A NEW EMTP TRANSFORMER MODEL BASED ON MODAL ANALYSIS

D.J. Wilcox **D.J. Leonard**

University College Galway
Faculty of Engineering
Galway, Ireland

ABSTRACT

The paper describes a new form of EMTP-compatible transformer model based on MODAL analysis. The MODAL model is shown to be conceptually different from the form of model proposed be Degeneff and to have much more in common with Wedepohl's MODAL transmission line theory. Given results, comparing predicted results with oscillographic records for a test transformer, show that the time-domain modal model is capable of accurately accounting for the frequency-dependent effects of practical transformers with high computational efficiency.

KEYWORDS
Transformers, Modal Analysis, EMTP, Transients

1 INTRODUCTION

Up to the present, EMTPs have been deficient in not incorporating a transformer model capable of accurately representing the behaviour of transformers at the high frequencies generally associated with electromagnetic transient phenomena (say from 200 Hz to 1 MHz). This is clearly a serious deficiency given the major role of transformers in any power network. In particular, it is not possible with present EMTPs to investigate with any confidence whether certain operating conditions, or the inception of certain network faults, could give rise to hazardous transient overvoltages within a transformer. Nor is it possible to properly account for transient interactions between parts of a system operating at different voltage levels.

The difficulty in establishing an adequate transformer model for EMTP implementation has been two-fold:

- Firstly, magnetic couplings within a transformer vary with frequency in a rather complex way as a result of eddy currents induced in the core. For example, at high frequencies, magnetic couplings are relatively weak since flux is unable to penetrate deeply into the core. Also, the eddy currents give rise to frequency-dependent damping effects which must be accurately represented if the magnitude of transient oscillations is to be accurately predicted. Hitherto, no one has been able to include proper account of frequency-dependent self and mutual inductances, and associated frequency-dependent resistances, in any time-domain model. The paper explains how this difficulty may be overcome, without compromise to accuracy, by the method of modal analysis.

- Secondly, the conventional approach of NODAL analysis [1] lacks sophistication and leads to a time-domain model whose structure is too inefficient to allow fast computation. This second difficulty is also overcome, as explained in the paper, by employing MODAL (as opposed to NODAL) analysis. This vastly increases computational efficiency in the same way that the FFT algorithim, by capitalizing on inherent structural features, is able to vastly increase the speed of calculations compared with direct implementation of the DFT algorithim.

In fact, the concept of MODAL analysis applied to transformers is similar to that routinely used in the representation of polyphase transmission lines for EMTP implementation. The advantage of MODAL analysis in the representation of transformers is however very much greater than for transmission-line representation inasmuch as a transformer is far more complex.

2 REVIEW OF THE THEORY OF MODAL ANALYSIS

In the case of polyphase overhead transmission lines [2], linear transformation by an eigenvector matrix Q (say) converts the following pair of (matrix-vector) simultaneous equations (the multiconductor Telegraphers' equations) :

$$\frac{dV}{dx} = -ZI \quad \text{and} \quad \frac{dI}{dx} = -YV \tag{1}$$

into the equations of n completely independent single-phase lines. Each of these is represented separately, in the time domain, for EMTP implementation. Interactions between phase and modal quantities are transacted by the MODAL transformation matrix Q (and its inverse). Success of the modal method rests in the fact that, for all practical purposes, the transformation matrix Q turns out to be independent of frequency and is therefore directly applicable in the time-domain.

In the case of transformers, behaviour is governed by the following pair of (matrix-vector) simultaneous integro-differential equations:

$$\frac{dV}{dx} = - \int_0^t Z(x,\tau)\, I(\tau)\; d\tau$$

and (2)

$$\frac{dI}{dx} = - \int_0^t Y(x,\tau)\, V(\tau)\; d\tau$$

where, as in equation (1), voltages and currents are represented by their Laplace transforms. If the transformer has N windings, then V and I will be vectors of dimension N. Correspondingly, $Z(x,\tau)$ and $Y(x,\tau)$ will be matrices of dimension $N*N$. Unlike the case of polyphase transmission lines (equation (1)), where Z and Y are independent of the longitudinal variable x, Z and Y featuring in equation (2) are very much dependent on the longitudinal variable. This makes their solution very much more difficult than the solution of the Telegraphers' equations. So much so, that no analytic solution has ever been found for equation (2) for a general case.

