# DIFFERENT METHODS FOR MODELLING THE TRANSIENT BEHAVIOUR OF 3-PHASE HIGH VOLTAGE POWER TRANSFORMERS AND DEVIATIONS IN THE RESULTS

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*Abstract* **In this work various models are derived to investigate the resonance behaviour of high voltage power transformers in different frequency ranges. A first method uses electrical networks consisting of lumped elements (capacitances, inductances, resistances, conductances) or transmission line elements. If the transformer's behaviour at the network terminals is of interest, the network can be derived by regarding the transformer as a "black box" quadripole, accomplished by using the Modal Analysis. A detailed transformer model is used to calculate the voltage shape inside the winding system. The elements of this model are determined from construction sketches. The second method is based on the Finite Element Method (FEM), which discretizes the whole winding system. In order to reduce the number of winding turns, equivalent material properties must be introduced.**

**Keywords** : Transformers, Transient Analysis, Modelling

## I. INTRODUCTION

In operation transformers are subject to various kinds of high voltage stresses, caused by lightning strikes, disconnecting operations or system disturbances. The rise times of the initiated travelling waves are in the range of ms to ns and correspond to frequencies in the range of kHz to several MHz. If the dominating frequency of a voltage surge corresponds to one of the eigenfrequencies of the winding system resonance excitations are caused. These cause high voltage stresses in parts of the winding system, which can lead to insulation faults like interturn faults, flashovers and short circuits. Since measurements can only be carried out at specific places in the winding system, e.g. at the transformer's regulation taps, numerical simulation is the only way to determine the high frequency performance of a transformer.

This paper develops different methods for the examination of the resonance processes. The methods' limits are shown as well as the deviations in their results. The application of the method is demonstrated by a model of a 110/6.3 kV distribution transformer. Fig.1 shows its with winding arrangement.

The winding system is composed of a low voltage, a high voltage and a tap winding. The tap winding is divided into two different parts, a coarse-step and a fine-step tap winding. The high voltage and the coarse-step tap winding are designed as interwound disc windings. The fine-step tap winding as well as the low voltage winding are constructed as multi-layer windings.

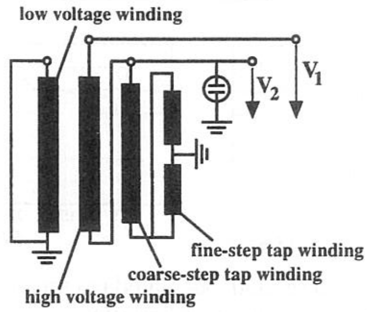


Fig. 1 : Test arrangement for 110/6.3 kV distribution transformer

## II. NETWORK MODELLING

### *A. Idealized Transformer Model*

A transformer can be modelled as a quadripole if the voltage and current time characteristics at the transformer's terminals are of interest, e.g. if just the transformer's interaction with other electrical facilities is examined. This quadripole has to have the same frequency characteristics as the transformer, which can be obtained by measurements.

The frequency dependence can be described by the response of the idealized transformer model to an arbitrary excitation, which leads to the transfer function H and the input admittance function Y :

$$H(j\omega) = \frac{V_2(j\omega)}{V_1(j\omega)} \qquad Y(j\omega) = \frac{I_1(j\omega)}{V_1(j\omega)} \qquad (1).$$

The excitation $V_1(t)$ and the response $V_2(t)$ obtained in the time domain can easily be transformed to the frequency domain by the Fast Fourier Transformation (FFT).

One method to develop a "black box" quadripole model is the modal analysis, which characterizes the examined system by three modal parameters: mode shape, eigenfrequency and damping. Its basic approach is to reduce the step response at a specific place in the system to its suboscillations. Each suboscillation is interpreted as the output signal at a capacitor of a R-L-C-series resonance circuit. As a result, for each resonance frequency a series resonance circuit is used.

