Dynamic Simulation of Transients in Synchronous Machines

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Abstract— **This paper presents the development of a synchronous generator model capable of simulating machine transient response to internal and external fault conditions based on a dynamic magnetic circuit model (MCM). The model includes multi-layer windings, damper bars, and magnetic saturation under both steady state and faulted conditions. The model also represents the internal connections, the machine ground connection, and the external power system.**

Simulation results for steady-state and terminal fault conditions are presented and compared to laboratory test data. The model can be used to represent internal transients that cannot be modeled using EMTP, such as: stator and rotor turn-to-turn faults, shorted stator turns, shorted rotor turns, phase to phase and phase to ground faults on the stator, and rotor ground faults.

Keywords— **transient magnetic circuit model (MCM), synchronous generators, terminal faults, internal faults, transferences**

I. Introduction

The synchronous machine models used in fault and transient simulations are written to model the response of the machine to external faults and system disturbances. The Park's transformation is commonly used to represent the machine with a "two axis model." This representation significantly reduces the computation burden of modeling the machine, while allowing for more detail in the external system. Such a model is used in the EMTP [1]. However, this model is unable to accurately model the transients resulting from internal faults.

This paper presents the development of a dynamic magnetic circuit based synchronous generator model. The model is capable of simulating machine transient response to internal fault conditions as well as modeling response to external power system transients. The model includes multi-layer windings, damper bars, and magnetic saturation under both steady state and faulted conditions. The model also represents the internal connections (including internal capacitances), the machine ground connection, and the external power system.

Simulation results for steady-state and terminal fault conditions will be presented and compared to laboratory test data. The model can be used to represent internal transients that cannot be modeled using conventional transient programs, such as: stator and rotor turn-to-turn faults, shorted stator turns, shorted rotor turns, phase to phase and phase to ground faults on the stator, and rotor ground faults.

II. Modeling Synchronous Generator Faults

The ability to model synchronous generator faults would be useful for the setting of generator protective relays [2], [3]. Third harmonic stator ground fault relays capable of protecting the last 3-15% of the winding are often set based on experimental measurements from a limited number of operating conditions. These settings are refined with operating experience with the machine. However, at present there is no good way to determine these fault levels and signatures prior to the commissioning of the machine.

There are several options for modeling internal faults in electric machines, including synchronous generators. These include: approximate electric circuit models [4], finite element analysis [5], [6], and dynamic magnetic models. Finite element analysis provides a very accurate representation of the machine, but is computationally intensive and is better suited for static studies rather than transient studies. Magnetic circuit models also provide a very accurate representation of the machine, but tend to exhibit the advantage of reduced 'dynamic' simulation time when representing a rotating machine. Both finite element analysis and magnetic circuit models require physical geometric data for the machine, which is available for utility scale synchronous generators since it is required for routine rewinds. A transient magnetic circuit model is also better able to accurately represent internal losses in the machine due to hysteresis and eddy currents.

Internal faults such as shorted turns on either the stator or rotor cannot be efficiently or satisfactorily modeled by either finite element analysis or the EMTP. While the EMTP is capable of modeling external line-to-line or line-to-neutral faults it cannot adequately model internal faults. Finite element analysis is capable of predicting internal fault response, but the time required for convergence of repeated 'dynamic' solutions is excessive. Therefore, a magnetic circuit model capable of simulating internal fault conditions more accurately than the EMTP and more efficiently than finite element analysis is needed. Faults that could be studied using a magnetic circuit model but cannot be analyzed using the two reaction model include: stator or rotor turn-to-turn faults, stator and rotor shorted turns, stator internal phase-to-phase faults, stator internal phase-to-ground faults, and rotor ground faults.