If rather severe approximations are made, then, as reported by Pirenne [3], it is possible to render the transformer equations soluble - leading to the classical results of Wagner [4], Rudenburg [5], Blume and Boyajian [6]. Unfortunately, the extent of the approximations required by each of the different methods is so great as to render these classical solutions to be of little value from a practical, quantitative viewpoint. Nevertheless, classical studies have proved valuable in providing qualitative insight into the general nature of transient phenomena in transformers

Recent analytical work involving one of the present authors [7,8,9] has shown that it is possible to solve the transformer equations, i.e. solve equation (2), to any required degree of accuracy using a subtle numerical approach. The proposed solution is applicable to all transformers, irrespective of the number of windings and irrespective of any frequency-dependent parameters. The only restriction is that the method is not able to take account of nonlinear phenomena, such as magnetic hysteresis. Fortunately, such nonlinear phenomena are generally considered to have insignificant effects at the high frequencies associated with electomagnetic transient phenomena.

Because frequency-dependent parameters are involved, MODAL solution of the transformer equations takes place in the frequency domain, leading to a frequency-domain transformer model referred to henceforth as the frequency-domain prototype. The task is then to convert this frequency-domain prototype into a time-domain counterpart with a view to EMTP implementation.

This process is no different from that of modelling overhead transmission lines for EMTP implementation. A model is first established in the frequency domain (generally involving frequency-dependent mutual couplings and losses) and a time-domain counterpart is then sought by one of a number of well-established approaches.

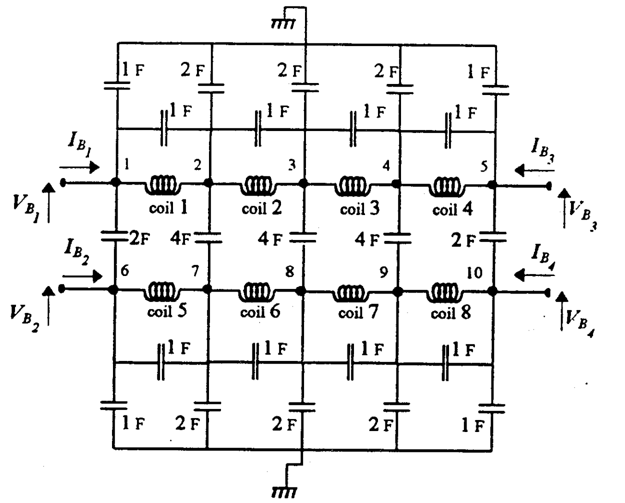


Fig.1 : *Simple transformer representation*

To highlight the basics of transformer modelling, Fig.1 shows a discrete representation of an illustrative 2-winding transformer. The fact that there are only two windings, and that the discretisation is course, is of no significance so far as the present general discussion is concerned. All that matters is that Fig.1 has the structural form of a general transformer representation. Note that there is no need for the capacitance to be uniformly distributed as it is in Fig.1, or for certain groups of capacitance to have the same value.

In general, all self inductances will be frequency dependent, generally reducing with frequency. Likewise, all mutual inductances, such as L_{14} (the mutual inductance between coil 1 and coil 4 in the figure), will generally be frequency-dependent in any practical transformer. The presence of longitudinal mutual inductances, such as L_{13}, in addition to lateral mutual inductances such as L_{15}, is the main distinguishing feature between transformers and multiconductor transmission lines.

In Degeneff's method of analysis [1], viz. NODAL analysis, currents are injected at each node to form an admittance equation which takes account of the self and mutual inductances involved, as well as of the admittances owing to the capacitance network. This approach is straightforward but suffers from one major drawback. This is that if frequency-dependent parameters are to be properly represented, there is no established method for accurately translating the admittance equation obtained in the frequency domain into the time domain.

