A comparison of frequency-dependent line models in connection with implicit segmentation schemes

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Abstract: This work analyses numeric and computational problems that appear when modeling short sections of transmission lines to reflect as accurately as possible the distributed nature of corona phenomena. A parametric study investigates the adequate selection of the section length in the context of the implicit segmentation scheme, taking into account the errors generated by the use of rational functions to synthesize the frequency response of the line parameters, and identifies the nature of numerical oscillations that are present in simulations of surge propagation under corona.

INTRODUCTION

It is well known that surge propagation in transmission lines is affected by the frequency dependence of the parameters and by corona phenomena. Due to the inherent difficulties in modeling both effects together, the most usual approach adopted in electromagnetic transients computations has been the subdivision of the line into short length segments; the non-linear "corona branches" are then connected to the section ends. In order to reflect correctly the space discretization, the sections must be made as short as possible. To reduce computer time requirements, an efficient recursive scheme for calculation of the voltage profile along the line has been successfully applied to the implementation of corona models in single- and multi-phase systems [1],[2],[3].

A critical aspect in the application of any discretization technique is the choice of section length. A compromise must be reached between the need to minimize the errors arising from the imperfect distribution of the corona phenomena and the computational cost, which will be directly dependent upon the number of line sections. In consequence, a clear understanding of the errors involved with this approach is necessary. The following sections provide an analysis of such errors and point out alternatives to reduce them.

COMPARISON OF FREQUENCY DEPENDENT LINE MODELS

Since modal analysis entails the reduction of multiphase transmission line equations to a single-phase problem for each mode, a comparison of different line models can be made on a single-phase basis. The models proposed by J. R. Marti [4] and by W.-G. Huang & A. Semlyen [5] were chosen due their common properties of providing good accuracy of simulation and relative facility of implementation. The strength of these methods arises from the efficient recursive scheme to solve convolution integrals that result from the frequency to time domain transformation of traveling wave equations [9].

Previous work on evaluation of Marti's model as applied to short line sections [6], has addressed the error build-up problem, the main source of which was identified as being the approximation of the line propagation function *A (ω)* in the frequency domain. The solution for this problem was the inclusion of extra poles and zeroes in the "flat" low frequency region. Figure 1 shows simulations of a 0.3x7μs surge propagation on a single-phase 220kV line subdivided in sections 15 and 500 meters long. It can be seen that the voltage profile is practically identical, for both the adopted lengths, which leads to the conclusion that the syntheses with the additional poles and zeroes for short sections does not introduce any detectable errors.

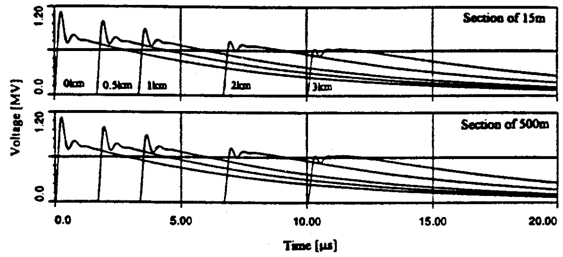

Fig. 1 - Surge propagation on Gary's line without corona: Marti's fd-line model, 15m and 500m sections.

The higher number of poles and zeroes will however increase the computational cost which is aggravated by the very large frequency spectrum, from 10^{-2} to 10^{11} Hz, this upper limit being of necessity very high for purely mathematical reasons. The synthesis of the propagation function $A(\omega)$ is very oscillatory in the high-frequency region and thus adequate fitting can only be achieved by extending the frequency range. Even though one should be aware of the limitations of Carson's formulas for ground return at such high frequencies, simulations of field tests have shown very good accuracy of representation [2]. This can be attributed to the fact that the frequency spectrum used for the fitting exceeded by far the highest and the lowest frequency contents of the surges applied in the actual tests [7].

An apparently conflicting solution to avoid excessive computational cost has been found in the observation that the "flat" region of $A(\omega)$ extends to frequencies up to 10^4 Hz; thus the fitting was modified to start from 10^4 Hz and to extend to 10^{11} Hz. Applying this procedure, a reduction from 35 to 20 poles was obtained and the results for the 220kV single-phase line are shown in figure 2. It is clear that that the results match very closely up to 3 km of propagation.