When considering a uniform solution method for transformers and electrical lines at high frequencies, e.g. using the travelling wave time-table, distributed parameters have to be used. The measured and calculated results for a 32/12 kV distribution transformer are depicted in Fig. 2,3. All functions show very good agreement.

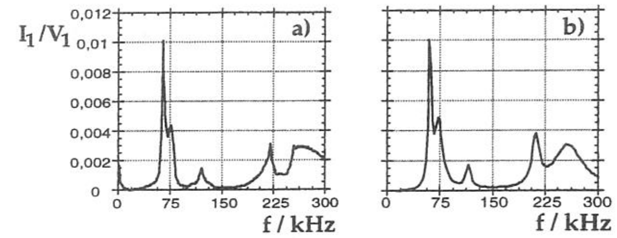


Fig. 2 : Measured (a) and calculated (b) real part of the input admittance function

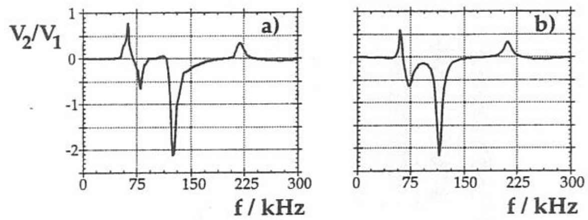


Fig. 3 : Measured (a) and calculated (b) imaginary part of the transfer function

An example for this idealized transformer model accomplished by the use of the modal analysis is shown in Figure 4. The application of modal analysis is limited by the resolution of high resonant frequencies. Further, the transient processes inside the winding system cannot be calculated.

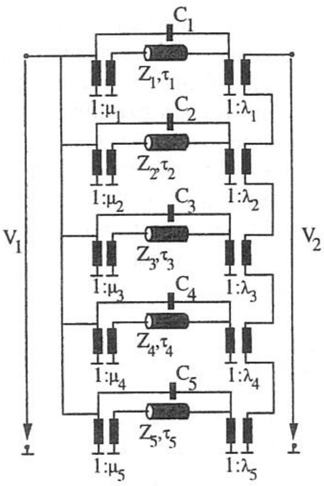


Fig. 4 : Idealized transformer network model

*B. Detailed Transformer Model*

If the voltage shape inside the transformer's winding system is of interest, a detailed network model has to be developed. To take into account the skin effect, proximity effect and the displacement of the magnetic flux within the transformer core, several results obtained by FEM calculations can be used. Alternately, as shown later in chapter II.B.5, results yielded by the network calculations can be used in the transformer's FEM model.

As a result of the transformer's non-linear and frequency-dependent parameters, the calculation of the transient and resonant behaviour of a transformer leads to a non-linear field problem. Due to the saturation of the transformer core and hysteresis, core losses and non-linear inductances occur. Further, eddy current effects lead to frequency-dependent parameters. To calculate the electrical and magnetic interactions the winding system has to be discretized into a corresponding network consisting of lumped elements and active devices. The equivalent network for a double disc unit is shown in Fig. 5.

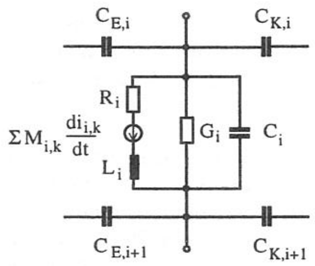


Fig. 5 : Equivalent network of a double disc unit

The resulting series capacitance of a double disc unit is represented by $C_i$. The coupling capacitance for the surrounding winding units is described by $C_{K,i}$ and the stray ground capacitances by $C_{E,i}$. The self inductance for an individual disc unit is symbolized by $L_i$ and the coupling inductances to other units by $M_{ik}$. The frequency-dependent resistance $R_i$ reproduces the eddy current losses in the conductor in high frequent magnetic fields. $G_i$ takes into account the frequency-dependent losses of the insulation. The coupling of the electrical and magnetic magnitudes is modelled by their effects voltage and current. The only way to perform direct coupling is the use of the FEM.