Although mathematically equivalent to Degeneff's admittance formulation, the method of MODAL analysis [7,8,9] sets up a completely different structure. Instead of injecting currents at the nodes, (e.g. nodes 1 to 10 in Fig.1), equations are set up in the form of a discrete approximation to equation (2). This gives a set of equations of the form :

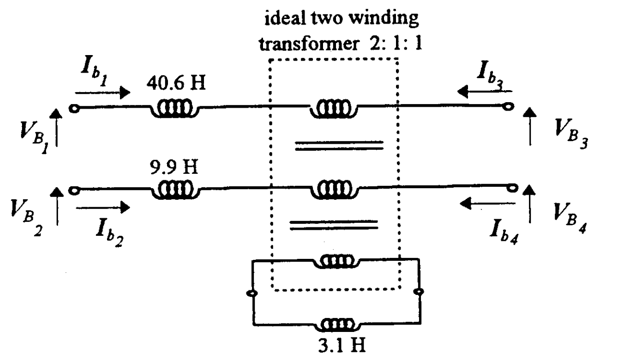
$$V(k) - V(k-1) = \sum_{m} Z(k,m)\, I(m)$$

$$I(k) - I(k-1) = \sum_{m} Y(k,m)\, V(m)$$

In MODAL analysis these equations are organised, as described in reference [7], to give a MODAL voltage equation. The advantage of the MODAL formulation is that simple linear transformations reduce an original representation such as that in Fig.1 to a set of decoupled resonant circuits.

Taking the case of the illustrative representation shown in Fig.1, with additional data as given in the appendix, MODAL analysis [7,8] reduces this original representation to a set of six decoupled resonant circuits, plus one purely capacitive circuit and one purely inductive circuit (inclusive of an ideal transformer). These circuits are shown in Fig. 2 and represent the transformer via the block diagram shown in Fig.3, where V_B is a vector representing the boundary voltages (i.e. the voltages at the winding terminals) and I_B represents the boundary currents. In the case of Fig.1:

$$V_B = \begin{bmatrix} V_S \\ V_R \end{bmatrix} = \begin{bmatrix} V_{B1} \\ V_{B2} \\ V_{B3} \\ V_{B4} \end{bmatrix} \quad \text{and} \quad I_B = \begin{bmatrix} I_S \\ I_R \end{bmatrix} = \begin{bmatrix} I_{B1} \\ I_{B2} \\ I_{B3} \\ I_{B4} \end{bmatrix}$$



(a)

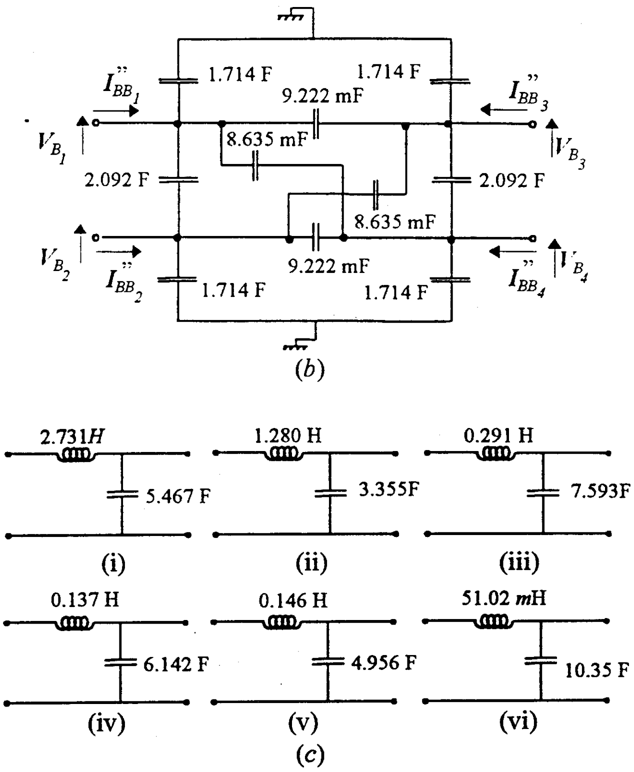


Fig.2 : *Circuit representation of MODAL components*
(a) Y_b *(b)* Y''_{BB} *(c)* Core