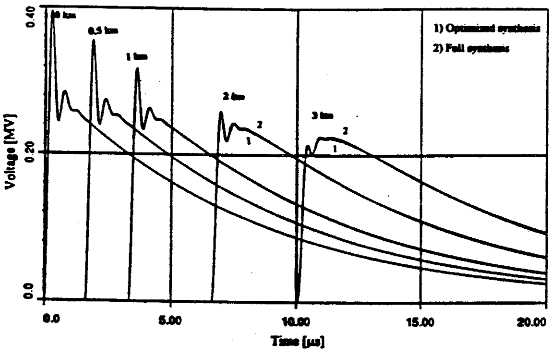

Fig. 2 - Surge propagation on Gary's line without corona: Comparison of synthesized functions of Marti's fd-line model, 15m sections.

The model proposed by Huang & Semlyen [5] represents in fact a suitable evolution of the traditional lumped **RLC** π-cascade circuit. The adopted approach overcomes the main limitations of π models such as the large storage requirements and the oscillations in the output due to **LC** loops resonant frequencies. In the Huang & Semlyen implementation, the geometric part of the longitudinal impedance is kept apart and combined with the geometric capacitance, to yield a travel delay for the line segment. The other part of the longitudinal impedance must be fitted by a cascade of parallel **RL** blocks, such as in the π model.

Figure 3 compares Marti's model and the Semlyen & Huang's model as implemented by Hamadani-Zadeh [8] in the EMTP. This model is an excellent alternative for frequency dependent line modeling of very short line sections because the synthesis of the series impedance $Z_s(\omega)$ can be performed in per unit length, which makes it possible to experiment different line sections without having to produce a new synthesis of the line functions. Another advantage, as shown in [5] is the reduction in the number of poles per section. This is achieved by spreading some lower-frequency **RL** blocks along the line, i.e. each section does not have all the blocks but only a smaller number of them. The frequency dependency is obtained when all the sections are considered together by distributing lower frequency blocks at larger intervals than higher ones.

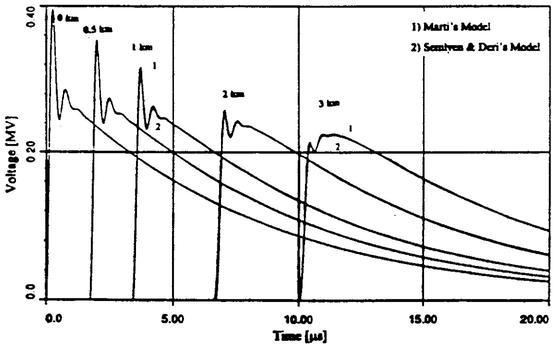

Fig. 3 - Surge propagation on Gary's line without corona. Marti's fd-line versus Semlyen & Huang's models 15m. sections.

The discussions of this section so far indicate that both Marti's and Semlyen & Huang models, are comparable both in terms of performance and of ease of implementation. The remainder of the section will consider some aspects related to multiphase systems and in this case the Marti's model will be used to represent frequency dependency, simply because the fitting routines and the corresponding input data files were already implemented in the EMTP program. This model will from now on be referred to as the "fd-line model", and the results can be expected to be almost identical if the Semlyem & Huang model was used instead.

Figure 4 shows the results of simulations and field tests performed by Gary [7], for the voltage surge on phase A of a three-phase 220kV line. Two representations were used in addition the cp-line model: the standard constant parameter line model available in the EMTP, hereafter referred to as the "cp-line" model and a so-called "mixed line" model, in which the earth mode is modeled using the fd-line model and the aerial modes are represented using the cp-line model. The pole numbers adopted for the syntheses of the earth mode for the "mixed line" are the same as for the corresponding cases of the fd-line model.

Comparing the results in Figs. 4a and 4b, it is clear that the fd-line model provides a better accuracy of representation than the cp-line model, the better performance of the former becoming more evident for longer distances of propagation. This was to be expected, since the cp-line model does not allow for the changes in the frequency spectrum as the surge propagates along the line [6]. However, these errors can be reduced by considering at least, the earth mode as frequency dependent, as shown in Fig. 4c. The "mixed line" can thus be justified when the curves shown in Fig. 5 are analyzed. This figure presents the modal voltages for the three kinds of representation.

