Tuning of Resonant Modal Transformer Models

M. Condon **D.J. Wilcox**

Department of Electronic Engineering
National University of Ireland, Galway
Ireland

Abstract – **With a view to optimal representation of large power transformers under general transient conditions, the paper examines the structure of MODAL models and their suitability for *a postiori* tuning to measured data. The paper proposes a methodology for tuning an initial modal model to fit test data which is obtained from a transformer after construction. A simple illustrative case helps to clarify the suggested procedure.**

Keywords: Transformers, MODAL models, Optimization.

I. INTRODUCTION

Recent work by the authors [1-4] has shown that it is possible to develop high-frequency transformer models from drawing-board data, just as transmission-line models are developed from drawing-board data (conductor heights, separations, soil resistivity etc.). The accuracy of frequency-dependent transformer modelling has been investigated previously for the case of test-windings on a 25kVA core [3] and the accuracy was found to be good. However, it is problematic as to whether such good accuracy could be achieved in the case of large power transformers using only relatively simple data.

Recognising this, the formation of accurate models for large power transformers should involve field test measurements. In particular, it is envisaged that such field test results would include admittance frequency-response results as measured at the transformer terminals.

The first section of the paper shows that the elements of a transformer modal model for a 150MVA power transformer maintain the same structural form identified previously [3-4] for the much smaller 25kVA transformer. The paper proceeds to adopt a variational approach in which key transformer parameters are varied in order to establish the effects on the model elements. The paper shows that the effects are quantitative rather than qualitative, i.e. the basic structure remains intact. It is this feature of MODAL models which makes them ideally suited to tuning.

The paper proceeds to develop a methodology whereby an initial MODAL model can be tuned to fit experimental data. In particular, since the core of the model consists of a set of independent *RLC* circuits, tuning of this part of the model simply amounts to adjusting the *L* and *C* values to give the correct resonant frequency and adjusting *R* to ensure the correct damping. The remainder of the model can be adjusted using various optimization techniques [6-8].

Unfortunately, the authors do not have a large power transformer for test purposes. However, the proposed procedure is applied to a simple test case involving the admittance spectra from a synthetic winding to demonstrate its validity.

II. TRANSFORMER MODAL MODEL

Fig. 1 shows the MODAL model developed for transformers in [1-4]. In the frequency domain, the model is an exact equivalent of an original lumped-parameter representation of a transformer. V_B is a vector comprising the voltages at the terminals of the individual windings. I_B is a vector representing the currents entering (or leaving) the terminals. V' are the voltages at the internal nodes of the discrete representation of a transformer.

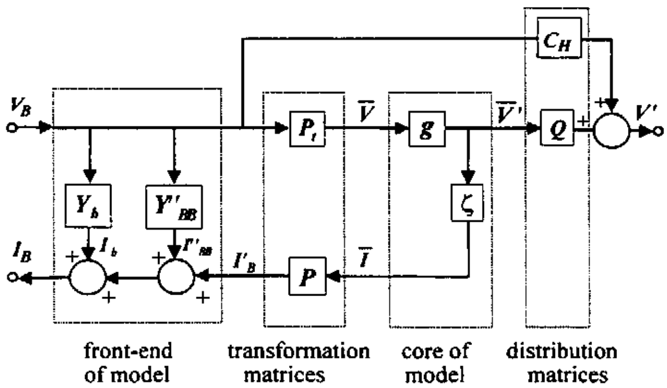


Fig. 1 *Transformer modal model*

The model has an admittance equation

$$I_B = \{Y_b + Y''_{BB} + P\zeta gP_t\}\ V_B = Y_B V_B \tag{1}$$

while the voltages at the internal nodes are given by

$$V' = (QgP_t + C_H)V_B \tag{2}$$

The matrices Q, P, ζ, g, P_t, Y_b, Y''_{BB} and C_H are defined in [1-4].