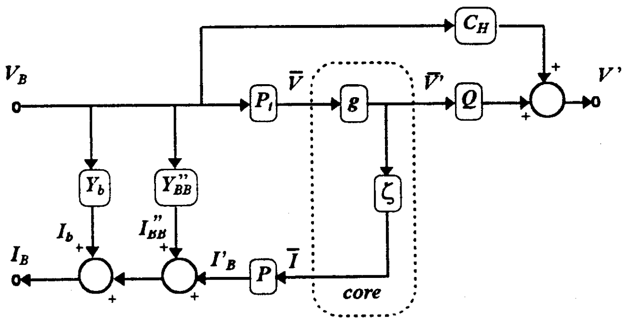


Fig.3: *The MODAL model*

The voltages at the internal nodes are the elements of the vector V'. For the particular representation shown in Fig.1

$$V' = \begin{bmatrix} V_2 \\ \vdots \\ V_4 \\ V_7 \\ \vdots \\ V_9 \end{bmatrix}$$

At this stage, Fig.3 is in the frequency domain (the frequency domain prototype). From Fig.3,

$$I_B = \{Y_b + Y''_{BB} + P\zeta g P_t\} V_B \qquad (3)$$

which is evidently an admittance equation. This equation describes the behaviour of the transformer seen as a black box with $2N$ external terminals (2 terminals for each of the N windings). If some windings are in fact grounded, connected in Δ, etc. then such connections are understood to be made externally by applying appropriate boundary conditions to equation (3).

Voltages at the internal nodes inside the box are given by

$$V' = \{ Q\, g\, P_t + C_H \} V_B \qquad (4)$$

i.e. the MODAL model is able to calculate these from the boundary voltages once the latter have been found by solving the admittance equation.

In the above illustrative case (Fig.1, data as given), all the transformer parameters are independent of frequency and all losses have been neglected. The upshot is that Y_b is purely inductive (its effects represented by the circuit of Fig.2(a)), Y''_{BB} is purely capacitive [Fig.2(b)]. Most importantly, for ANY transformer subject to the above restrictions, the transformation matrices P and P_t, and the distribution matrix Q, and C_H are all purely real and independent of frequency.

3 TIME-DOMAIN MODAL MODELLING

The matrices P, Q and C_H, for the illustrative case of Fig.1 (with data as given) are, as noted above, purely real and completely independent of frequency. In fact, caculations give :

$$P = \begin{bmatrix} -0.8358 & -0.2095 & -0.2795 & -0.1050 & 0.0781 & 0.0753 \\ 0.4831 & 0.8983 & 0.1675 & 0.0617 & -0.2857 & -0.0910 \\ -0.8358 & -0.2095 & 0.2795 & -0.1050 & -0.0781 & 0.0753 \\ 0.4831 & 0.8983 & -0.1675 & 0.0617 & 0.2857 & -0.0910 \end{bmatrix}$$

$$Q = \begin{bmatrix} -0.4869 & 0.2435 & -0.7062 & -0.4360 & -0.2959 & 0.0759 \\ -0.7112 & 0.2967 & 0.0000 & 0.6367 & 0.0000 & -0.0608 \\ -0.4869 & 0.2435 & 0.7062 & -0.4360 & 0.2959 & 0.0759 \\ -0.0732 & 0.4575 & -0.0361 & -0.1991 & -0.6422 & -0.4851 \\ -0.0965 & 0.6122 & 0.0000 & 0.3677 & -0.0000 & 0.7171 \\ -0.0732 & 0.4575 & 0.0361 & -0.1991 & 0.6422 & -0.4851 \end{bmatrix}$$

and

$$C_H = \begin{bmatrix} 0.17589 & 0.09197 & 0.00922 & 0.00864 \\ 0.03924 & 0.03219 & 0.03924 & 0.03219 \\ 0.00922 & 0.00864 & 0.17589 & 0.09197 \\ 0.09197 & 0.17589 & 0.00864 & 0.00922 \\ 0.03219 & 0.03924 & 0.03219 & 0.03924 \\ 0.00864 & 0.00922 & 0.09197 & 0.17589 \end{bmatrix}$$