III. Transient Magnetic Circuit Modeling

The first major work devoted to magnetic circuit modeling is the classic book by Roters [7] who derived the permeances needed to represent the spatial properties of many typical types of electrical machines. However, these parameters were only used to refine the standard motor design procedure. The duality between electrical and magnetic circuit modeling, exploited in this paper, has been recognized for many years [8]. In the late 1960's Laithwaite [9] and Carpenter [10] further developed the duality of magnetic circuits. Of particular interest is the model proposed by Carpenter, based on magnetic currents, where magnetic current is defined as the time rate of change of the magnetic flux. The resulting equivalent circuit is intuitively pleasing in its similarity to electric circuits. Magnetic transference, discussed later in this paper, was introduced by Lathwaite in [9].

Beginning in the early 1980's, Ostovic [11] - [13] applied magnetic equivalent circuits to electric machine transient and steady-state analysis. Recent developments in the magnetic circuit modeling of various types of electric machines provide a basis for the analysis of internal faults in synchronous generators using an equivalent magnetic circuit based approach. In 1990, Slemon [14] developed an equivalent magnetic circuit model for a synchronous machine based on nodal analysis methods. More recently, Xiao and Slemon, et.al. [15] developed and implemented an equivalent magnetic circuit model for a synchronous machine capable of simulating external L-L and L-G faults, motor start-up, and saturation. The model presented in [14] and [15] has two limitations. First, the accuracy in modeling saturation is limited to prevent increased complexity of the solution. Second, the magnetic circuit model in [14] is based on node equations rather than flux loop equations. The latter simplifies the model by enabling use of half-wave symmetry conditions that require only a single pole to be modeled rather than a pole-pair, decreasing computational complexity and overhead.

A magnetic circuit model of a synchronous reluctance machine based on flux loop equations was developed by Law in [16]. Busch included the ability to model magnetic saturation in the reluctance machine without numerical instability, [17], [18], [19]. Lockwood [20] developed an equivalent magnetic circuit model for an induction machine. Incorporating knowledge of synchronous reluctance machine stator winding characteristics from [16], induction machine rotor winding characteristics from [20], and saturation modeling methods from [17], [18], into the existing Magnetic Circuit Model (MCM) program allowed for the development of a model capable of simulating transients in synchronous machines.

A. Basic Concepts

The MCM solves a combination of magnetic differential equations (MDE) and electrical differential equations (EDE). These equations are constrained by magnetic algebraic constraint equations (MAC) and electrical algebraic constraint equations (EAC). The basic differential equations are:

- Faraday's Law: $N\dot{\Phi} = V - rI$ (EDE)
- Leakage Inductance Voltage Equation: $L\dot{I} = E$ (EDE)
- MMF Loop Equations: $\Gamma\dot{\Phi} + R\Phi = NI$ (MDE)

where N = number of turns, Φ = magnetic flux, V = voltage, r = resistance, I = current, L = inductance, R = reluctance, and Γ = transference (defined below, a magnetic circuit analog to inductance).

These equations link the electrical and magnetic differential equations needed to represent the machine. The MMF loop equations necessary to cover a magnetic core with multi-turn/multi-layer windings and an airgap lead to a matrix structure similar to a Y_{bus} matrix in an electric power system. Each loop is only coupled to adjacent loops, so the matrix tends to be quite sparse.

The basic algebraic equations are Kirchhoff's Current Law (EAC) and Kirchoff's Voltage Law (EAC). These include things like phase belt constraints (i.e., the current leaving one winding section of phase "A" must equal the current entering the next section of phase "A"), and line-to-line voltage constraints. A magnetic circuit version of Ohm's law in the form of $R\Phi = NI$ or $R\Phi = 0$ forms the principle magnetic algebraic constraint equation (MAC).

An external power system model can be added through additional electrical algebraic and differential equations. The electrical equations are implemented in a manner similar to that used in the EMTP [21].

