Analysis of Torsional Torques of Big Turbine-Generator Shafts

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Abstract - **In this work the mathematical model for calculating and analyzing the torsional torques of big turbine-generator shafts that occur during various kinds of disturbances in the power network is described. On the basis of the mathematical model the software package tested on an appropriate example is designed. Special attention is dedicated to torque analyzis at malsynchronization and at high-speed reclosure. It is shown that in the certain cases, the torsional torques of the shaft become several time larger then the rated ones, even of the torques occurring at three-pole short-circuit on generator bus-bars. It is also performed the spectral analysis of the time course of the torsional torques.**

Keywords: Turbogenerator, Malsynchronization, High-speed reclosure, Steam turbine, Torsional torques, Modelling

I. INTRODUCTION

Mechanical construction of multistage steam turbine and generator is very sensitive to torsional tensions because of big shaft lengths. Any kind of disturbances causes torsional oscillations of turbine and generator rotors which impose additional shaft strains. This additional torsional strain of the shaft depends on the moment of inertia of individual turbine stages, shaft characteristics, type of disturbance and parameters of protective relays. Several cases of failures of the turbine-generator shaft were noticed in the world [1 - 3] and almost all the failures were assigned to the additional strains due to oscillations.

Researches beginning in [4] are continued in this work. Special attention is dedicated to torsional oscillations of turbine and generator shafts occurring at malsynhronization of generator and at automatic reclosure after short-circuits in network (three-phase, double-phase, double-phase to ground and single-phase fault). The mentioned disturbances were analysed in the cases of voltage, angle and velocity mismatches related to system and for different times of automatic high-speed reclosure and their impact to intensity of torsional torques of each shafts.

For a long time, autmatic high-speed reclosure (AR) is accepted praxis in many countries. The aim is to return the system in previous state in the order to save stability of system, from one hand, and to minimise the time of consumers interrupting supply, from the other hand. Autmatic high-speed reclosure , as a rule, ensures quick synchronity restitution. During disturbance, the generator has asynchronous operation that affects to velocity and voltage angle changing. Significant currents and electromagnetic torques may occur at switch reclosure which are dangerous on system elements in electrical and mechanical view. Specially dangerous is unsuccessfully AR. Then turbine-generator shaft accepts new stroke and because of it existing oscillations may increase considerably. Shaft oscillations, torsional torques and strains increasing depends on AR time and fault duration. All of it should be studied to avoid possible damages.

Calculation is implemented at example of the generator connected through the transformer and double line to infinite network. Next chapter shows mathematical model of problem and calculation results.

II. MATHEMATICAL MODEL

The electromechanical system consisting of a multistage turbine and generator *G* connected through a transformer *T* and power line *L* on the infinite network, Fig. 1, is considered. The faults at the beginning of the line (location k) were analysed. The elements were described by dq0 components where the d and q axes were related to the machine rotor.

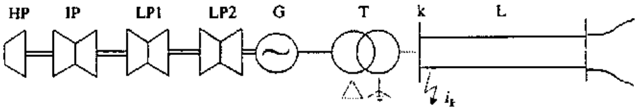

Fig. 1 Considered system

A. Generator Model

The synchronous generator is described by fluxes and voltage equations using the standard Park's model [4 - 6] with one damping contour in each axis.
The equations of the electromagnetic balance are:

$$[U] = [R][I] + \frac{d}{dt}[\Phi] + \omega[\Phi] \tag{1}$$

$$[\Phi] = [L]\cdot[I] \tag{2}$$

- equations of rotor motion:

$$\frac{d\omega}{dt} = \frac{T_T - T_e}{J_1} = \frac{T_T - (\Phi_d i_q - \Phi_q i_d)}{J_1} \tag{3}$$

$$\frac{d\theta}{dt} = \omega \tag{4}$$

In the above equations the elements of the matrices $[\Phi]$, $[L]$, $[U]$, $[R]$ and $[I]$ are flux, inductance, voltage, resistance and current variables, respectively.