As transient excitations in transformers usually occur only in one phase, it is sufficient to model only one of the transformer's phases. Furthermore, comparisons between the FEM analysis of single phase and three phase models only show slight differences, which also proves a single phase approach.

*B.1 Calculation of the Capacitances*

Ground and coupling capacitances represent the electrical fields between different disc units and between disc units and ground, respectively. The insulation between the turns is constructed of multiple layers consisting of transformer board, oil or an alternate mixture of both. These materials are represented by an equivalent dielectric constant. By adapting the equation of a plate capacitor together with a correcting factor that takes into account the "edge effect", the ground and coupling capacitances can easily be obtained. The resulting series capacitance $C_i$ is yielded by additional application of the energy conservation law. Today, for the purpose of voltage linearization, interwound disc windings are usually used. This results in a higher capacitive energy storage and consequently in higher equivalent serial capacitances $C_i$.

*B.2 Calculation of the Inductances*

The self-inductances take into account the stray field of each conductor within the winding system. To determine the self-inductances at higher frequencies the knowledge of the iron core flux is necessary. As shown later, a FEM application proves that at frequencies above 10 kHz the magnetic flux is completely displaced from the interior of the iron core. As a result the transformer coils can be regarded as air-core reactors. Besides using analytical equations the self and mutual inductances can be determined by numerical methods. These are based on the specification of the magnetic vector potential by means of the extended Biot-Savart law.

*B.3 Calculation of the Parallel Conductances*

The parallel conductances $G_i$ account for the dielectrical frequency-dependent losses in the insulation material of the transformer, which are mainly affected by space charge polarization processes. By measurement of the materials' $\tan\delta(f)$ the parallel value $G_i$ is calculated by the equation

$$G_{ik} = 2\pi f \cdot C_{ik} \cdot \tan\delta(f) \qquad (2).$$

*B.4 Calculation of the Series Resistances*

Due to the transient processes within the winding system the current density is nonuniformly distributed across the conductor's cross section, leading to a considerable increase of the damping if the skin depth reaches the conductor's dimensions. These losses are a result of the skin effect (the influence of the conductor's self-field on itself) and the proximity effect (the influence of the magnetic field of nearby conductors). The skin and proximity effect losses can be calculated separately and superposed afterwards.

For modelling the frequency dependent damping resistances in time domain-simulations, it is necessary to use equivalent networks instead [1,2]. The first method is based on Foster circuits which consist of parallel R-L blocks connected in series (Fig.6). The real part of the impedance at the terminal has to match the resistance shape in the winding. For this purpose, the resistances and inductances - the parameters of this network - have to be determined iteratively.

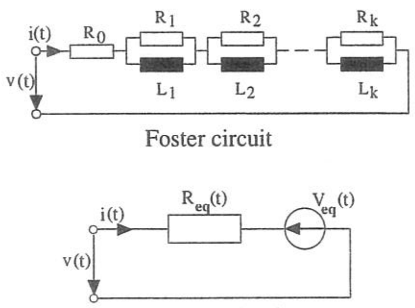


Fig. 6 : Equivalent networks for modelling frequency dependent resistances in the time domain

With the analysis program EMTP/ATP Thevenin circuits can also be used. These circuits consist of a time varying resistor $R_{eq}(t)$ connected in series to a time varying voltage source $V_{eq}(t)$. The values of the two components are determined during the calculations in TACS. Using Thevenin circuits instead of Foster circuits reduces the total transformer network size considerably. The disadvantage is that extensive calculations have to be carried out in advance.

*B.5 Calculation of the Skin and Proximity Effect Losses*

The analytical calculation of skin effect losses can only be performed for simple arrangements. Hence, numerical methods have to be used to determine the current distribution accross the conductor's cross section and the skin effect losses. As shown in chapter III.A good results are obtained by the application of the FEM. These results can easily be used in the network model.