III. STRUCTURAL FEATURES OF THE MODEL

Eqn. 1, the admittance equation for the modal model, can be split into three distinctive parts, that dominated by Y_b, that dominated by Y''_{BB} and that where the term $P\zeta gP_t$ is at its most dominant.

Y_b has the identifiable physical significance of completely representing any transformer in the case where capacitive effects are suppressed (or ignored). In particular, it accounts for normal transformer action (turns ratio, leakage reactance). It follows that Y_b is necessarily inductive in character (albeit lossy and frequency-dependent) and hence dominates at low frequencies.

Y''_{BB} is always purely capacitive if the capacitances of the original lumped-parameter representation of the transformer are assumed to be lossless and frequency-independent. It corresponds to the input admittance to a transformer when all the inductive couplings are suppressed. Hence, the Y''_{BB} matrix swamps out all other effects at high frequencies.

The term $P\zeta g P_t$ models the resonant effects which occur during the transition from the low-frequency behaviour of the transformer to the high-frequency behaviour of the transformer.

Knowledge of the frequencies at which particular terms in the model dominate is of great benefit in a tuning procedure since it is only necessary to consider those terms when tuning the model to match the exact spectra in the corresponding frequency range (Section VI).

Eqn. 2 defines the relationship between the voltages at the internal nodes along the transformer and those at the boundary terminals. It can be rewritten as

$$V' = C_H V_B + QgQ^{-1}(C_L - C_H)V_B \qquad (3)$$

where P_t in eqn. 2 is replaced by

$$P_t = Q^{-1}[C_L - C_H] \qquad (4)$$

As shown in [2], C_L may be identified as the low frequency component of the solution and corresponds to the quasi-final distribution of classical transformer theory. I.e. as $s \to 0$, where s is the Laplace transform parameter,

$$V' = C_L V_B \qquad (5)$$

Similarly, C_H may be identified as having the physical significance of corresponding to the initial distribution of classical analysis. I.e. as $s \to \infty$

$$V' = C_H V_B \qquad (6)$$

g is a diagonal matrix whose elements are defined as modal transfer functions. As identified in [4], the character of the modal transfer functions is that of lightly-damped second-order low-pass filters. The elements of g model the natural resonances of the transformer and are approximated using transfer functions of the form

$$g_j(s) = \frac{\omega_j^2}{s^2 + 2\delta_j \omega_j s + \omega_j^2} \qquad (7)$$

It is straightforward to realise transfer functions of the form of eqn. 7 with simple RLC circuits as shown in [4] where the capacitance values, C_j, may be determined from the diagonal matrix ζ since $\zeta = sC$. (The subscript j denotes the jth mode).

Each column of Q spatially distributes the corresponding modal output voltage, $\overline{V_j}'$, to the internal nodes of the discrete transformer representation.

IV. 150MVA TRANSFORMER MODEL

In this section, the nature of the elements of the modal model for a 220kV/66kV 150MVA disk-winding transformer is established. The high-voltage winding consists of two 664-turn windings wound in parallel. The low-voltage winding consists of 200 turns. Details of the transformer core are given in the Appendix. The goal in this section is to confirm that the general characteristics of the key elements of the modal model for a large power transformer are identical to those identified previously for a 25kVA transformer [4]. Furthermore, in order to be suitable for tuning, it is vital that the basic structure of the elements of the model remains unchanged when input design data values are varied. Since accurate capacitance values are crucial to the accurate modelling of resonant effects, the present section includes results clarifying the consequences of using incorrect capacitance values. As a test case, inter-winding and ground capacitance values which are incorrect by a factor of two have been used. Other test cases confirm the findings.

Fig. 2 shows the amplitude spectra of the modal transfer functions (the elements of g) for modes 1-3. Fig. 3 shows the corresponding amplitude spectra when the capacitance values are altered.