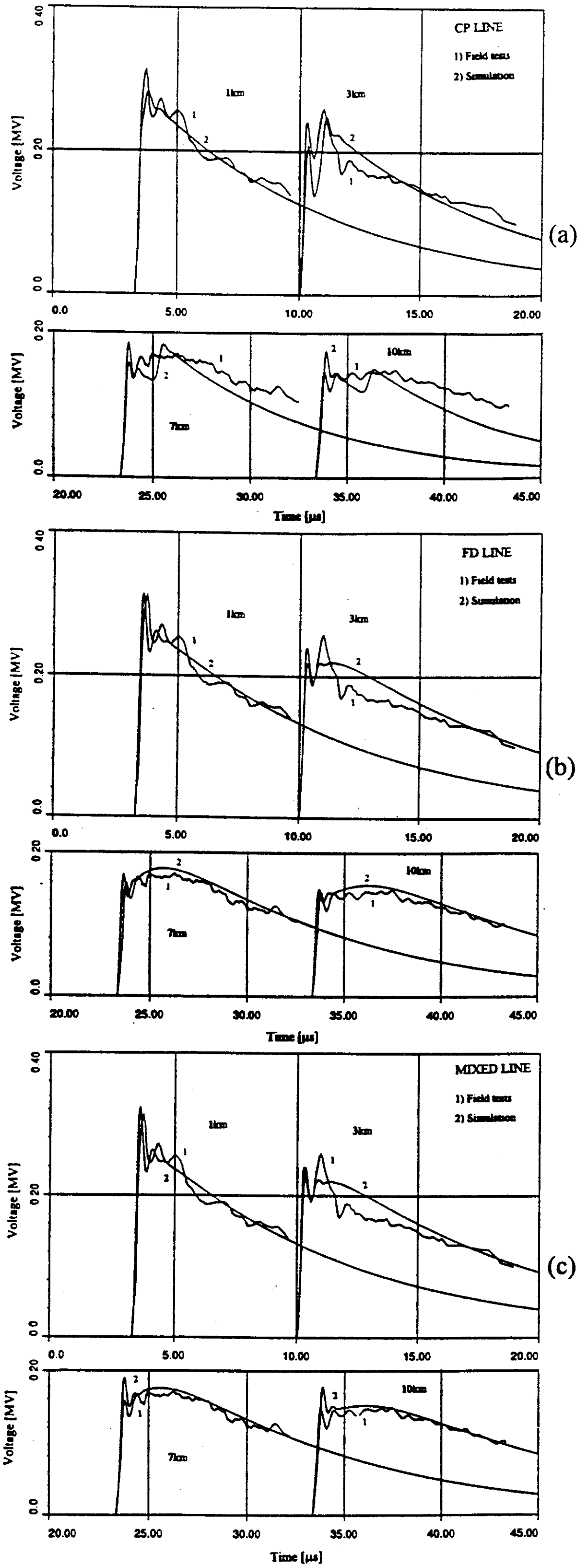

Fig. 4 - Surge propagation on Gary's line without corona. Field tests versus computer simulations.

It can be seen from fig. 5 that the biggest differences between the fd-line and the cp-line models occur in the earth mode. Table 1 shows the relative computer timings for the three representations, and it is seen that whereas the mixed line increases the CPU time by 67%, the full fd-line time requirement is almost 4-fold.

CONSTANT PARAMETERS	FREQUENCY DEPENDENT	MIXED LINE
1.00	3.99	1.67

Table 1 - Relative CPU times for different line models.

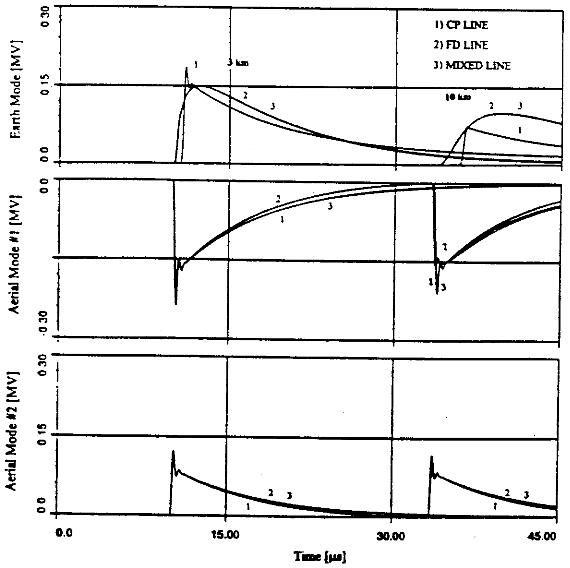

Fig. 5 - Surge propagation on Gary's line without corona: Modal components given by three alternative models.

