GEOMETRICAL TRANSFORMER MODEL INCLUDING HYSTERESIS

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ABSTRACT

Modelling of the hysteresis effect of transformer magnetic core material is presented in this paper. The hysteresis model is incorporated into a three-phase transformer model which takes also saturation effect into consideration and is validated via the simulation of inrush currents during energisation, where hysteresis plays an important role.

Key-words: three-phase transformer modelling, hysteresis effect, inrush currents

1. INTRODUCTION

Accurate transformer modelling for electromagnetic transient studies has been one of the most difficult problems, researchers have been dealing with, the major difficulty being the accurate representation of the combination of all three nonlinear phenomena characterising the transformer core magnetic material, i.e. [1]:

- saturation
- hysteresis
- eddy currents

Concerning the hysteresis modelling, several efforts have been made, focusing either on the physical treatment of the phenomenon, or on the macroscopic observation of it. The former dominates in magnetic material behaviour studies [2-6], whereas the latter particularly characterises transformer modelling as a power system component [7-12]. Preisach hysteresis model [2-5] is considered an accurate representation of the phenomenon introducing, however, considerable complicity and also demanding many data not easily available. Up to date, few efforts to simplify the model so that it demands less data, while maintaining its accuracy, have been presented [3,6]. Furthermore, it is only recently that some efforts of implementing the original or an altered version of Preisach hysteresis model into single-phase transformer modelling have been published [9]. However, to the best of the authors' knowledge, no equivalent effort in the three-phase transformer modelling has been made.

In this paper, a simplified Preisach but accurate hysteresis module is developed. This module is incorporated in the geometrical transformer model (GMTRAN), a three-phase transformer model based on the analysis of its magnetic core circuit [13,14]. The resultant transformer model, the main part of which is an inductance matrix, [L], representing the mutual phase coupling, depends on the B-H hysteresis loop, which is common for the entire core, instead of the λ-i curves. The latter vary from phase to phase, due to the asymmetrical nature of the 3-phase core magnetic circuit, as this is reflected in the excitation currents. [13,14]

The three-phase transformer model developed is used in the simulation of transformer energisation characterised by transient inrush currents.

2. MODIFIED PREISACH MODEL

The hysteresis module presented in this paper is based on the well-known Preisach hysteresis model [2-5]. More specifically, the core material is considered to be composed of a very large number of dipoles. Each dipole has a rectangular hysteresis loop corresponding to only two magnetisation states, i.e. positive and negative saturation (Figure 1). The resultant magnetisation of the entire material is obtained from the status of the majority of dipoles.

Figure 1. Hysteresis loop of an elementary dipole
In particular, the input-output relationship between $H$ and $B$ is defined as \([2-5]\):

$$ B = \int_{a,b} \varphi(a,b) \gamma(a,b) \, da \, db $$

(1)

where, $\varphi(a,b)$ is a weight function being non-zero within the limits of the major hysteresis loop which corresponds to an isosceles right-angled triangle. (Figure 2). Axis $a$ corresponds to increments of $H$, whereas $b$ to decrements of $H$. The triangle is symmetrical with respect to the axis $a=b$, corresponding to the symmetry of the major hysteresis loop with respect to the origin. This means that $\varphi(-a,-b)=\varphi(a,b)$. Operator $\gamma(a,b)$ is equal to $+1$ if it corresponds to an infinitesimal area $dadb$ of dipoles positively magnetised, while it is equal to $-1$ in case of an area negatively magnetised.

