TURBINE-GENERATOR SET TRANSIENTS CAUSED
BY UNBALANCED SHORT-CIRCUITS

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ABSTRACT
In the paper transient phenomena at unbalanced short
circuits are analyzed. The emphasis is given to the
analysis of torsional torques in the shafts of a
turbogenerator set. Measurements on a laboratory model
of a turbine-generator including a 4-pole synchronous
generator and a DC motor drive connected by a torque
transducer and a flexible joint are carried out. Adding a
flexible joint natural frequencies similar to those of large
turbogenerator sets are obtained. The results of the
measurements are compared to the results of calculation
by a mathematical model of a synchronous generator
completed with the equations of motion of the partial
masses of the turbine-generator set. Measured and
calculated results show that significant torsional strains
are present at unbalanced short circuits close to the
generator.

1. INTRODUCTION
Transient phenomena in a turbine-generator set are a
function of the occurrences in the complex system of the
turbine-generator-power system. Faults in the power
system can cause torsional oscillations and eventually
fatigue life expenditure of the shafts of the turbine-
generator set. As the power of the generator units
tends to increase, more attention is paid to the problems
of monitoring and calculation of torsional torques in the
shafts of a turbine-generator set. The shafts of
turbogenerator sets are susceptible to low-frequency
excitation torques because of the large inertial masses of
the rotors of the generator and turbines and their poor
ability to damp low-frequency oscillations. These
problems are smaller in hydrogenerator sets because of
greater shaft stiffness and relatively small inertia of a
turbine comparing to the inertia of a generator.

Subsynchronous oscillations have been intensively
investigated in the past twenty years because they have
cause several severe damages of turbogenerator-set
shafts. Subsynchronous oscillations are the topic of a
number of papers classified by IEEE Working Group [1].
Special attention is given to the subsynchronous
resonance in series-capacity compensated systems [2].
Most of the published papers dealing with the problems
of torsional oscillations analyze symmetrical faults,
predominantly a three-phase short circuit, either simple
or combined with the fault clearing and high-speed
reclosing. Most frequently analysis is carried out for a
turbogenerator [3], and rarely for a hydrogenerator set.

According to [4], strains in the coupling zones of the
turbine-generator shaft, are the largest at subsynchronous
resonance, then at the out-of-phase synchronization and
three-phase fast reclosing after a two-phase short circuit.
Strains caused by generator terminal short circuits are in
sixth place by severity. Although not the most severe
from the standpoint of shaft fatigue per incident, they
occur more often than some more severe disturbances.

In the past few decades efforts have been made to
develop a different mathematical model for calculation of
electrical and mechanical transients of a turbine-generator
set caused by electrical disturbances. However, there still
exists a need for more practical measurements of
mechanical transients which could contribute to the
evaluation of different mathematical models. Field
measurement of mechanical transients caused by such
severe disturbances, which could confirm the results
obtained by mathematical modelling, is quite impossible
to carry out on a large generator-set. What remains is
measurement on a smaller generator-set or on a
laboratory model.

In the paper transients in a turbine-generator set during
unbalanced short circuits are analyzed. Measuring of
electrical and mechanical transients has been carried out
on a laboratory model of an isolated synchronous
generator. The results of measuring are compared to the
results of calculations by the mathematical model and by
analytical expressions for calculating transients in
synchronous machines.

2. ANALYTICAL EXPRESSIONS FOR
CURRENT AND ELECTRICAL TORQUE
Approximate expressions for calculating steady-state and
maximum short circuit current of an unloaded
synchronous generator are well-known. It is also known
that the largest steady-state armature current occurs at
phase-to-neutral short circuits, somewhat smaller at
two-phase short circuits and the smallest at three-phase
short circuits. The situation with maximum current, interesting because of large mechanical strains in windings which they cause, is somewhat different. The maximum value of the alternating component of an armature current can be theoretically calculated (for short circuit from no-load condition) as:

\[ I'_{\text{max}} = \frac{\sqrt{2}}{x''} \frac{U}{U} \quad (A) \quad (1) \]

for three-phase short circuit;

\[ I'^{''}_{\text{max}} = \frac{\sqrt{3}}{x'' + x_2 + x_2} \quad (A) \quad (2) \]

for two-phase short circuit and

\[ I'^{''}_{\text{max}1} = \frac{3\sqrt{2}}{x'' + x_2 + x_0} \quad (A) \quad (3) \]

for phase-to-neutral short circuit where:

- \( U \) - generator’s phase voltage before short circuit (V);
- \( X' \), \( X_2 \), \( X_0 \) - generator’s subtransient, inverse and zero-sequence reactance per phase (Ω).