In this case (with reference to Fig.3) the MODAL model can be implemented directly in the time domain as follows. If the boundary voltages and currents are known from previous calculations at time $t = t$, then the three components of equation (3) convert to the time domain as follows. The first part, $I_b = Y_b\ V_B$ converts to the time domain as

$$i_b(t+\Delta t) = G_b\ v_B(t+\Delta t) + i_{b\mathrm{hist}}(t) \qquad (5)$$

which is simply an EMTP implementation of the circuit of Fig.2(a). The capacitive network of Fig.2(b) is similarly implemented directly by the EMTP.

The third component, $I'_B = P\,\zeta\, g\, P_t\ V_B$, may be incorporated into EMTP form by a process of MODAL decomposition similar to that used in the representation of polyphase transmission lines. The boundary voltages in the time domain convert to a set of MODAL voltages by the transformation

$$\bar{v}(t) = P_t\, v_B(t) \qquad (6)$$

Note that this equation originated in the frequency domain as $\bar{V} = P_t\, V_B$ but converts directly into the time domain by virtue of the fact that P_t is purely real and completely independent of frequency.

The action of the diagonal matix g, corresponds to applying each modal voltage to its corresponding MODAL network (see Fig.2(c)). E.g. the second MODAL voltage (second element of $\bar{v}(t)$) is applied to the circuit specified by Fig. 2(c)(ii). The action of the modal circuit can be modelled by the EMTP in the usual way. Boundary currents are obtained from the modal currents (the currents flowing in the modal circuits) by applying the transformation.

$$i'_B(t) = P\,\bar{i}(t) \qquad (7)$$

This process leads to a standard time-domain equation for representing the component $I'_B = P\,\zeta\, g\, P_t\ V_B$. Viz.

$$i'_B(t+\Delta t) = G'_B\, v_B(t+\Delta t) + I'_{B\mathrm{hist}}(t) \qquad (8)$$

as detailed in reference [10].

This completes conversion of the frequency domain prototype into the time domain for the illustrative case.

Once the overall admittance equation has been solved, the voltages at the internal nodes are determined by

$$v'(t+\Delta t) = Q\,\bar{v}'(t+\Delta t) + C_H\, v_B(t+\Delta t) \qquad (9)$$

where the vector $\bar{v}'(t+\Delta t)$ is obtained as detailed in reference [10].

4 MODELLING OF PRACTICAL TRANSFORMERS

The procedure described in the previous section applies to all lossless transformers with frequency-independent parameters. In practice, of course, transformers are not lossless nor do they have frequency-independent parameters. However, it turns out that very little modification is required to fully account for the realities of practical transformers.

To demonstrate this, Fig.4 shows a set of test windings on a 25kVA core. The eight inner sections are connected in series to form one winding and the eight outer sections are connected in series to form a second winding. Full details of this test set-up are given in reference [9].

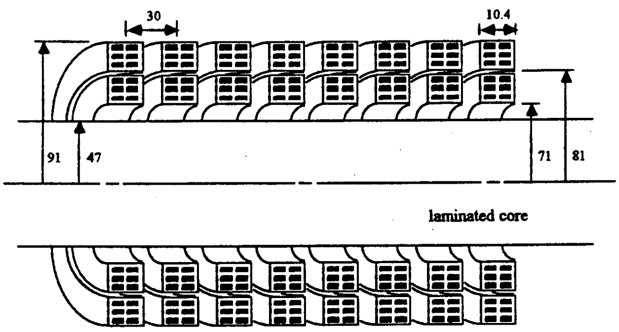


Fig. 4 : *Test windings on 25 kVA core (dimensions in mm)*

As established in reference [11], the matrices P and Q in the MODAL model (Fig.3), as well as the matrix C_H, turn out to be real and frequency-independent for all practical purposes. It is also the case that Y''_{BB} is purely capacitive if, as is normally assumed, dielectric losses in the insulation are considered to be negliable. It is also the case, under the same conditions, that the diagonal matrix ζ in the core of the model is also purely capacitive.