Alternatively, a not so radical approach for modeling line modes of propagation that are less affected by earth return (aerial modes), consists in using low order approximations [13],[14] in $Z_c(\omega)$ and $A(\omega)$ synthesis. Again a significant reduction in computation time can be archived.

EFFECTS OF LINE SUBDIVISION IN SIMULATION RESULTS

The Marti's fd-line model discussed in the previous section have been interfaced with nonlinear corona models using an efficient implicit segmentation scheme described in [3]. Two corona models were selected for the simulations: a piecewise linear representation of the charge cycles and Suliciu's dynamic model [6]. The scheme was validated using Gary's measurements of surge propagation under corona on a three-phase 220kV line, and it was shown that good accuracy of representation was obtained. Considerable savings in CPU times and data preparation requirements were also achieved. It is apparent from these studies that the subdivision in short segments provide an adequate space discretization for the distributed nature of corona. However the choice of length for the line sections both in the above and previous studies was based on purely empirical considerations; different authors have adopted a wide range of values, starting from as small as 3m [15], to as long as 350m [10]. It was thus decided to investigate the nature of the errors involved in the discretization process and therefore to attempt to derive some criteria to assist in the choice of section length.

Two main sources of errors were identified: changes in the rate of rise of wave-front and high-frequency oscillations. It is relevant to stress at this stage that these errors are not caused by the line segmentation by itself. As pointed out in [11], the actual division in sections does not introduce any discontinuities in the line and therefore the line propagation characteristics are not affected. Both errors appear during calculations above the corona threshold, when the corona branches are connected to the nodes, thus introducing discontinuities in the surge impedance of the line. The procedure adopted to analyze the various aspects can be summarized in the following steps:

i. a single-phase line was divided in sections of equal length, ranging from 3m up to 60m long; for each length the fitting routines were run;
ii. the fd-line model was tested for each length using the procedure described in Fig. 1, to ensure that the model was accurate and that the actual section length was not introducing any errors or oscillations;
iii. an impedance equal to the line surge impedance at infinite frequency $Z_c(\infty)$ was inserted at the source end, to avoid the generation of multiple reflected waves;
iv. a voltage step was applied at the sending end and the responses were evaluated using the piecewise linear corona model.

Rate of rise of wave-front:

Fig. 6 shows the different responses of the line to the voltage step input. It is evident that the section length has a direct effect on the rate of rise of the wave-front, the effect becoming more pronounced when the section length exceeds 30m.

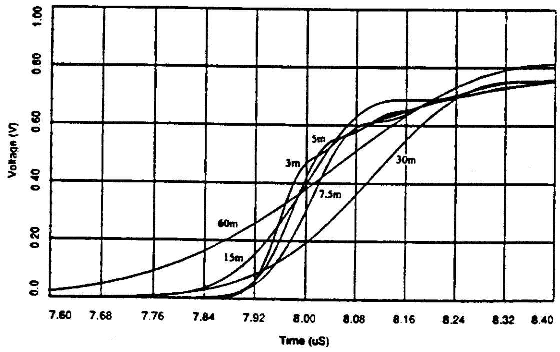

Fig. 6 - Step response under corona at 2 km from voltage source. Effect of section length on the wave-front's rate of rise.

In order to understand the nature of the phenomena, consider the simplified model of Fig. 7. Each section in this scheme is represented by its characteristic impedance at infinite frequency $Z_c(\infty)$ and the corona shunt branch by a constant capacitance C_{cor} , obtained from charge cycles of the conductors. The step response of the node k voltage has been predicted analytically by Portela as [12]:

$$v_k(t_k) = \{ 1 - \exp(- t_k / \eta \}, \qquad (1)$$

$$0 < t_k < 2\tau ,$$

where:

- t_k is the time measured from the instant that the surge reaches node k,
- $\eta = Z_c(\infty).C_{cor}/2$ is the time constant of the exponential response, and
- τ is the section travel time.

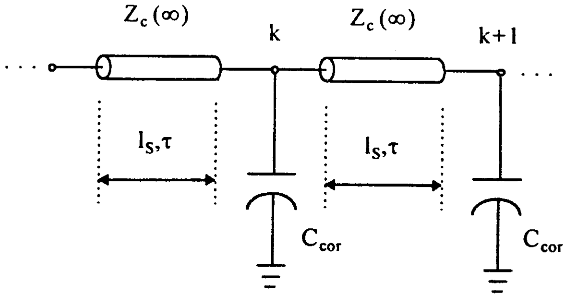

Fig. 7 - Schematic representation for two adjacent sections.