B. Model of transmission elements

Transformer and power line, through which the generator is connected to a power network, are presented with the complete series of resistances R and inductivities L.

The equations of voltage balance in RL branch expressed by dq0 components are:

$$u_{1d} - u_{2d} = L\frac{di_d}{dt} + \omega L i_q + R i_d \tag{5.1}$$

$$u_{1q} - u_{2q} = L\frac{di_q}{dt} - \omega L i_d + R i_q \tag{.5.2}$$

$$u_{10} - u_{20} = L\frac{di_0}{dt} + R i_0 \tag{5.3}$$

C. Modelling of fault state

The fault state is modelled by the equations of voltage and current balance written for the fault location according to the 1-st and 2-nd Kirchoff's law. The equations of the boundary conditions characterising the type of fault were added to these ones [6].

D. Model of mechanical system

Rotors of the multistage high power steam turbines are completely separated, and shafts are connected by the couplings. This enables turbine-generator system, although with distributed masses, to be treated as a discrete torsional system consisting of the stiff discs interconnected with elastic shafts. To each turbine and generator rotor one disc corresponds. The discs possess a mass that is acquired through moment of inertia J, while the shafts are assumed to be massless but posses the feature of elasticity that is acquired through stiffness coefficient K. The stiffness coefficients of the shaft system depend on shaft dimensions and properties of material of which the shafts are made.

For complete model presentation, in order to study torsional oscillations, the damping effect must be taken into consideration. There are two types of damping: internal and external. In both types of damping it is assumed that the damping coefficients D for specified disc or shaft are constant and that the corresponding torques linearly depend on relative angular velocities. Assuming this supposition about damping, the differential equations describing the torsional oscillations of the system become linear, so that their solution becomes easier.

The complete equivalent of the mechanical system with four turbines and one generator masses with damping is given on Fig. 2.

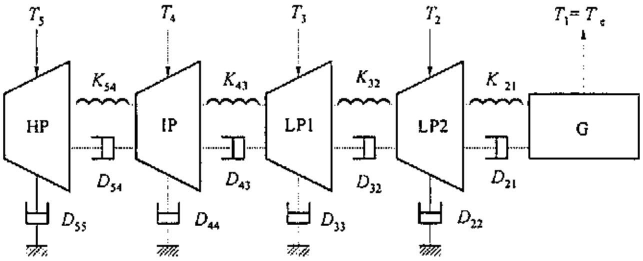

Fig. 2 Model of mechanical system

The external turbine torques T_i and electromagnetic torque of generator T_e act on this system. In the stationary state, at constant speed, the turbine torque is balanced with electromagnetic torque of the generator, and due to property of shaft elasticity the discs of individual masses are rotated for the angle θ_i measured in relation to reference synchronous axis.

Movements of certain system discs from Fig. 2 are described with the system of second order differential equations [2, 3, 4, 7]:

$$[J]p^2\theta + [D]p\theta + [K]\theta + [T] = 0 \tag{6}$$

This system may be reduced on two systems of first-order differential equations as:

$$p\theta_j = \omega_j \tag{7.1}$$

$$\begin{aligned} p\omega_j = \{ & T_{j,in} - T_{j,out} \\ & + D_{j-1,j}(\omega_{j-1} - \omega_j) + D_{j,j+1}(\omega_{j+1} - \omega_j) - D_{j,j}\omega_j \\ & + K_{j,j+1}(\theta_{j+1} - \theta_j) + K_{j-1,j}(\theta_{j-1} - \theta_j)\} / J_j; \qquad j = 1,\ldots,5 \end{aligned} \tag{7.2}$$

For the mechanical model on Fig. 2 is:

$$D_{10} = D_{65} = K_{10} = K_{65} = 0$$

For the complete computation of the electromagnetic and mechanical transients at fault in the system, the system of differential equations (1) and (7) should be solved simultaneously. In that way, the system of 17 first-order differential equations are obtained for the tree-phase and double-phase faults, and 19 differential equations for single-phase and double-phase to ground fault. This system can be solved by any of the numerical methods for solving the system of differential equations. In this work the fourth order Runge-Kutta method is used, and the results of the calculation are given in the remainder of this text.