Using a method developed by Dietrich the increase of the serial resistance caused by the proximity effect can be determined. The most important equations are the following:

$$R(f) = k \cdot R_0 = k \cdot \left(w \cdot l_m \,/\, \kappa \cdot a \cdot b\right) \qquad (3)$$

$$\text{where} \quad k = g\left(f, \Delta H_x, \Delta H_z, a, b, w, \kappa\right) \qquad (4)$$

The dc resistance is symbolized by $R_0$, while w describes the number of turns within the respective double disc unit and $l_m$ the average length of one turn. The dimensions of a singular conductor are respresented by a and b. $\Delta H_x$ and $\Delta H_y$ symbolize the radial and axial field strength gradient, obtained by using a numerical method. The increase of the resistance caused by the proximity effect is shown in Fig.7. The proximity effect losses in the considered frequency range are approximately five times higher than the losses caused by the skin effect. This is proved by FEM calculations performed for the 110/6.3 kV distribution transformer.

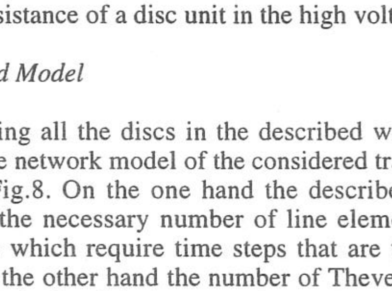


Fig. 7 : Resistance of a disc unit in the high voltage winding

*B.6 Detailed Model*

Discretizing all the discs in the described way yields the single-phase network model of the considered transformer, as shown in Fig.8. On the one hand the described method is limited by the necessary number of line elements for high frequencies which require time steps that are too small for EMTP. On the other hand the number of Thevenin or Foster circuits is limited mainly because of numerical problems.

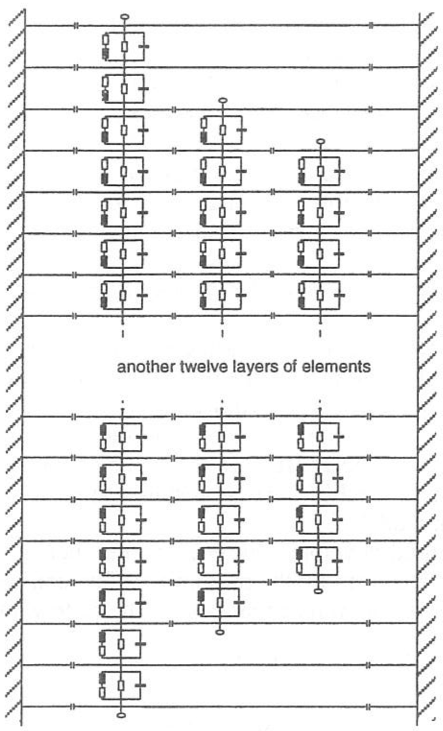


Fig. 8 : Detailed network model

## III. FINITE ELEMENT MODELLING

In order to investigate the influence of the skin effect on the transformers' resonance behaviour at high frequencies different FEM models have to be developed. Especially the core and the conductors have to be simulated by separate models. The obtained characteristics are then combined in a three dimensional three phase model of the 110/6.3 kV distribution transformer.

### *A. FEM Model of the Conductor*

Due to symmetry it is sufficient to discretize one quarter of the conductor using three dimensional elements. The conductor's elements are surrounded by air elements to a cylindrical outer model boundary, where the magnetic vector potential is constrained to zero. At the symmetry plains no boundary conditions are applied. As a result, the magnetic field is forced to be perpendicular to these borders (Neumann condition). The eddy current distribution of the conductor is shown in Fig.9 for 10 kHz and 1 MHz. The displacement of the current density can easily be recognized. As mentioned before, the obtained results are also used for the calculation of the series resistance in the network model.