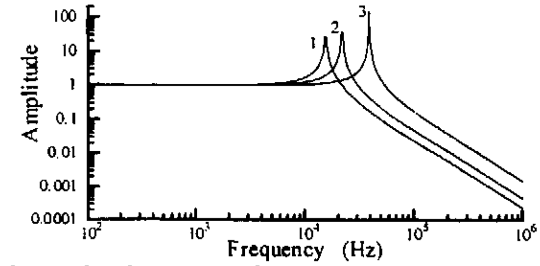


Fig. 2 *Amplitude spectra of modal transfer functions*

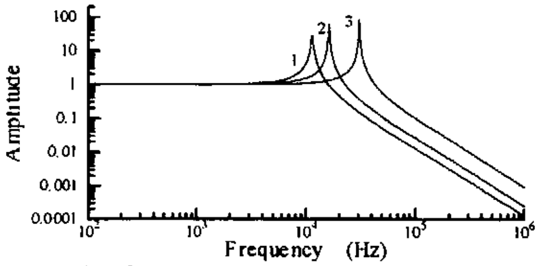


Fig. 3 *Amplitude spectra with inter-winding and ground capacitance values doubled from base case*

What is important to note regarding Figs. 2 and 3 is that the amplitude spectra of the modal transfer functions follow the same general nature regardless of the accuracy of the input data. (The same is true of the phase characteristic of the modal transfer functions). The amplitude spectra are initially unity before rising to a peak after which they decay to zero. This allows the same type of circuit approximation (eqn. 7) to be used regardless of the actual transformer dimensions or details.

The elements of the modal distribution matrix, Q, may be shown to be, to all intents and purposes, real and independent of frequency. Fig. 4 shows the distribution associated with the first mode along both the high and low voltage windings. Superimposed on this is the corresponding modal distribution when the ground and inter-winding capacitance values are doubled (dashed line). For this mode and all other modes, the modal distributions defined by the columns of Q are well-behaved functions. Furthermore, the distributions are only affected to a minimal extent by changes in the capacitance values (as illustrated here) or by changes in the impedance formula parameters. This is important since the ultimate goal is to find simple approximating functions to these distributions with the function variables determined from an optimization routine.

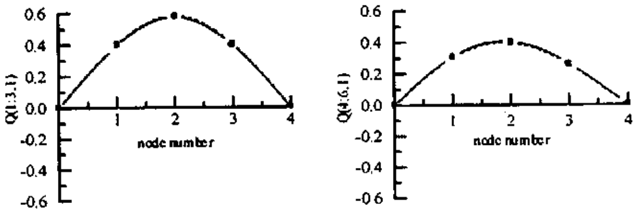


Fig. 4a *Modal distributions along HV windings*

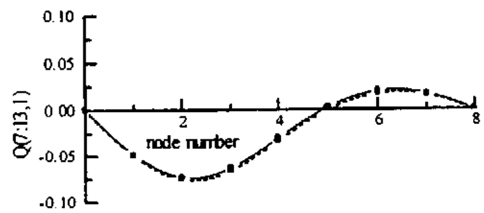


Fig. 4b *Modal distributions along LV winding*

The distributions defined by the columns of the C_H matrix correspond to the high frequency or initial distribution of classical theory. Fig. 5 shows the high-frequency distribution along the windings when a unit of voltage is applied to the sending-end of the high-voltage parallel windings with all other terminals grounded. Again, the distributions maintain the same generic nature despite the changes in the transformer data.