III. NUMERICAL EXAMPLES AND RESULT ANALYSIS

Torsional torques calculation, according to presented model, is implemented on the example of turbine-generator shaft where generator *G* is connected through transformer *T* and double line *L* to infinite bus, Fig. 1. The Generator operates at it's bus-bars with rated power P=0.85 (basic power equals rated apparent one) at power factor cosφ =0.85 and voltage U=1. Parameters of the system are given in Appendix.

The calculation is performed for the following disturbances - faults:

1. Malsychronization of generator
2. Tree-phase fault (3p)
3. Double-phase to ground fault (2p+g)
4. Double -phase fault (2p)
5. Single-phase fault (1p)

In this work will be presented just a few of results relating to malsychronization from non-loaded state and to unsuccessfully AR.

Malsynchronization causes mismatches of generator velocity, voltage and angle in comparison with the same network quantities. Any quantity mismatch can have difficult consequences on generator and whole system. Fig. 3 shows phase currents amplitudes and electromagnetic and torsional torques amplitudes of each shafts in dependence on malsynchronization angle. Current gets maximal value at angle 180^0, when the voltages are opposite, that is expected. However, maximal values of electromagnetic and torsional torques appear at lower and mutually different angles. So, for example, electromagnetic torque gets maximum at angle 128°, torsional torque T_{21} at angle 112^0, T_{32} at angle 109^0, T_{43} at angle 114^0 and T_{54} at angle 113^0. It should be noticed that usually in literature is assumed angle 120^0 as critical. This example and many others considered by authors shows that maximum of torques appears in the range of angles from 110^0 to 140^0, which depends on generator parameters.

Time functions of phase currents and torques at malsynchronization angle of 128^0 are shown in Fig. 4 and Fig. 5.