### *B. FEM model of the Core*

In order to simulate the displacement of the magnetic flux in the transformer core a two dimensional model is developed, as displayed in Fig. 10.

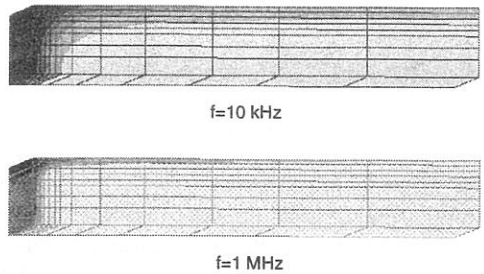


Fig. 9 : Skin effect at 10 kHz and 1 MHz

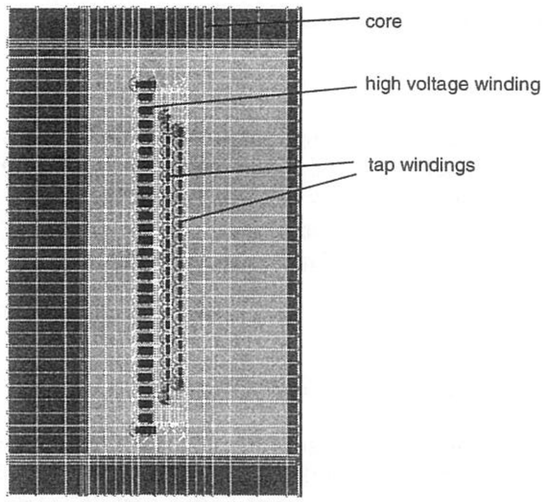


Fig. 10 : Model for analysing the frequency dependent magnetic field in the transformer core

If a transfromer core consists of steel layers with thickness h, conductivity $\sigma$ and permeability $\mu$, the damping and the inductivity change with increasing frequency :

$$R \approx L_{DC} \cdot \frac{1}{h}\sqrt{\frac{2\omega}{\sigma\mu}} \qquad L \approx L_{DC} \cdot \frac{1}{h}\sqrt{\frac{2}{\omega\sigma\mu}} \tag{5}$$

Since the transformer core consists of laminated steel the conductivity in the model is reduced in the horizontal direction in such a way, that the same changes in damping and inductivity are obtained. Fig.11 shows the flux as a function of the core radius.

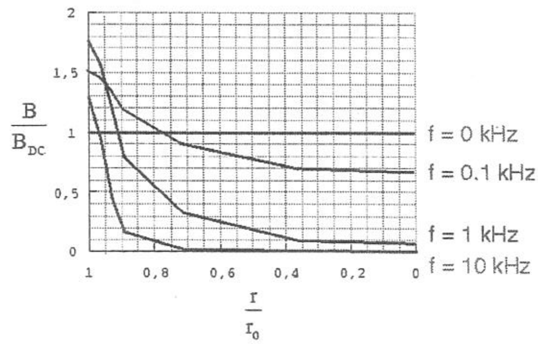


Fig. 11 : Magnetic flux in the core at different frequencies

At 10 kHz the magnetic flux within the transformer core is almost completely displaced out of the core which confirms the calculated approach for the network inductances in regarding the transformer coils as air-core reactors.

*C. FEM model of the Transformer*

Based on the given geometrical data a complete 3-phase model of the 110/6.3 kV distribution transformer is developed. Since it is impossible to model each turn, because of the outrunning number of elements, the equivalent material method has to be used [3].

The basic idea is to increase the relative permeability constant and the relative dielectricity constant in the same ratio as the number of winding turns is reduced. As a result, the propagation velocity is reduced and the travelling time is kept constant as well as the reflection factor.

First, a single phase of the transformer is modelled to compare the calculated results to available measurements. The transformer core is modelled using penta elements with anisotropic conductivity considering the influence of the laminated steel. The low voltage winding is reproduced by a conductive cylinder consisting of quad elements.