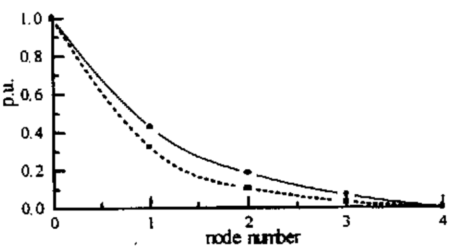


Fig. 5a *C_H distributions along the HV windings*

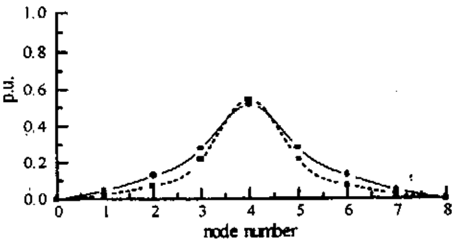


Fig. 5b *C_H distributions along the LV winding*

The columns of the C_L matrix define the low-frequency distributions along the transformer windings. Fig. 6 shows the low-frequency distribution along the windings when a unit of voltage is applied to the input terminal of the HV parallel windings with all other terminals grounded. Superimposed on this is the distribution obtained when impedance formula parameters, ρ and α, [5] are halved (dashed line).

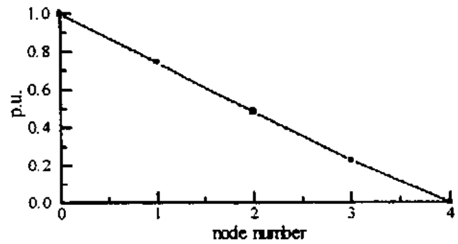


Fig. 6a *C_L distributions along HV windings*

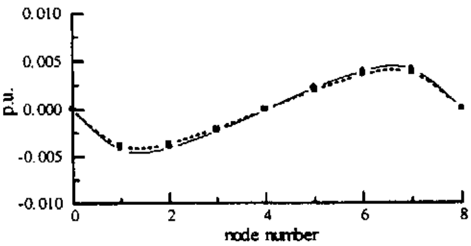


Fig. 6b *C_L distributions along LV winding*

The above results clarify the structural nature of the MODAL model for large power transformers. Each distribution is seen to have a basic structural format which is important in that it facilitates a corresponding generic function approximation thereby making the model structure ideally suited to tuning.

V. OPTIMIZATION TECHNIQUES

In order to tune a model, it is first necessary to define an error function or cost function which when minimised defines an optimal set of model parameters. The function used in the paper is

$$f = \sqrt{\frac{\sum_{n=1}^{N}\left(|F_n| - \left|\overline{F_n}\right|\right)^2}{\sum_{n=1}^{N}|F_n|^2}} \qquad (8)$$

where F_n represent the exact or measured data points while $\overline{F_n}$ represent the estimated or simulated data points. N is the number of data points.

Having determined the error function, it is necessary to minimise the function. Different procedures are required depending on whether the function is univariate or multivariate. In addition, there may be certain constraints on the variables e.g. it may be necessary to impose a constraint that all variables are positive if they correspond to capacitance or inductance values or if a pole is to be in the left-half of the plane in the s-domain. Numerous methods exist for minimising functions. However, in this work the Golden Section search and parabolic interpolation methods [8] are used for one-dimensional minimisation while Quasi-Newton methods and the Nelder-Mead algorithm [6-8] are used for multidimensional minimisation. Where constrained minimisation is required Active Set Methods are used [6].

VI. TUNING PROCESS FOR A MODAL MODEL

In this work, the measured data available for use is taken as the set of admittance measurements made at the transformer terminals. Hence, the first requirement in the process involves obtaining the exact admittance spectra for each of the elements of the Y_B matrix in eqn. 1. For example, $Y_B(1,1)$ corresponds to the input admittance of a transformer when all the other terminals are grounded.

To help explain and illustrate the tuning methodology, a lossless single winding will be considered when describing the technique.

The first step in the process involves approximating Y_b. In the case of a lossless model, $Y_b = 1/sE$ where E is purely real. Hence, the elements of E can be obtained by applying a univariate optimization algorithm to the low frequency section of each of the admittance spectra. In a lossy case, the elements of Y_b are approximated as shown in [4] using RL circuits. The RL values are obtained by applying a multidimensional optimization routine to the low-frequency section of the admittance spectra.