Maximum current arises at phase coinciding to the position of the rotor axis in the moment of short circuit, and can be calculated for all short circuits approximately as:

\[ I'^{''}_{\text{max}2} = 1.8 I'^{''}_{\text{max}} \quad (4) \]

According to above expressions the largest maximum armature current occurs at phase-to-neutral short circuits, somewhat smaller at three-phase short circuits and smallest at two-phase short circuits.

In some specialized literature [6,7] comprehensive analytical expressions for calculating armature currents, with electrical damping included, are given. These expressions enable time response of the current from the moment of short circuit to be calculated with relatively good precision if constant rotational speed is supposed.

The largest field current (for independent excitation) can be expected at three-phase short circuits, somewhat smaller at two-phase short circuits and smallest at phase-to-neutral short circuit, which can be shown by comprehensive expressions, from [7] for example.

Maximum value of the alternating component of electrical torque can also be calculated from expressions:

\[ m_{k3} = \frac{u^2}{x''} \quad (pu) \quad (5) \]

for three-phase short circuit;

\[ m_{k2} = 2.6 \frac{u^2}{x'' + x_2} \quad (pu) \quad (6) \]

for two-phase short circuit and

\[ m_{k1} = 2.6 \frac{u^2}{x'' + x_2 + x_0} \quad (pu) \quad (7) \]

for phase-to-neutral short circuit where torque \( m_1 \), voltage \( u \) and reactance \( x'' \), \( x_2 \) and \( x_0 \) are expressed per unit.

Maximums occur 1/4 of period after the moment of short circuit at three-phase short circuit or 1/3 of period at two-phase and phase-to-neutral short circuits. At unbalanced short circuits the alternating component oscillates with basic and double frequency.

3. MATHEMATICAL MODEL OF SYNCHRONOUS GENERATOR

The base of the mathematical model is a set of differential equations of a synchronous generator represented with 2d-1q rotor coils and equations of motion of turbine-generator set masses together with relating turbine governor.

Voltage equations in \( dq0 \) rotating axes are modified for different unbalanced short circuits according to [8,9]. Because of limited space, only modelling of short circuits from no-load conditions is presented in the paper (Appendix I). The modified differential equations of a synchronous generator in \( dq0 \) coordinates originate from short circuits’ zero conditions in \( abc \) coordinates. By substituting expressions for transformation of currents and voltages from the \( abc \) to \( dq0 \) system and further modification equations similar to those for three-phase short circuits are obtained. However, these expressions include variable coefficients inevitable in analysis of the unbalanced short circuits.

4. MATHEMATICAL MODEL OF TURBINE-GENERATOR SET

In order to compute torsional strains in the shafts of a turbine-generator set, its rotational motion should be described by an equation of motion for each of its rotor part. In case of the largest turbogenerator sets, it is usually sufficient to take 5 concentrated masses into account: rotor of a high pressure turbine (HP), intermediate turbine (IP), two low pressure turbines (LP1, LP2) and a generator (G). The differential equation system can be presented (per unit) as:

\[ \frac{d\varphi_i}{dt} = \omega_i \quad (8) \]

\[ T_m \frac{d\omega}{dt} = m_n + C_{i+1}(\varphi_{i+1} - \varphi_i) - C_i(\varphi_i - \varphi_{i-1}) + D_{i+1}(\omega_{i+1} - \omega_i) - D_i(\omega_i - \omega_{i-1}) - D_{\omega} \omega \quad (9) \]

where index \( i \) is an integer from 1 to 5 indicating turbine stages HP (\( i = 5 \)), IP (\( i = 4 \)), LP1 (\( i = 3 \)) and LP2 (\( i = 2 \)).
The equation of motion of the generator rotor \((i=1)\) involves electrical torque \((m_{11} = m_{el1})\). In the equations \(\varphi_i\) (rad) is rotational masses twist angle, \(\omega_i\) (pu) angle speed difference, \(T_{mi}\) (pu) mechanical time constant, \(C_i\) (pu) coefficient of torsional stiffness and \(D_{ai}\) (pu) coefficient of damping.

Torque at coupling zones between two concentrated rotational masses can be calculated (per unit) as:

\[
m_{ai} = C_i' (\varphi_i - \varphi_{i-1})
\]

(10)

For the purpose of mathematical modelling, the laboratory model is described with three masses: the DC-motor mass, mass of flexible joint together with the torque transducer and the generator mass.