![Figure 2. Preisach triangle corresponding to major hysteresis loop](image)

A considerable simplification is accomplished by assuming the weight function $\varphi(a,b)$ to be \([3,6]\):

$$ \varphi(a,b) = \varphi_1(a) \varphi_2(b) $$

(2)

This simplification reduces the number of data required to the major hysteresis loop and the initial magnetisation curve. Thus, the separate weight functions $\varphi_1(a)$ and $\varphi_2(b)$ are determined through an iterative preprocessing (off-line) procedure using the normalised derivatives of the curves mentioned above\([3,6]\). Moreover, every operating point can be estimated by using only the last operating point and the last reversing point, i.e. a point where the derivative $dB/dH$ changes sign. The total error introduced is proven to be satisfactorily small, especially regarding the area included by a minor loop and therefore the corresponding losses\([6]\).

Furthermore, the method described so far can be used to determine the magnetic flux density corresponding to a given magnetising force. However, in transformer modelling studies it is the voltage, or equivalently the magnetic flux density that is applied to it while the magnetising force is its response to this excitation. Hence, the procedure described is reversed by the application of an iterative algorithm according to which a magnetising force corresponding to the magnetic flux density applied is repetitively sought for.

### 3. THREE PHASE TRANSFORMER MODELLING

The transformer model in which the hysteresis module is incorporated, is obtained from the analysis of the core magnetic circuit \([13-14]\). In this way, an inductance matrix $[L]$ is formed taking inherently into account the non-symmetrical mutual phase coupling amongst the windings, under positive- and zero-sequence conditions. In addition, the non-linear behaviour of the transformer core due to saturation is represented as described in \([14]\). Moreover, winding resistances as well as interwinding and stray capacitances are separately included in the model \([13-14]\).

The hysteresis module described above is applied for the calculation of the operating point on the $B$-$H$ plane of every limb of the core magnetic circuit. Figure 3.

![Figure 3. Magnetic flux densities in 3-leg core case](image)

More specifically the set of equations of a transformer can be written as follows:

$$ \frac{d}{dt} \lambda = y - [R] \lambda $$

$$ \lambda = [L] \dot{y} $$

$$ [L] = [L(\mu(H(B(\lambda)))]) $$

where, from the hysteresis module, the magnetising forces are calculated and hence the operating permeabilities of all the core limbs. In this way, the inductance matrix can be calculated and finally the magnetising currents can be updated for the time step considered.

The hysteresis losses depend on the area included within the hysteresis loop trajectory. Whenever a reversing point is identified, i.e. when a new hysteresis loop is introduced the corresponding losses...
have to be updated. The hysteresis losses are represented by non-linear resistances, the value of which are updated according to the introduction of a new reversing point.

There are several methods used to solve this system of equations which can be stiff [11,14]. In this work, the fourth-order Runge-Kutta method with variable step is applied yielding satisfactorily good results.

Concerning initial conditions, due to the non-linearity, they are difficult enough to be accurately determined without resorting to the trial and error approach.

The B-H characteristic curve of an actual magnetic core often exceeds the limits of the major hysteresis loop and becomes a single line.

In case of a point exceeding the major hysteresis loop a saturation module is activated without altering the last reversing point information. The calculation of the magnetising force corresponding to the magnetic flux density of each core limb is more simple as the B-H relationship is a single-valued one [14]. When the operating point enters again the restricted major loop area the hysteresis subroutine is called in turn.

4. STUDY CASE

The transformer model described above is validated by the computer simulation of the inrush current phenomenon. This phenomenon occurring during the energisation of a no-loaded transformer is characterised by a transient current absorbed by the transformer. The peak value of this no-load current can be up to several times the rated one [15-18], while the corresponding waveform comprises a wide spectrum of odd and even order harmonics, the amplitude of which decays with time [15-18]. The severity of this phenomenon, which is due to the non-linear magnetising impedance of the transformer is determined by the following parameters [15]:

- the core magnetic material characteristic curves, i.e. the B-H non-linear relationship.
- the remanence fluxes of all the phase limbs remaining stored in the core after the last disconnection of the transformer from the system.
- the moment of energisation, i.e. the relevant phase angle of the system voltage at the instant that the corresponding switch connects the transformer to system.