The second step involves identifying the elements of the Y''_{BB} matrix. If dielectric losses are ignored, $Y''_{BB} = sC_{BB}$. Hence, the elements of C_{BB} can be obtained by applying a univariate optimization algorithm to the high-frequency section of each of the admittance spectra.

The resonant frequencies are adjusted by changing the values of the L_j and C_j (i.e. the values of the elements of the modal circuits) since $\omega_j = 1/\sqrt{L_j C_j}$. (In a lossy case, the values of R_j can be varied to adjust the damping of the resonant peaks).

The final step involves identifying the elements of the Q, C_H and C_L matrices. Note that P_t is defined as $Q^{-1}(C_L - C_H)$. Hence, having defined Q, C_L and C_H, P and P_t (transpose of P) are automatically defined. It is required that each column of Q, C_H (if the capacitance representation is in error) and C_L (if the impedance parameters are inaccurate) be approximated by a relatively simple function with a minimal number of variables. The type of function is not crucial. However, suitable functions can be identified based on knowledge of the behaviour of a particular transformer structure. For example, the columns of Q for a uniform single winding are quasi-sinusoidal in nature. Hence, in the illustrative case of a lossless winding, the jth column of Q could be approximated as $K_{qj}\sin(j\pi x/l)$, where l is the axial length of the winding and x measures distance along the winding where x=0 is at the sending-end terminal of the winding.

For a lossless winding with a uniform capacitance distribution, the initial voltage distribution along the winding when the remote end of the winding is grounded can be theoretically evaluated as

$$v(x) = \sinh(\gamma(l-x))/\sinh(\gamma l) \quad (9)$$

where $\gamma = \sqrt{C/K}$, C is the capacitance to ground per unit length and K is the capacitance between the turns per unit length. As a consequence of this, a function of this type or one similar in nature, can be used to approximate the elements of C_H when uniform capacitance distributions are considered. Other exponential-type functions can be used when non-uniform distributions are involved.

The matrix C_L corresponds to the quasi-final distribution of classical theory and is determined by the inductance network. The distributions defined by the columns of C_L are very nearly linear in nature for a uniform single winding.

Having decided on approximating functions for the model elements, the unknown variables can be obtained using a multidimensional optimization strategy.

VII. ILLUSTRATIVE EXAMPLE

Consider a six section lossless winding details of which are given in the Appendix. This is defined to be the reference model. Note that the capacitance representation involves capacitances between each and every other node.

However, suppose that an initial modal model was formed on the assumption that only series capacitance between adjacent nodes was to be taken into account. The series capacitance per section was taken as 400pF and the shunt capacitance per section was taken as 300pF. Fig. 7 shows the exact admittance spectra obtained from the defined reference model when the correct capacitance representation inclusive of all capacitive effects is used. Superimposed on these are the admittance spectra obtained when the capacitance representation inclusive only of series capacitance between adjacent nodes is used. Fig. 8 shows the corresponding unit-step responses at the remote end of the winding.

Using the procedure outlined in Section VI, the initial model can be tuned so as to match the exact spectra. Fig. 9 shows the amplitude spectra from the adjusted model (dashed line) superimposed on the exact spectra (solid line). Fig. 10 shows the corresponding step responses. Note that for this case the first column of C_H is approximated by a function of the form $e^{-(\gamma i^r)}$ where i is the node along the winding. The second column of C_H is identical to the first but in reverse order. The columns of Q are approximated as $K_{qj}\sin(j\pi x/l)$. The correct resonant frequencies are obtained by adjusting the values of C_j. A univariate optimization algorithm is used to obtain the values of E and C_{BB}. Table 1 in the Appendix gives the values of the tuned variables. It is clearly evident from the results shown in Figs. 9 and 10 that remarkable accuracy has been achieved for this particular case.