B. Non-Linear Saturation Modeling

When attempting to model magnetic saturation of the iron comprising the stator and rotor cores in [17], [18],difficulties arose with flux density oscillations between parallel magnetic flux loops. Much of the flux would pass through the loop that was saturated more deeply in one iteration and then pass through the neighboring coil on the next iteration with a continual exchange. These oscillations were eliminated with the addition of eddy current loops (and subsequent energy dissipation), which added damping. The initial solution in [18], [19] used a lumped eddy current loop approximation to solve the problem. A more accurate representation can be achieved through the addition of a magnetic transference element. Using Fig. 1 the following differential equation can be defined:

$$N_{ec}\dot{\Phi}_2 \quad = \quad -r_{ec}I_{ec} \quad \text{(EDE)} \qquad (1)$$

along with the following algebraic equation:

$$R_{21}\Phi_1 + R_{22}\Phi_2 \quad = \quad N_{ec}I_{ec}(\text{MAC : Mag. Ohm's}) \qquad (2)$$

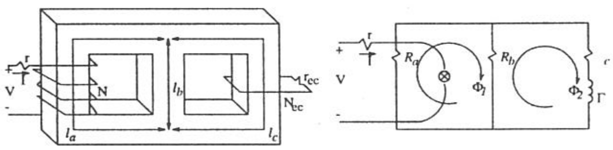

Fig. 1. Two Loop MCM with Transference

Solving for I_{ec} yields:

$$I_{ec} \quad = \quad -\frac{N_{ec}}{r_{ec}}\dot{\Phi}_2 \qquad (3)$$

and hence

$$N_{ec}I_{ec} = -\frac{N_{ec}^2}{r_{ec}}\dot{\Phi}_2 \quad (4)$$

Substituting N_{ec} into the MAC yields:

$$-\frac{N_{ec}^2}{r_{ec}}\dot{\Phi}_2 = R_{21}\Phi_1 + R_{22}\Phi_2 \quad (5)$$

Defining the *Transference*, Γ, as:

$$\Gamma = \frac{N_{ec}^2}{r_{ec}} \quad (6)$$

then the MAC becomes the magnetic differential equation (MDE):

$$\Gamma\dot{\Phi}_2 + R_{21}\Phi_1 + R_{22}\Phi_2 = 0 \quad (7)$$

The model is completed with the addition of leakage flux Φ_l as an inductance, resulting in the following sets of differential equations with unknowns Φ, I, and E:

$$\begin{aligned} \Gamma\dot{\Phi} &= NI - R\Phi \quad \text{(MDE)} \\ N\dot{\Phi} &= E - rI \quad \text{(EDE)} \\ \mathrm{L}\frac{dI}{dt} &= V - E \quad \text{(EDE)} \end{aligned}$$

C. Model Implementation

The equations above form the basis for an equivalent machine model based on magnetic circuit flux loops. A detailed development of such a model for a synchronous reluctance machine is presented in [16], [19]. Enhancements to the model presented in [16] will be the topic of a more detailed paper in the future. The resulting set of sparse non-linear differential equations is solved using an implicit iterative predictor-corrector method. In addition to solving for the loop fluxes, the program also calculates machine terminal voltages and currents from algebraic constraint equations given in detail in the next section. The differential and algebraic equations are solved simultaneously at each iteration, such that the algebraic constraints are met at every step. This makes it relatively simple to add internal and external faults, since they represent modifications to the electrical constraint equations as will be described later.

The topology used to represent the magnetic circuit model for a synchronous machine is generated via a user interface developed by Law for a synchronous reluctance machine [16]. A graphical user interface was developed to allow the user to input the primitive reluctance topology of the machine. This interface was modified to allow for the addition of transferences for elements passing through magnetic material.

The topology description created using the graphical interface contains the following information:

- Stator, Rotor and Airgap Locations
- Topology Grid and Nodes
- Primitives Reluctance and Transference Elements
- Boundary Conditions/Symmetry
- Winding Locations/Phases

The reluctance and transference primitive magnetic elements include the following detailed information:

- Dimensions: Length, Width, Area
- Material Permeability, Saturation
- Type: Air, Airgap, Stator Iron or Rotor Iron

The program has the ability to automatically locate the loops in a magnetic circuit topology node list description and generate the differential magnetic matrix equations. The reluctances and transferences are linked to both self and mutual elements in the matrix. Similarly, the electric circuit matrix equations are generated from a description of the machine windings. The numerical integration scheme is based on the implicit trapezoidal method, resulting in electrical equations similare to those used in the EMTP.