It is seen that the time constant is directly proportional to C_{cor} and thus to the section length. Therefore, for this particular line it is clear that the maximum length that should be used for step propagation is 30m, since the discretization of the corona capacitance C_{cor} for longer sections will introduce substantial deviations at the wave-front.

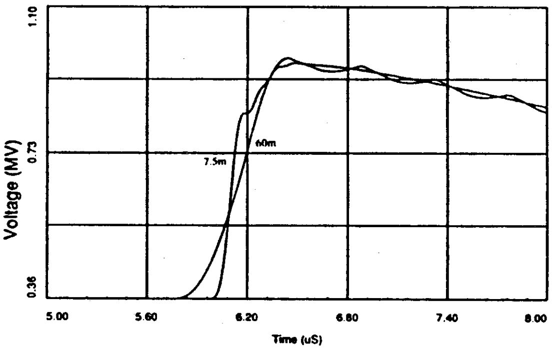

Fig. 8 - Surge response under corona at 1.5 km from point of injection. Static corona model with line sections of 7.5m and 60m.

The same procedure described above was repeated, except that now in step **iv.** a 0.3 / 7μs surge was applied and both the piecewise linear and the Suliciu's model were used for comparison. It is seen from Fig. 8 that the sudden connection of the corona capacitance brought by the piecewise linear model is not as pronounced as when the step input was applied, but is still very substantial. The results obtained with Suliciu's model are shown in Fig. 9. Since this model provides a dynamic description of the charge cycles, the insertion of the additional capacitance above the corona threshold is very gradual that is, there is no sudden change in the corona capacitance as occurs when the piecewise model is used. The effect on the rate of rise of the wave-front can therefore be expected to be less pronounced, and is in this case almost negligible. Thus longer sections can be adopted when Suliciu's model is used, but it is nevertheless important to know the errors involved.

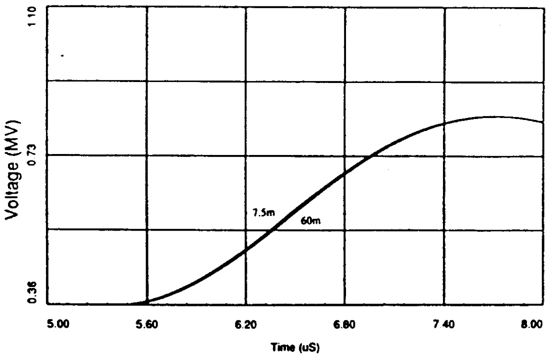

Fig. 9 - Surge response under corona at 1.5 km from point of injection. Dynamic corona model with line sections of 7.5m and 60m.

High frequency oscillations in the simulation results

The presence of spurious oscillations in calculations of electromagnetic transients has been a problem since the early stages of development in this field. The trapezoidal rule of integration is possibly the simplest integration scheme available and is well known for its numerical stability. It can however perform badly in several circumstances, e.g. in cases where the voltage across a capacitance in a circuit is forced to change abruptly. The trapezoidal rule solution for the current flowing through the capacitor can be shown to oscillate about the correct result. This is a classic example of purely numerical oscillations. Some situations arise due to improper or idealized modeling, some are a consequence of discretization of distributed parameter components and some are due to linearization of nonlinear characteristics, such as hysteresis.

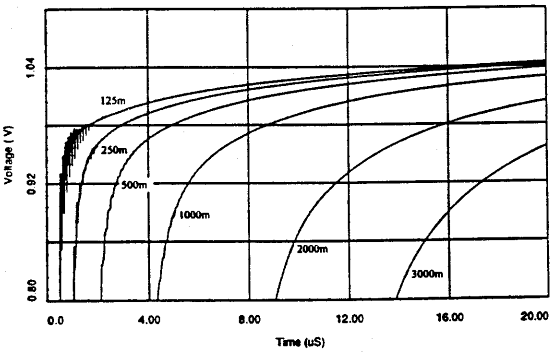

Fig. 10 - Step profile under corona on Gary's line to earth mode of propagation with line sections of 15m.

Figures 10 to 11 show details of the step response for each mode of the 220kV three-phase line under corona. The piecewise linear corona model was used for the simulations. It is seen that high frequency oscillations are generated as soon as the corona threshold is reached; and as propagation proceeds along the line, the oscillations are gradually damped out. It is clear that the damping is more pronounced in the earth mode, as was to be expected since this mode is more affected by the skin effects on the conductors and earth return than the aerial modes.