5. LABORATORY MODEL

Measurements have been carried out on a laboratory model of the turbine-generator set. The laboratory model included a 4-pole synchronous generator 25 kVA, 400/231 V, 50 Hz, driven by a DC motor 17 kW. The synchronous generator data are given in the Appendix II. The DC motor was supplied from a SIMOREG compact converter which included speed regulation circuits. The excitation of the generator was supplied from an independent source of constant voltage.

![Fig.1 Illustration of laboratory model:](image)

1-tachogenerator; 2-DC motor; 3-flexible joint; 4-flange 1; 5-flange 2; 6-synchronous generator; 7-torque transducer; 8-base

The generator and the DC motor were connected at opposite sides of a torque transducer T 30 FN with torque rated at 2000 Nm. Measurements were taken using a 16-channel high performance data acquisition card PCL-818 controlled by the "Asystant" software package on a personal computer. Current signals were taken by current sensors, and the speed at the drive side was measured by a tachogenerator. In addition, all electrical values were also measured using classical measurement instruments.

Figure 1 shows the laboratory model with a torque transducer. On the drive side a flexible joint is implemented. With the implementation of a flexible joint torsional natural frequencies similar to those of large turbogenerator sets were obtained.

6. RESULTS OF MEASUREMENTS AND COMPARISON WITH CALCULATED RESULTS

A number of measurements and calculations have been carried out for different ratios of excitation and loading conditions at which unbalanced short circuits were applied. All measurements have been carried out at the isolated generator. The results of measurements are compared to the calculated results. All presented results refer to the unbalanced short circuits of the nominally excited generator from no-load conditions so they can be compared to the analytical expressions regularly relating to the short circuits from no-load conditions.

6.1. Two-phase short circuit

Figures 2 to 4 show the calculated time response of electrical torque, and measured and calculated time responses of torsional torque for two-phase short circuit.

![Fig.2 Electrical torque at two-phase short circuit - calculated](image)

![Fig.3 Torsional torque at two-phase short circuit - calculated](image)

![Fig.4 Torsional torque at two-phase short circuit - measured](image)
6.2. Phase-to-neutral short circuit

Figures 5 to 7 show the calculated time response of electrical air-gap torque, and measured and calculated time responses of torsional torque for phase-to-neutral short circuit.

Fig. 5  Electrical torque at phase-to-neutral short circuit  
- calculated

![Graph](image1)

Fig. 6  Torsional torque at phase-to-neutral short circuit  
- calculated

![Graph](image2)

Fig. 7  Torsional torque at phase-to-neutral short circuit  
- measured

![Graph](image3)

6.3. Two-phase-to-neutral short circuit

Figures 8 to 10 show the calculated time response of electrical air-gap torque, and measured and calculated time responses of torsional torque for two-phase-to-neutral short circuit.

Fig. 8  Electrical torque at two-phase-to-neutral short circuit  
- calculated

![Graph](image4)

Fig. 9  Torsional torque at two-phase-to-neutral short circuit  
- calculated

![Graph](image5)

Fig. 10  Torsional torque at two-phase-to-neutral short circuit  
- measured

![Graph](image6)
6.4. Comparison of the results

From the presented time responses it can be seen that the natural frequency of the mechanical system of the laboratory turbine-generator set is approximately 33 Hz. This is the frequency of oscillations that occurs at step change of the external excitation torque.

Table 1 Comparison of calculated and measured maximum current

<table>
<thead>
<tr>
<th>Short circuit type</th>
<th>Compl. math. model (pu)</th>
<th>Analytical expression</th>
<th>Measurement (pu)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Three-phase</td>
<td>10.05</td>
<td>849</td>
<td>16.6</td>
</tr>
<tr>
<td>Two-phase</td>
<td>9.6</td>
<td>716</td>
<td>14.0</td>
</tr>
<tr>
<td>Phase-to-neutral</td>
<td>13.2</td>
<td>827</td>
<td>16.2</td>
</tr>
<tr>
<td>Two-phase-to-neutral</td>
<td>11.9</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 2 Comparison of electrical torques calculated by complete mathematical model and by analytical expressions

<table>
<thead>
<tr>
<th>Short circuit type</th>
<th>Compl. math. model</th>
<th>Analytical expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>Three-phase</td>
<td>7.01</td>
<td>9.24</td>
</tr>
<tr>
<td>Two-phase</td>
<td>9.59</td>
<td>11.7</td>
</tr>
<tr>
<td>Phase-to-neutral</td>
<td>7.44</td>
<td>9.55</td>
</tr>
<tr>
<td>Two-phase-two-neutral</td>
<td>7.96</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 3 Comparison of calculated and measured torsional torques