Rotation of the machine requires updating the topology of the airgap of the magnetic circuit and the corresponding matrices. The airgap will only change topologically after the machine has been allowed to rotate for several discrete numerical time steps. The matrix need only be re-factored when the model topology changes.

IV. Synchronous Generator Modeling

The magnetic equivalent model of the synchronous machine is represented by the reluctance and transference circuit topology shown in Fig. 2. The topology is based on loop flux methods developed by Law [16]. Figure 2 shows a single pole of the machine. For normal operation of the machine, symmetry conditions allow a single pole to represent the entire machine. This is only true if flux loop equations are used as opposed to flux node equations, which require a two pole model.

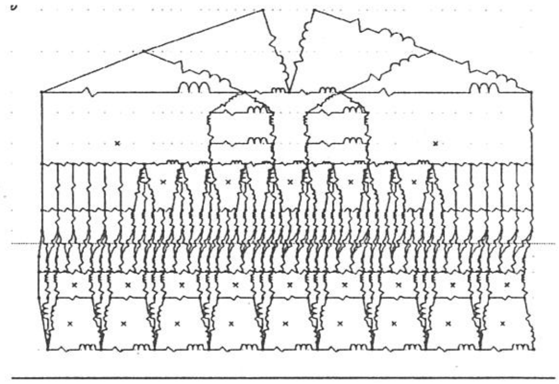

Fig. 2. Synchronous Machine Primitive Diagram

The rotor iron and interpole elements are above the horizontal dashed line in Fig. 2, with the "resistors" representing reluctance and the "inductors" representing transference. The rotor damper bars are also included in this region. The air gap elements are the reluctance elements crossing the dashed line. These elements do not have any transference since there is no iron to carry eddy currents. The stator iron elements are below the horizontal dashed line. The X's represent currents flowing through physical conductors (stator windings, field windings, and damper bars).

The current and voltage winding constraints will vary with machine configuration. For example, the six pole machine used for experimental tests had:

- Six coil-sides per phase per pole
- Three coil-sides model the return current in the adjacent pole using symmetry
- Series coil-side voltages sum to yield line-to-neutral voltages

The basic stator winding configuration is shown in Fig. 3, which shows the overlapping phase belts for a single phase (for a two pole machine).

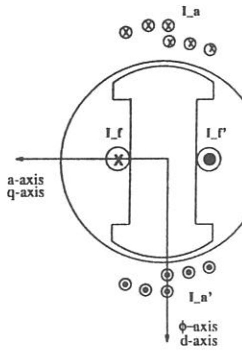

Fig. 3. Stator Winding Configuration for a Two Pole Machine

The algebraic constraint equations results from applying Kirchhoff's laws. For example, $I_a + I_b + I_c = 0$ (Three-Phase Constraint), and $I_{d5} = -(I_{d1} + I_{d2} + I_{d3} + I_{d4})$ (Damper Winding Constraint). Other constraints include: $I_{F1} = - I_{F2}$ (Field Winding Constraint) $I_a = 6I_{a1}$ (Sum of Coil Sides) and $V_a = V_{a1} + V_{a2} + V_{a3} + V_{a4} + V_{a5} + V_{a6}$ (Stator Phase Belt Constraint)

V. Fault Modeling

As stated earlier, electrical short circuits or open circuits can be represented by changes to the electrical equations. For example, a terminal line-to-line fault can be represented with:

1. $I_b + I_c + I_{bs} + I_{cs} = 0$ (Fault Current Constraint)
2. $V_{bc} = 0$ (Line-to-Line Fault Voltage Constraint)
3. $V_{bn} = V_{cn}$ (Line-to-Neutral Fault Voltage Constraint)

and an internal shorted turn fault can be represented with:

1. $I_{c3s} = - I_{c4s}$ (Shorted-Coil Current Constraint)
2. $V_{c4s} - V_{c3s} = 0$ (Shorted-Coil Voltage Constraint)
3. $V_{cn} = V_{c3} - V_{c4} + V_{c1} + V_{c2} + V_{c5} + V_{c6}$ (Coil-Side Voltage Constraint)

VI. Simulation Results and Experimental Verification

The fault modeling capability of the magnetic circuit model program was tested by comparing simulation results to experimental results obtained from a 20HP, 220V synchronous motor. The 6 pole machine has a rated field voltage of 125 V_{dc}, and has a salient pole rotor. The machine was recently rewound to add taps for creating internal faults and provide access to the neutral. The machine was ungrounded in the initial tests.

A. Step Change in Applied Field Voltage

The first test was to step the applied field voltage from zero to one half of the rated level with the machine turning at rated speed and with the phase leads open circuited. The terminal voltage of the machine will exhibit an exponential rise as the field current increases. The experimental line-line voltages are shown in Fig. 4(a). Notice that the machine takes about one second to approach steady-state operation. The same test was simulated using the MCM and this is shown in Fig. 4(b). The line-to-line voltages exhibit a similar exponential increase.

The synchronous machine has concentrated stator windings rather than sinusoidally distributed windings, so the ac output voltage will exhibit a distinct slot ripple. The slot voltage ripple from the MCM simulation compares favorably with the experimental results as shown in Fig. 5.

B. External Phase-to-Phase Fault

The next test was the application of a line-to-line fault at the machine terminals. The measured line-to-line voltages are shown in Fig 6(a) and the MCM simulated line-to-line voltages are in Fig 6(b).

The line currents on the faulted phases are shown in Fig. 7(a) and the MCM results are shown in Fig. 7(b). Notice that both plots exhibit similar subtransient and transient time constants. Simulation of 1.5 seconds of real time required approximately six hours to run on an HP C110 workstation.

VII. Conclusion

This paper has presented the development of a synchronous generator model capable of simulating machine transient response to internal fault conditions based on a dynamic magnetic circuit model. The model includes multi-layer windings, damper bars, and magnetic saturation under both steady state and faulted conditions. The model also represents the internal connections, the machine ground connection, and the external power system. Simulation results for steady-state and terminal fault conditions have been presented and compared favorably to laboratory test data.

The model can be used to represent internal transients that cannot be modeled using EMTP, such as: stator and rotor turn-to-turn faults, shorted stator turns, shorted rotor turns, phase to phase and phase to ground faults on the stator, and rotor ground faults which will be presented in later work.

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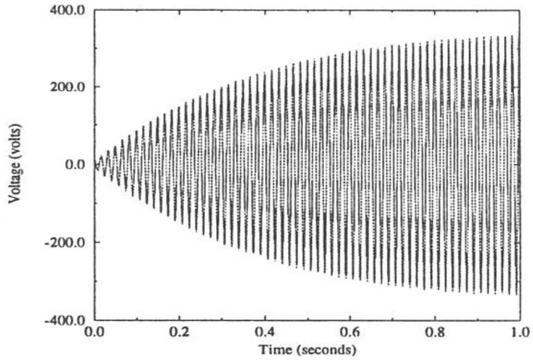
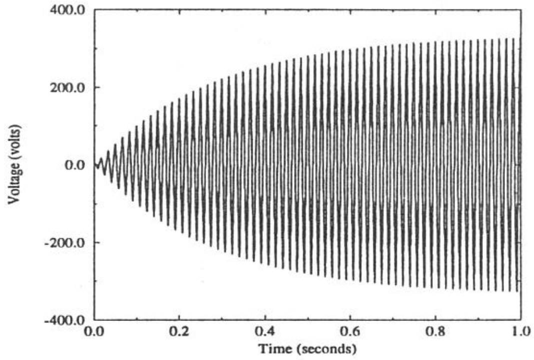


(a) Experimental line-to-line voltages



(b) MCM line-to-line voltages

Fig. 4. Line-to-line terminal voltage with field voltage stepped from zero to one-half rated