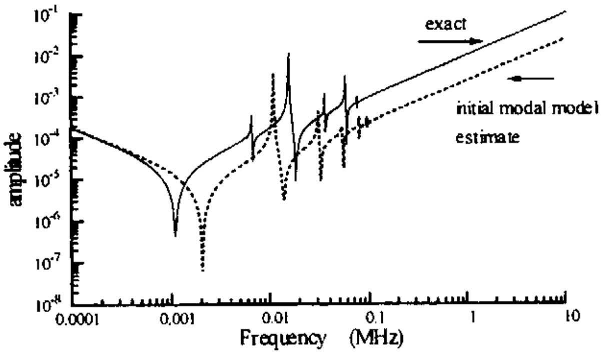


Fig. 7a *Amplitude spectra of $Y_B(1,1)$*

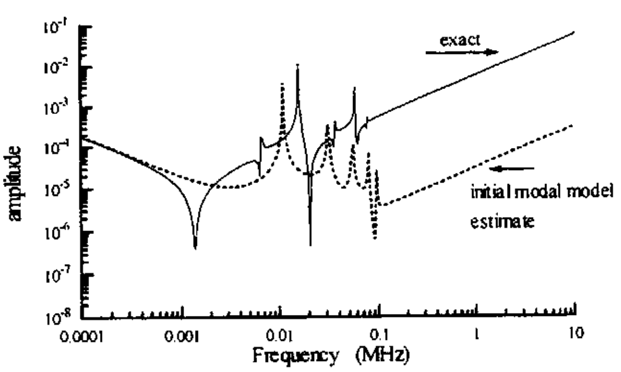


Fig. 7b *Amplitude spectra of $Y_B(1,2)$*

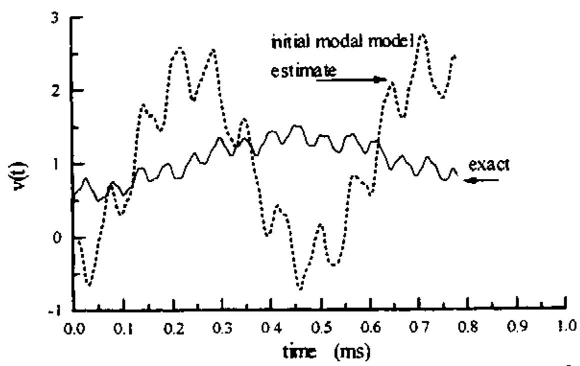


Fig. 8 *Open-circuit step-responses at remote-end of winding*

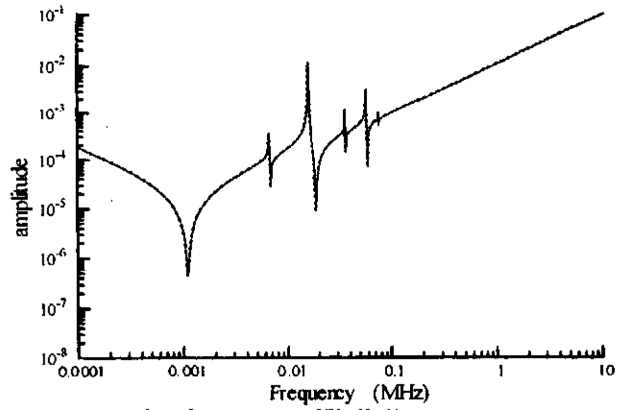


Fig. 9a *Amplitude spectra of $Y_B(1,1)$*
——— Exact ············ Adjusted model

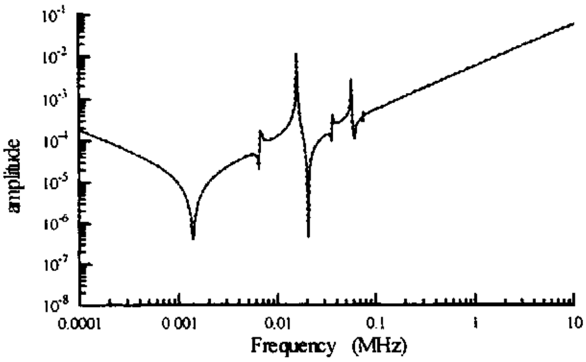


Fig. 9b *Amplitude spectra of $Y_B(1,2)$*
——— Exact ············ Adjusted model

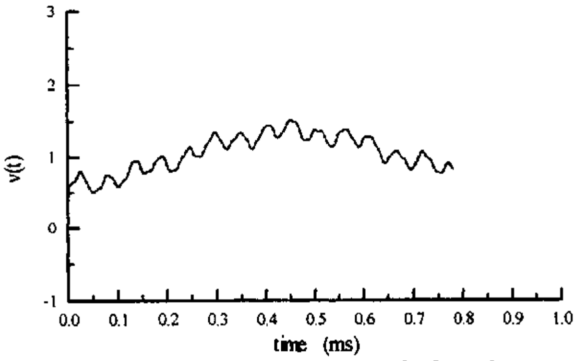


Fig. 10 *Step-responses at remote-end of winding*
——— Exact ············ Adjusted model

VIII. CONCLUSIONS

The paper has investigated the nature of a modal model for a large power transformer. It has been shown, at least for a disk-winding transformer, that the character of the model remains unchanged from that established in previous work for a 25kVA transformer [3-4].

The paper proceeds to examine the effects on the elements of the MODAL model of varying physical parameters of the transformer. The effects are seen to be such that the model maintains its general structure, thereby clarifying the inherent suitability of the modal model structure for tuning. The paper proposes a tuning methodology which is shown to be effective for a simple illustrative test case.

In principle, the modelling technique described in the paper is applicable up to quite high frequencies. In practice, it would appear that natural frequencies, directly associated with the windings, are heavily damped beyond about 1MHz – at which stage the windings themselves exhibit a largely capacitive nature which can be identified by the proposed method. It is acknowledged that making measurements at high frequencies could prove problematic. In real life situations, resonant transient overvoltages will be governed as much by external conditions as conditions internal to the transformer (e.g. capacitances and inductances associated