Calculations have shown that the low voltage winding's influence on the potential distribution inside the transformer is negligible, because it is grounded under test conditions. As a result, the number of elements can be reduced remarkably by leaving it out. By using the equivalent material method the number of turns of the high voltage winding is reduced from 553 to 26. Each turn is built from 12 quad elements, so that they occupy the same space as the original winding. The distance between two layers of quad elements is greater than the distance between two discs of the real high voltage winding. For unchanged capacitances the relative dielectricity constant of the insulating elements is increased. The coarse-step and the fine-step tap winding are modelled by 12 line elements for each turn. Their number of turns is reduced in the same ratio as used at the high voltage winding.

The low voltage, high voltage and tap windings are separated from each other by many layers of different insulating materials, such as oil, paper or transformer board. The single phase model's windings are shown in Fig.12.

Based on this single phase model a three phase transformer model is developed. It consists of 10445 elements and 8681 grids and is depicted in Fig.13, without its air envelope. At the boundary of the air envelope the time-integrated scalar potential is set to zero as well as the magnetic vector potential.

As an ac analysis of the complete transformer model shows, the first maximum at the connection of the high voltage winding and tap winding occurs at 53 kHz. Superposing the results of the skin and proximity effect calculations show that the conductors' conductivity at this frequency is about 500 times higher than the direct current value. Hence, the conductivity of the elements representing the conductors is reduced by this factor for direct transient analysis.

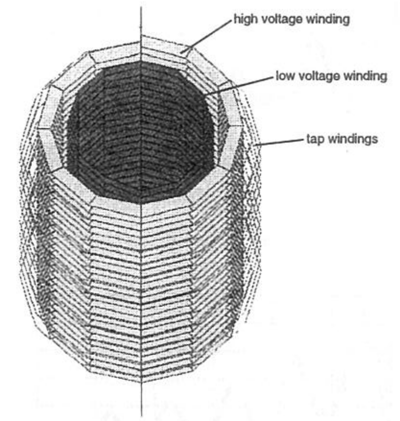


Fig. 12 : Single phase transformer model

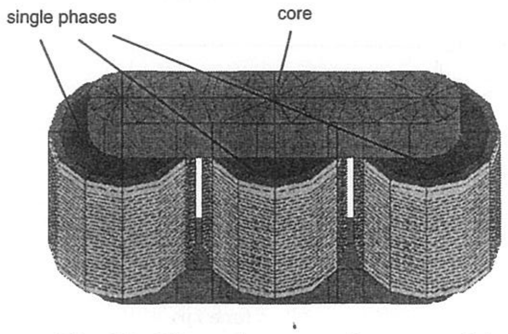


Fig. 13 : Three phase transformer model

A great disadvantage of solution sequences operating in the time domain is that it is impossible to take into account the frequency dependent material parameters of the transformer's conductors and insulation. In order to model frequency dependent materials ac analysis have to be performed. In addition to the changes of the magnetic flux distribution in the iron core and to the resistivity and inductance of the windings, a frequency dependence of the losses in the insulating materials due to a frequency dependent $\tan\delta$ can be used.

The excitation has to be transformed into the frequency domain using the Fast Fourier Transformation (FFT). After the multiplication of the transfer function, obtained by the ac analysis, and the FFT of the excitation function, the inverse FFT can be employed to transform the product to the time domain leading to the desired voltage distribution. In principle the FEM method can be used for the whole frequency range. Its only limitation is the restricted number of elements.

## IV. CALCULATED AND MEASURED RESULTS

Fig.14 shows the result obtained by measurements. As excitation a 1.2/50 μs lightning impulse test voltage was used. Fig.15 shows the results yielded by the use of the detailed network transformer model and by the FEM model. The results show a good agreement.