<table>
<thead>
<tr>
<th>Short circuit type</th>
<th>Compl. math. model (pu)</th>
<th>Measurement (Nm)</th>
<th>Measurement (pu)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Three-phase</td>
<td>3.73</td>
<td>712</td>
<td>4.47</td>
</tr>
<tr>
<td>Two-phase</td>
<td>3.50</td>
<td>641</td>
<td>4.03</td>
</tr>
<tr>
<td>Phase-to-neutral</td>
<td>2.63</td>
<td>497</td>
<td>3.12</td>
</tr>
<tr>
<td>Two-phase-to-neutral</td>
<td>3.49</td>
<td>599</td>
<td>3.76</td>
</tr>
</tbody>
</table>

Table 1 shows that measured peak values of the armature currents and the ones calculated by complete mathematical model are in good agreement. Values calculated by simplified analytical expressions (1), (2), (3) and (4) are 30 to 50% larger than measured values. The main reason for the discrepancy is that a relatively small synchronous generator whose electrical time constants are small has been examined, so in expression (4) a factor significantly smaller than 1.8 should be used.

Table 3 shows that the calculated torsional torques are somewhat smaller than the measured ones: 16% at three-phase short circuit, 18.6% at two-phase short circuit, 15.7% at phase to neutral short circuit and 7.2% at two-phase-to-neutral short circuit.

7. CONCLUSION

The developed mathematical model makes it possible to calculate the electrical and torsional torques of large turbogenerator sets during transient phenomena after different electrical faults. The presented examples of time responses after short circuits and maximum results displayed in the tables show that the largest torsional strains occur at three-phase short circuit, somewhat smaller at two-phase short circuit and the smallest at phase-to-neutral short circuit. Electrical torque is however largest at two-phase short circuit and smallest at three-phase short circuit while at phase-to-neutral it is only slightly larger. The cause of these disproportions is the dominance of the 50 Hz-alternating electrical torque at three-phase short circuit which has more influence on shaft oscillations with a natural frequency of 33 Hz it being closer to 50 Hz than it is the case with the 100 Hz-alternating torque occurring at unbalanced short circuits. Regardless of the distinctions in the calculated and measured results, it can be claimed that the results of the calculations by the complete mathematical model are in good agreement with the measured results and that the mathematical model can be used for calculating torsional torques in large turbogenerator-sets.

REFERENCES

Appendix I
Mathematical modelling of unbalanced short circuits

a) Two-phase short circuit

Zero conditions for two-phase to neutral short circuit from no-load conditions in abc coordinates are:

\[ u_a = u_b \quad i_a = -i_b \quad i_c = 0 \]  

(11)

The substitution of expressions for transformation of currents and voltages from abc to dq0 system in (13) results in zero conditions in dq0 coordinates:

\[ u_d = u_q - u \cdot \cos(\gamma - \frac{\pi}{3}) - u_i \cdot \sin(\gamma - \frac{\pi}{3}) = 0 \]  

\[ i_d = i_q - i \cdot \cos(\gamma - \frac{\pi}{3}) - i_i \cdot \sin(\gamma - \frac{\pi}{3}) = 0 \]  

(12)

By the substitution of synchronous generator circuits equations and further modification of the (14), equations similar to those for three-phase short circuits are obtained:

\[ \frac{d\psi_d}{dt} + \omega \cdot \psi_d + r \cdot i_d = 0 \]  

\[ \frac{d\psi_q}{dt} + \omega \cdot \psi_q + r \cdot i_q = 0 \]  

(13)

where:

\[ \psi_d = a_2 \cdot i_d + b_2 \cdot (i_f + i_d) + c_2 \cdot i_k \]  

\[ \psi_q = a_2 \cdot i_q + e_2 \cdot (i_f + i_q) + c_2 \cdot i_k \]  

(14)

\[ a_2 = [x - y \cdot \cos(2\gamma - \frac{\pi}{3})], \quad b_2 = \frac{x_{ad}}{2} - [1 - \cos(2\gamma - \frac{\pi}{3})] \]  

\[ c_2 = \frac{x_{ad}}{2} [1 + \cos(2\gamma - \frac{\pi}{3})], \quad d_2 = \frac{x_{ad}}{2} \cdot \sin(2\gamma - \frac{\pi}{3}) \]  

\[ e_2 = -\frac{x_{ad}}{2} \cdot \sin(2\gamma - \frac{\pi}{3}), \quad x = \frac{x_{ad} + x_{qf}}{2}, \quad y = \frac{x_{ad} - x_{qf}}{2} \]  