In this paper, the simulation is performed making use of the parameters of a 2 kVA, 220/127 V, 3-leg Ye/Y laboratory 3-phase transformer. As shown in Figure 4, the no-loaded transformer is supplied from a source with an internal impedance through a power switch.

Figure 4. System of the study case considered

In Figure 5 the B-H characteristic curves of the transformer considered, are depicted.

Figure 5. Hysteresis loop and Magnetisation curve of the study case

At first, the transformer operates in no-load conditions, absorbing the steady-state excitation current. At 42 ms, the switches disconnect the transformer. However, every phase is electrically interrupted when the instantaneous value of its current passes through zero. In this way, the remanent fluxes are determined more accurately than in [15-17]. On the other hand, no arc phenomena in switch are considered. The interval during which the transformer remains disconnected often varies from few minutes up to several weeks. For practical reasons, in this case, at 254 ms, the switches are forced to reconnect the transformer to the network.

5. DISCUSSION

The most interesting waveforms of the simulated phenomenon are depicted in Figures 6-8, i.e. the winding linkage fluxes, the currents absorbed and the winding magnetising forces respectively. A qualitative explanation of these results follows:

Up to 42 ms, the transformer operates in steady-state. The corresponding excitation currents, Figure 7, differ from phase to phase. More specifically, the current of the two outer limbs are almost the same but
Figure 6. Linkage fluxes of the three transformer primary windings

Figure 7. Excitation currents of the transformer

Figure 8. Magnetising forces of the primary transformer windings

Figure 9. (i) Linkage flux, (ii) Excitation current and (iii) Magnetising force of phase “a”
(only saturation-no hysteresis)
differ from that of the centre limb in regards of the harmonics included and the waveform amplitude. Obviously, this is due to the asymmetrical nature of the core magnetic circuit [14].

As it is mentioned above, the three phases are not interrupted simultaneously. Thus, phase “b” clears first at 43.8 ms, i.e. 1.8 ms after the mechanical disconnection, followed by phase “a” at 44.8 ms and finally by phase “c” at 50.0 ms. It is worth noting that when two phases have been interrupted, current circulates through the third phase and earth. When the transformer is interrupted, i.e. the excitation currents equal zero in Figure 7, the energy stored in the windings is slightly dissipated due to the core losses. Hence, the remanent fluxes do not have a constant value, while the magnetising forces do not turn immediately to zero. As shown in Figures 6 and 8 linkage fluxes fluctuate around the remanent flux value, while the magnetising forces fluctuate around values almost zero.

When the transformer is reconnected to the network, the remanent flux at this instant acts as a DC offset to the sinusoidal linkage flux waveform. This DC component may drive the material deep into the saturation region, depending on the conditions mentioned above. Hence, the excitation currents are considerably increased as already known [15-17], while as it can be seen in Figure 7 they do not decay at a constant rate. It has been shown, that the hysteresis loops do not play a critical role to the evolution of the inrush current phenomenon [12]. However, the DC component, which depends on the initial conditions can not be accurately estimated unless the dynamic hysteresis effect is taken into consideration. Otherwise, the remanent fluxes have to be assumed [15-17]. In case, that only the saturation effect is included, as shown indicatively for phase “a”, in Figure 9, the remanent flux remains completely constant. The excitation currents do not differ considerably from those of Figure 7 where both hysteresis and saturation are included, which agrees with the conclusion of [12]. However, the magnetising forces are not driven to zero, which is due to the disregard of the hysteresis loop. In addition, as saturation effect is not related to any losses, the latter are not represented in any way.

6. CONCLUSIONS

In this paper a three-phase transformer model including hysteresis and saturation is presented. The hysteresis module presented is based on an simplified but fairly accurate version of the well-known Preisach hysteresis model and can easily be combined with saturation representation. The resultant transformer model is tested in simulating transformer energisation. The simulated results are very reasonable and in general more reliable than those provided by a model including only saturation.

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