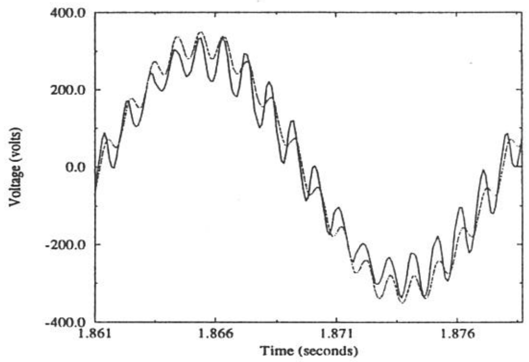
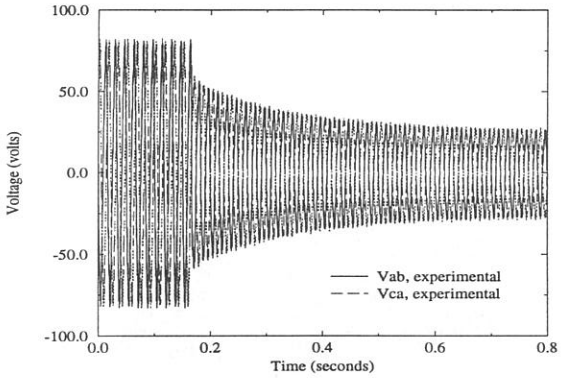
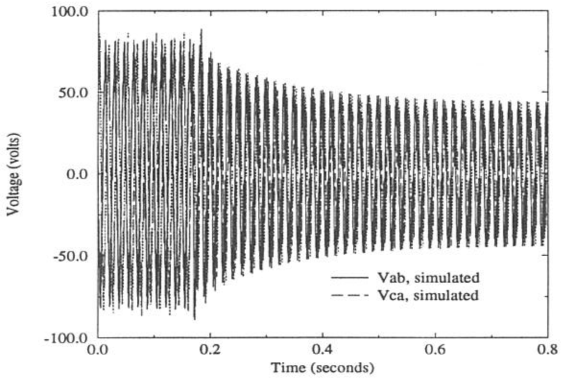


Fig. 5. Comparison of experimental and simulated slot voltage ripple

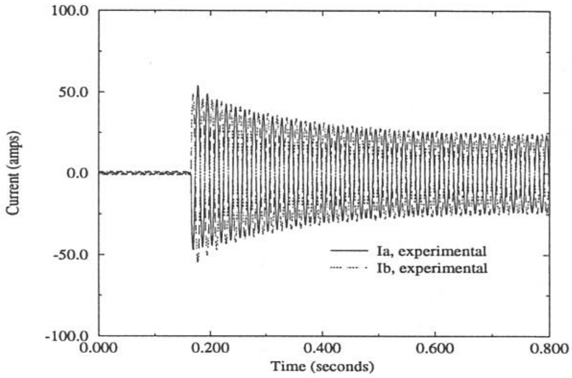


(a) Experimental line-to-line voltages

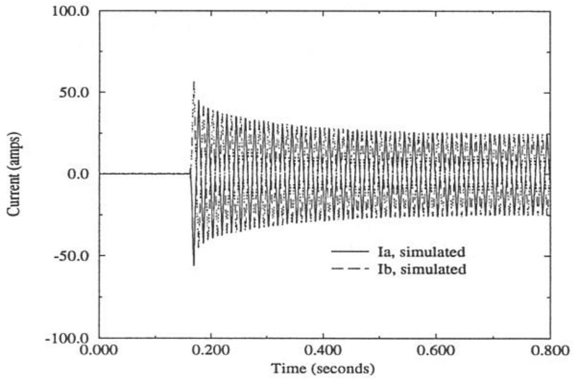


(b) MCM line-to-line voltages

Fig. 6. Line-to-line voltage for line-to-line fault at terminal



(a) Experimental line currents



(b) MCM line currents

Fig. 7. Line currents for line-to-line fault at terminal