with substation busbars, incoming feeders, etc.). The authors have not tried to model these latter effects and acknowledge that accurate prediction of transient effects at very high frequencies would need more than just an accurate transformer model. Also, wave propagation effects through bushings etc. would need accurate modelling. In short, the bandwidth attainable from the model can only finally be determined by seeing up to what frequency the proposed tuning method would be effective in practice if reliable high-frequency test measurements were available.

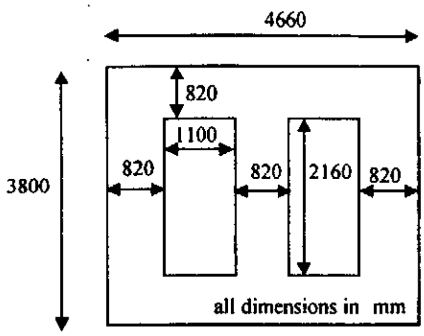
The structure of the model allows for non-linear effects associated with the iron core to be included. Indeed, it is a feature of the model structure that it actually detaches low-frequency behaviour from resonant effects at high frequencies. It is simply a matter of "bolting" any acceptable low-frequency model onto the FRONT-END of the model structure. Low-frequency effects are in fact modelled by Y_b in the given model. The model then effectively connects a series of admittances in parallel with Y_b to account for high-frequency effects. It may be noted that the lowest natural resonant frequency of a practical power transformer would appear to be about 10kHz at which value non-linear effects may be reasonably neglected. Thus, the attachment of any satisfactory non-linear low-frequency EMTP model in lieu of the authors' Y_b would extend the range of frequencies from very low to quite high (circa 1MHz).

It is worth noting that in a real transformer the number of natural modes can be established by simply counting the humps in a measured terminal admittance frequency response. If, for example, there were seven such humps (clearly distinguishable before the dominant capacitive effects take over) then the transformer could be modelled to sufficient accuracy using a minimum of eight sections. Increasing the number of sections simply increases the level of accuracy.

