal inductances and capacitors, because (8) is generally not solvable for $R = 0$. However, in reality, there is no such thing as a lossless conductivity, and the method has shown to be very effective up to very high impedance angles.

Lastly, it should be mentioned that the proposed algorithms are obviously not the only sensible choices to select the parameter Ω for a single element. It would, for instance, be possible to directly select Ω such that the exponential in (7) coincides with (5) in many cases, leading to an almost exact solution for the transient of at least a single element, but sometimes causing a greater shift in time constant. Such considerations may need to be addressed in future research. Also, the choice of 1% as maximum allowed time constant deviation may still be improved upon. In some cases, this could potentially be done by an a-priori estimate of the relative error if $\Delta\tau$ is known, using approximations such as, for instance, but not limited to (17).

V. Conclusions

A numerical integration algorithm that is compatible with EMTP-like methods was presented in this paper. It is based on a damped Trapezoidal rule, but also uses impedance correction in order to improve its performance at the presumed to be known nominal frequency of a network, which makes it particularly relevant for e.g. power system protection. The resulting deviation of time constant was discussed and a way to keep it below a certain threshold was given. Also, different integration methods (that is, with different damping) were used for separate elements, leading to more flexibility when addressing issues such as transient accuracy and numerical oscillations. Results in simulations as well as in applications in industrial settings by one of the author's company have consistently shown an increase in accuracy compared to the normal Trapezoidal rule. The major downside of this method seems to be its dependence on the nominal frequency.

Future work on this method may include discussing an implementation for three-phase coupled elements, which in turn entails a more complicated search for an ideal damping parameter for each element. A suitable choice of the latter might also be applied to a normal damped Trapezoidal rule, where instead of the time constant shift the error in the approximation of the exponential in the frequency space as in (5)-(7) may be minimized.

The relevance of this method can also be seen in its future application in industrial software focusing on power system protection, where more practical results in different testing scenarios will be observed and may be presented in future papers.

VI. References

[1] L. F. R. Ferreira, B. D. Bonatto, J. R. Cogo, N. C. de Jesus, H. W. Dommel, J. R. Marti, "Comparative Solutions of Numerical Oscillations in the Trapezoidal Method used by EMTP-based Programs", International Conference on Power System Transients (IPST) 2015

[2] W. Gao, E. Solodovnik, R. Doubal, G. Cokkinides, A. Meliopoulos, "Elimination of Numerical Oscillations in Power System Dynamic Simulation", Eighteenth Annual IEEE Applied Power Electronics Conference and Exposition, 2003, Vol. 2, pp. 790-794

[3] J. Dommel, "Electromagnetic Transients Program (EMTP) Theory Book", Branch of System Engineering, Bonneville Power Administration, 1994 Oscillations in the Trapezoidal Method used by EMTP-based Programs", International Conference on Power System Transients (IPST) 2015

[4] M. Michalik, E. Rosolowski, "Simulation and Analysis of Power System Transients", Wroclaw University of Technology, 2011, pp. 35-45

[5] N. Watson, J. Arrillaga, "Power Systems Electromagnetic Transients Simulation", The Institute of Engineering and Technology, 2001, pp. 99-121

[6] J. R. Marti, J. Lin, "Suppression of Numerical Oscillations In The EMTP", IEE Transactions on Power Systems, Vol. 4., No. 2, May 1989, pp. 739-747

[7] A. Ametani (ed.), "Numerical Analysis of Power System Transients and Dynamics", The Institution of Engineering ad Technology, London, 2015, pp. 53-54

[8] "2.161 Signal Processing: Continuous and Discrete", Massachusetts Institute Of Technology OpenCourseWare, Fall 2008

[9] L. Prikler, H. K. Hoidalen, "ATPDraw Users' Manual", version 5.6, November 2009, pp. 38-40

[10] "Numerical Oscillations in EMTP-Like Programs", ECE 524, Session 39, Spring 2014, pp. 1-14

[11] A. Meliopoulos, G. Cokkinides, G. Stefopoulos, "Quadratic Integration Method", International Conference on Power System Transients (IPST) 2005