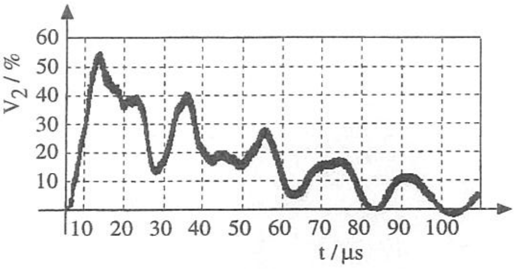


Fig 14 : Measurement results

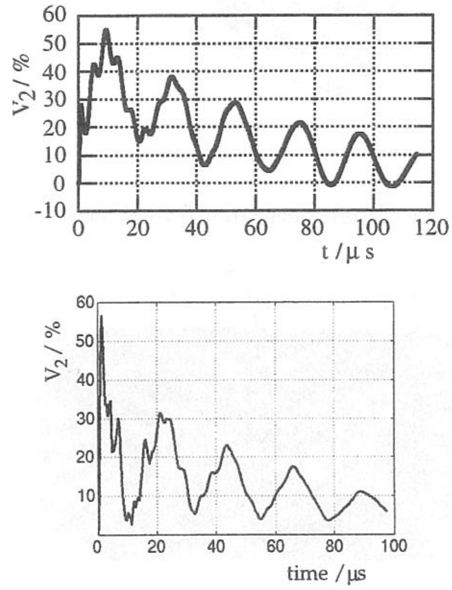


Fig. 15 : Calculation results, network model (above) - FEM model (below)

By application of the FEM method in the frequency domain the potential distribution of the windings are obtained. Fig. 16 shows first the potential distribution of the high voltage winding where x/l = 0 corresponds to the excited end of the high voltage winding and x/l = 1 to the connection of the high voltage winding and the coarse-step tap winding.

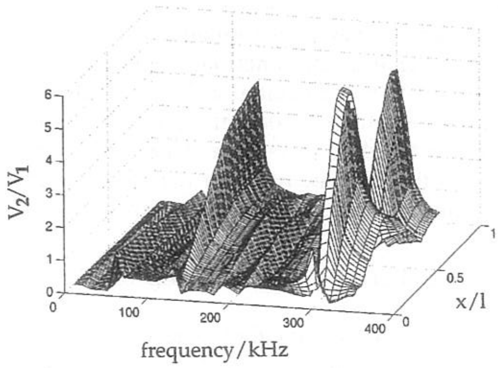


Fig. 16 : Potential distribution of the high voltage winding

Fig.17 shows the potential distribution of the coarse-step tap winding where x/l = 0 corresponds to the connection of the high voltage winding and the coarse-step winding and x/l = 1 to the connection of the coarse-step and the fine-step winding. The first maximum at the endangered intersection of both windings occurs at a frequency of 53 kHz.

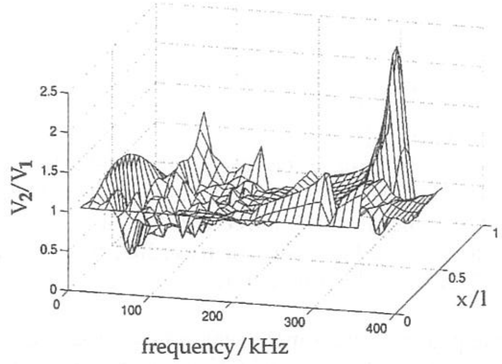


Fig. 17 : Potential distribution of the coarse-step tap winding

## V. CONCLUSION

This paper describes idealized and detailed network models and a detailed FEM model for the numerical simulation of power transformers. The obtained results show the applicability of network methods and FEM simulations. The serial resistance increase caused by the proximity effect is calculated by means of network approaches, that caused by the skin effect by means of finite element calculations. Furthermore, the displacement of the magnetic flux in the core at high frequencies was demonstrated. The comparison of results obtained by measurements with those obtained by numerical simulations shows good agreement. Finally, for the determination of the frequencies and the location at which the winding system is most endangered, an ac analysis using the FEM was performed.

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