Finally, to conclude, the principal advantage of the proposed MODAL model approach (based as it is on standing waves and corresponding natural resonant frequencies) is that it is enormously more efficient than directly modelling the elements of a multiterminal admittance matrix (just as Wedepohl's modal decomposition method is more efficient in modelling a six-phase line, say, than direct modelling in the phase domain).

IX. APPENDIX

150MVA Transformer core



Core parameters ρ, α [5] are set to 0.5Ωm and 8×10^{-8}, respectively. The ratio of axial to radial permeability is 10. Relative permeability in the axial direction is 343.

Parameters of illustrative 6-section lossless winding

Inductance values	*Capacitance Values*
L_{ii}=0.270H i=1,2,...,6	C_{ii}=300pF i=1...5 C_{00}=C_{66}=150pF
$L_{12}=L_{23}=L_{34}=L_{45}=L_{56}$=0.255H	$C_{01}=C_{12}=C_{23}=C_{34}=C_{45}=C_{56}$=400pF
$L_{13}=L_{24}=L_{35}=L_{46}$=0.235H	$C_{02}=C_{13}=C_{24}=C_{35}=C_{46}$= 360pF
$L_{14}=L_{25}=L_{36}$=0.216H	$C_{03}=C_{14}=C_{25}=C_{36}$= 340pF
$L_{15}=L_{26}$=0.199H	$C_{04}=C_{15}=C_{26}$= 328pF
L_{16}=0.183H	$C_{05}=C_{16}$=320pF C_{06}=316pF

Table 1: Values of tuned variables

γ	*1.1196*	L_j	C_j ($\times10^{-8}$)
r	*0.8768*	*0.5038*	*0.1131*
K_{q1}	*1.7705*	*0.0366*	*0.2752*
K_{q2}	*0.3565*	*0.0076*	*0.2530*
K_{q3}	*0.8195*	*0.0026*	*0.2871*
K_{q4}	*0.2224*	*0.0015*	*0.2884*
K_{q5}	*0.6318*		
$\boldsymbol{E}(1,1)$	*0.1175*	$\boldsymbol{C_{BB}}(1,1)$	0.1601×10^{-8}
$\boldsymbol{E}(1,2)$	*-0.1175*	$\boldsymbol{C_{BB}}(1,2)$	-0.0926×10^{-8}

X. REFERENCES

[1] D.J. Wilcox, "Theory of transformer modelling using modal analysis", *IEE Proc. C*, **138**, March 1991, pp. 121-128

[2] D.J. Wilcox and T.P. McHale, "Modified theory of modal analysis for the modelling of multiwinding transformers", *IEE Proc. C*, **139**, Nov. 1992, pp. 505-512

[3] D.J. Wilcox, W.G. Hurley, T.P. McHale and M. Conlon, "Application of modified modal theory in the modelling of practical transformers", *IEE Proc. C*, **139**, Nov. 1992, pp. 513-520

[4] D.J. Wilcox, M. Condon, D.J. Leonard and T.P. McHale, "Time-domain modelling of power transformers using modal analysis", *IEE Proc.- Electr. Power Appl.*, **144**, March 1997, pp. 77-84

[5] D.J. Wilcox, W.G. Hurley and M. Conlon, "Calculation of self and mutual impedances between sections of transformer windings", *IEE Proc. C*, **136**, Sept. 1989, pp. 308-314

[6] P.E. Gill, W. Murray, and M.H. Wright, "*Practical optimization*", Academic Press Inc., London, 1981.

[7] P.R. Abdy and M.A.H. Dempster, "*Introduction to optimization methods*", Chapman and Hall, London, 1974

[8] W.H. Press, B.P. Flannery, S.A. Teukolsky and W.T. Vetterling, "*Numerical Recipes in C*", Cambridge University Press, 1988