(15)

b) Phase-to-neutral short circuit

A similar procedure as for two-phase short circuit is applied where starting from:

\[ u_a = 0 \quad i_b = i_c = 0 \]  

(16)

the following expression is obtained:

\[ u_d \cdot \cos\gamma + u_q \cdot \sin\gamma = -u_0 \]  

\[ i_d \cdot \cos\gamma + i_q \cdot \sin\gamma = 2i_0 \]  

(17)

and further modifications brings to:

\[ 3r \cdot i_{d0} + \frac{d\psi_{d0}}{dt} + \omega \cdot \psi_{d0} = 0 \]  

\[ 3r \cdot i_{q0} + \frac{d\psi_{q0}}{dt} + \omega \cdot \psi_{q0} = 0 \]  

(18)

where

\[ \psi_{d0} = a_{x0} \cdot i_{d0} + b_{x0} \cdot (i_{f0} + i_{d0}) + c_{x0} \cdot i_{k0} \]  

\[ \psi_{q0} = a_{x0} \cdot i_{q0} + d_{x0} \cdot (i_{f0} + i_{d0}) + e_{x0} \cdot i_{k0} \]  

(19)

\[ a_{x0} = x_0 + 2x_2 \cdot \cos 2\gamma \]  

\[ b_{x0} = x_{x0} \cdot (1 + \cos 2\gamma), \quad c_{x0} = x_{x0} \cdot \sin 2\gamma \]  

\[ d_{x0} = x_{x0} \cdot \sin 2\gamma, \quad e_{x0} = x_{x0} \cdot (1 - \cos 2\gamma) \]  

(20)

c) Two-phase-to-neutral short circuit

A similar procedure for two-phase to neutral starting from:

\[ u_a = 0 \quad u_b = 0 \quad i_c = 0 \]  

(21)

and consequently:

\[ u_d \cdot \cos(\gamma + 2\pi/3) + u_q \cdot \sin(\gamma + 2\pi/3) = 2u_0 \]  

\[ i_d \cdot \cos(\gamma + 2\pi/3) + i_q \cdot \sin(\gamma + 2\pi/3) = -i_0 \]  

(22)

brings to:

\[ 3r \cdot i_{d0} + p \cdot \psi_{d0} + \omega \cdot \psi_{q0} = 0 \]  

\[ 3r \cdot i_{q0} + p \cdot \psi_{q0} + \omega \cdot \psi_{d0} = 0 \]  

(23)

where

\[ \psi_{d0} = a_{x0} \cdot i_{d0} + b_{x0} \cdot (i_{f0} + i_{d0}) + c_{x0} \cdot i_{k0} \]  

\[ \psi_{q0} = a_{x0} \cdot i_{q0} + d_{x0} \cdot (i_{f0} + i_{d0}) + e_{x0} \cdot i_{k0} \]  

(24)

\[ a_{x0} = 2x_0 + 2x_2 \cdot \cos 2\gamma \]  

\[ b_{x0} = x_{x0} \cdot (1 + \cos 2\gamma), \quad c_{x0} = x_{x0} \cdot \sin 2\gamma \]  

\[ d_{x0} = x_{x0} \cdot \sin 2\gamma, \quad e_{x0} = (1 - \cos 2\gamma) x_{x0} \]  

(25)

Appendix II
Generator data

\[ \begin{array}{ll}
S_b = 25 \text{ kVA} & x_0 = 2.05 \text{ pu} \\
U_b = 400 \text{ V} & x_q = 0.808 \text{ pu} \\
I_b = 36.1 \text{ A} & x_f = 0.172 \text{ pu} \\
\phi_r = 0.8 & x_t = 0.1082 \text{ pu} \\
t_0 = 1500 \text{rev/min} & T_{s0} = 21.7 \text{ s} \\
f = 50 \text{ Hz} & T_s = 0.0072 \text{ s} \\
T_{\text{me}} = 0.341 \text{ s} & T_{\text{m0}} = 0.283 \text{ s} \\
C_{\text{SGT}} = 1193.8 \text{ pu} & C_{TT} = 23.3 \text{ pu} \\
D_{\text{SGT}} = 0.025 \text{ pu} & D_{TT} = 7 \text{ pu} \\
\end{array} \]