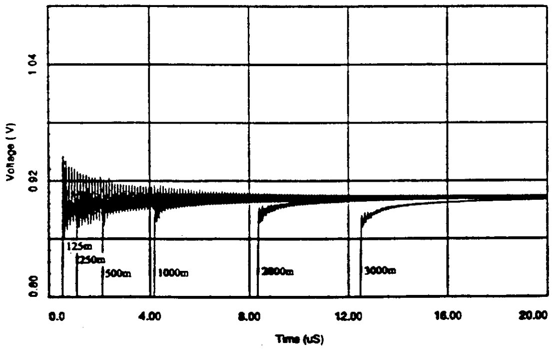

Fig. 11 - Step profile under corona on Gary's line to aerial modes of propagation with line sections of 15m.

The simple model of Fig 7 can be used to analyze the nature of the phenomena: the oscillations are not numerical in nature; they are a consequence of multiple reflections at the interconnecting nodes where the presence of the corona branches introduces discontinuities on the line surge impedance. The oscillation frequency can be expected to correspond to twice the section travel time τ and the amplitude will depend on the reflection coefficients at the section ends. Thus, if the section was reduced to half its length, the oscillations would have twice the frequency and smaller amplitudes as defined by the reflection coefficients. Table 2 gives the peak amplitude values, introduced by the high frequency oscillations on the earth mode, for different section lengths and shows how these values are attenuated for different points of measurement of the surge along the line. It is seen that the peak amplitude increases almost in direct proportion to the section length. For sections longer than 30m, the amplitude after 2 km of propagation is still substantial, corresponding to more than 7% of the step input.

Point of Measurement (meters)	Section Length (meters)				
	5	7.5	15	30	60
125	2.0	3.5	7.8	10.9	20.1
250	1.4	2.0	3.5	7.2	10.6
500	-	2.0	4.3	7.2	9.2
1000	-	2.0	4.3	7.2	8.6
2000	-	-	4.3	7.2	7.8

Table 2 - Relation between maximum amplitude of high frequency oscillation and unit step in percent.

A numerical technique to reduce the oscillations amplitude is to use a critically damped method such as the backward Euler instead of the trapezoidal rule to solve the equations. Figure 12 shows the results for the same mode as in Fig. 11 when the backward Euler method was used to solve the corona model equation. It is seen that the amplitude is considerably smaller than before, but since they are originated by the discretization of the distributed corona capacitance, the damping introduced by the numerical scheme is artificial and cannot eliminate the oscillation completely.

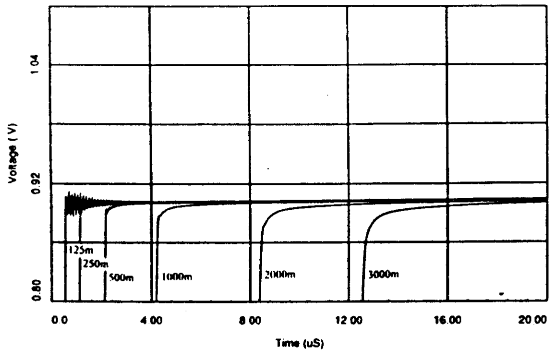

Fig. 12 - Step profile under corona on Gary's line to aerial modes of propagation with line sections of 15m. Backward -Euler integration rule.

The figures in Table 2 would be considerably smaller if Suliciu's model was used instead of the piecewise linear model. As pointed out in the previous section, Suliciu's model is dynamic and thus the insertion of the corona capacitance in the branches is very gradual.

Conclusions

This paper has made a comparative study of frequency dependent line models in the context of the implicit segmentation scheme, which was previously proposed to provide an efficient interface to incorporate corona models in electromagnetic transients calculations.

It has been shown that the best compromise can be achieved using the "mixed-line" model, in which the aerial modes are represented using the constant parameter line model and the earth mode is represented using the fd-line model. This model has been proposed based on the observation that the earth mode is much more affected by frequency dependence of the parameters than the aerial modes. Moreover, the fd-line syntheses can be made to cover the full frequency spectrum or an alternative reduced order fitting which starts at 10^4 Hz , to take advantage of the initial "flat" behavior of the propagation function $A(\omega)$.

The effects of discretization on the rate of rise of the wave-front and as a source of numerical oscillations were analyzed. It has been shown that both effects impose an upper limit on the section length, and that they are more pronounced when sudden changes are introduced by modeling of phenomena such as corona.

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