Thus, differences between a lossless transformer with frequency-independent parameters and a practical transformer are accounted for within the matrices Y_b and g in the MODAL model of Fig.3.

In the particularly simple case of the test windings shown in Fig.4, it turns out that the frequency dependencies associated with Y_b can be represented by simple R-L ladder networks replacing the inductive elements of Fig.2a. Further work is in progress to find an efficient general representation. Note that Y_b is purely inductive in character and so there are no complex resonances to account for in this part of the model. In fact Y_b corresponds to a full model of a transformer when all capacitive effects are supressed [10].

It now only remains to model the MODAL resonant circuits. MODAL resonant circuits in practical cases are found [11] to be different from those in Fig.2(c) by including resistance. Also, all resistances and inductances (R and L) are frequency dependent in a general case. Fortunately, it turns out that it is only necessary to represent R and L accurately in the vicinity of the resonant frequency. Fig.5 shows the amplitude responses of representative MODAL transfer functions obtained from the frequency-domain. It is seen in Fig.5 that each can be accurately modelled by a transfer function of the form

$$g_k(s) = \frac{\dfrac{1}{L_k C_k}}{s^2 + s\,\dfrac{R_k}{L_k} + \dfrac{1}{L_k C_k}} \qquad (10)$$

with

$$\zeta_k = s\,C_k$$

where the subscript k denotes the k^{th} MODAL circuit. Values for R_k, L_k and C_k are obtained by a simple least squares fit and apply to a simple RLC circuit.

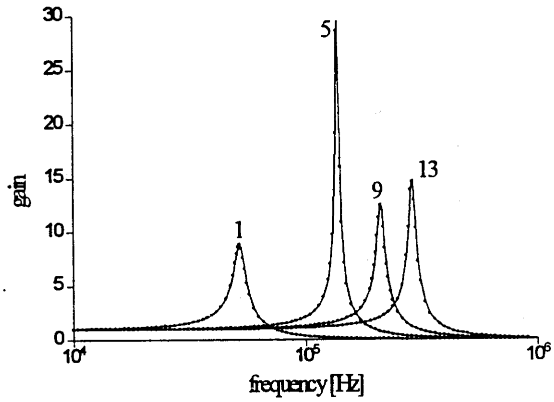


Fig.5 : *Amplitude spectra of the modal transfer functions (modes k = 1,5,9,13)*

.... sample values of exact $g_k(j\omega)$

——— least-squares approximation to eqn. (10)

5 PRACTICAL RESULTS

Fig.6a shows the unit step response halfway down the primary winding of the test transformer (Fig.4) as predicted by the time domain MODAL model for case 1 boundary conditions (the step being applied at one end of the primary with all other terminals open circuit). Fig.6b shows the corresponding test result. The accuracy is quite remarkable, showing that the time domain model is correctly representing damped resonant phenomena in a real transformer. Figs 7a and 7b give results for the quite different case 2 where the two terminals of the secondary winding are both grounded.

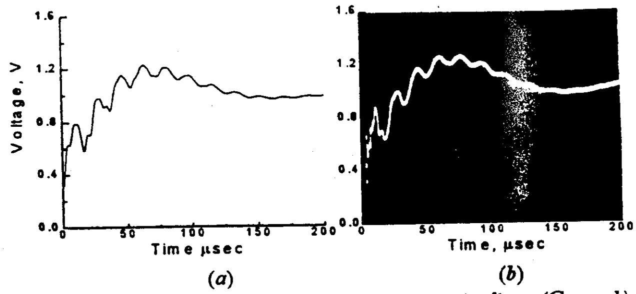


Fig.6 : *Voltage halfway down primary winding (Case: 1)*

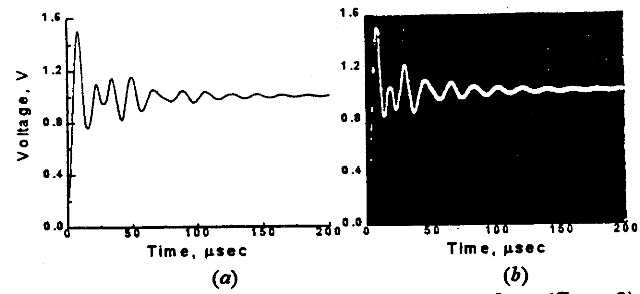


Fig.7 : *Voltage halfway down primary winding (Case: 2)*

6 CONCLUSIONS

The paper has shown that it is possible to accurately account for the frequency-dependent behaviour of practical transformers using a time-domain (EMTP-compatible) model based on MODAL analysis. The MODAL model also has the advantage of having an inherently efficient structure for computation purposes.

7 REFERENCES

[1] DEGENEFF,R.C.:'A general method for determining resonance in transformer windings', *IEEE Trans. on Power Apparatus and Systems*, 1977, Vol.PAS-96, (2), pp.423-430

[2] WEDEPOHL, L.M.: "Application of matrix methods to the solution of travelling-wave phenomena in polyphase systems ", *Proc. IEE*, 1963, 110, pp. 2200-2212

[3] PIRENNE, M.J.:'Theorie general des phenomenes oscillatoires dans les enroulements de transformateurs', *Revue Generale de l'Electricite*, 1940, (January), pp.19-29 and 53-61

[4] WAGNER, K.W.:'The process of an electromagnetic wave in a coil with capacity between turns', *Electrotechnik und Maschinenbau*, 1915, Vol.33, pp.89-107

[5] RUDENBERG, R.:'Performance of traveling waves in coils and windings', *Trans. AIEE*, 1940, Vol.59, pp. 1031-1040

[6] BLUME.L.F. and BOYAJIAN,A.:'Abnormal voltages within transformers', *AIEE Trans.*, 1919, Vol.38, pt. I, pp. 577-620

[7] WILCOX, D.J.:"Theory of transformer modelling using modal analysis", *Proc.IEE*, Vol.138, Pt.C, No.2, 1991, pp.121-128

[8] WILCOX, D.J. and McHALE, T.P.:"Modified theory of modal analysis for the modelling of multiwinding transformers", *Proc.IEE*, Vol.139, Pt.C, No.6, 1992, pp.505-512

[9] WILCOX, D.J., HURLEY, W.G., McHALE, T.P., and CONLON,M.:"Application of modified modal theory in the modelling of practical transformers", *Proc.IEE*, Vol.139, Pt.C, No.6, 1992, pp.513-520

[10] WILCOX, D.J and LEONARD, D.J.:"The nature of modal time-domain transformer models", *Proc.IEE*, (in press)

[11] WILCOX, D.J., McHALE, T.P. and LEONARD, D.J.: "Time-domain modelling of power transformers using modal analysis", *Proc.IEE*, (in press)

APPENDIX

Data for illustrative case

Self inductances :

$$L_{11} = L_{22} = L_{33} = L_{44} = 4.0 \text{ H}$$
$$L_{55} = L_{66} = L_{77} = L_{88} = 1.1 \text{ H}$$

Mutual inductances :

$$L_{12} = L_{23} = L_{34} = 3.4 \text{ H}$$
$$L_{56} = L_{67} = L_{78} = 0.9 \text{ H}$$
$$L_{13} = L_{24} = 2.9 \text{ H}$$
$$L_{57} = L_{68} = 0.7 \text{ H}$$
$$L_{14} = 2.5 \text{ H}$$
$$L_{58} = 0.5 \text{ H}$$
$$L_{15} = L_{26} = L_{37} = L_{48} = 0.6 \text{ H}$$
$$L_{25} = L_{36} = L_{47} = L_{16} = L_{27} = L_{38} = 0.4 \text{ H}$$
$$L_{17} = L_{28} = L_{35} = L_{46} = 0.3 \text{ H}$$
$$L_{18} = L_{45} = 0.1 \text{ H}$$

Note that for all mutual inductances, $L_{ij} = L_{ji}$