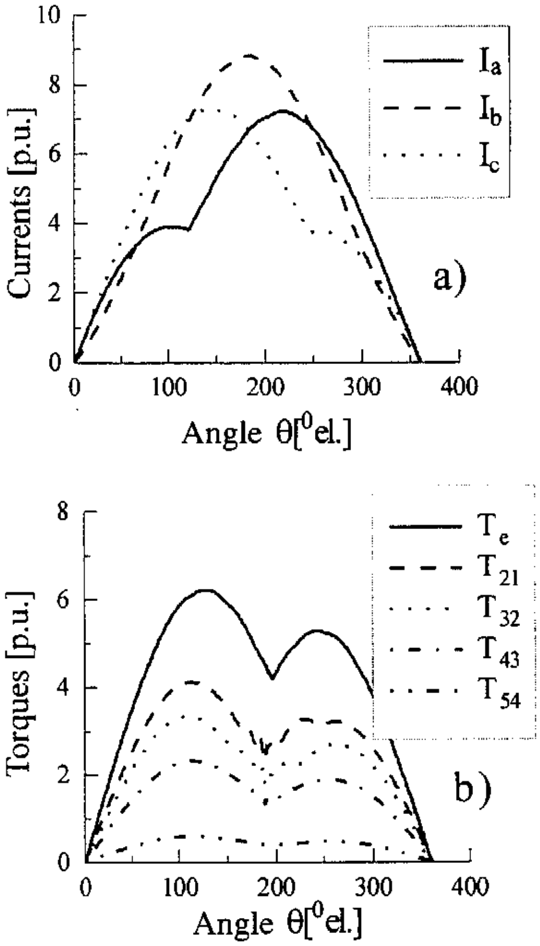

Fig. 3 Dependence of currents (a) and torques (b) on malsynchronization angle

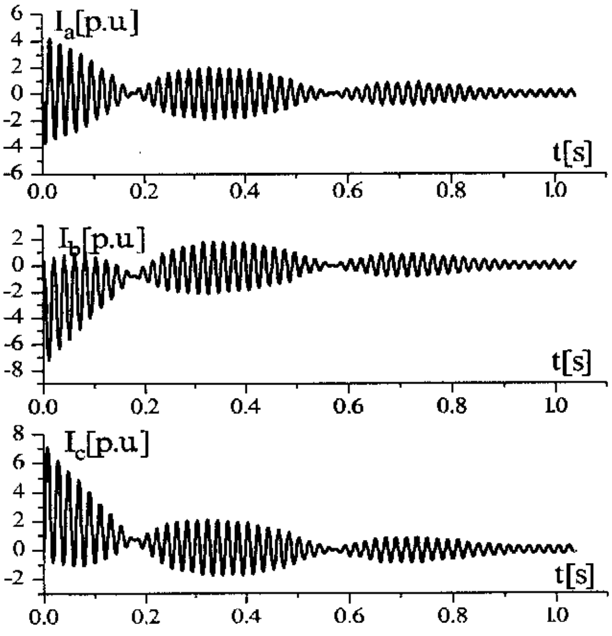

Fig. 4 Time function of phase currents at malsynchronization angle θ=128^0

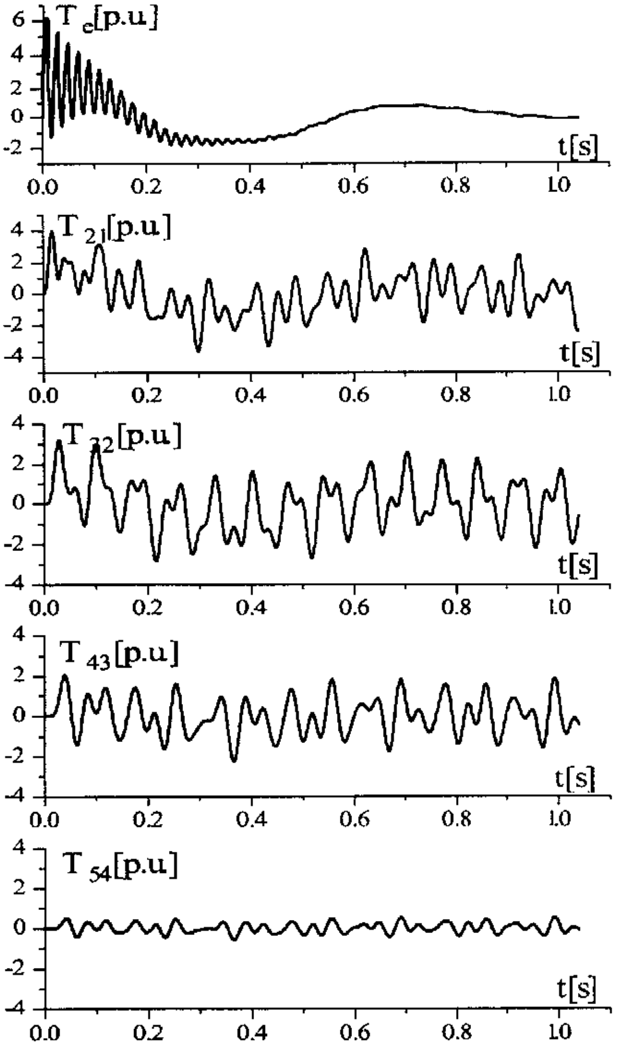

Fig. 5 Time function of torques at malsynchronization angle $\theta=128^0$

Velocity mismatch impact to maximal values of currents and torques is shown on Fig. 6. It is supposed that fault appears at the beginning of one line and the switches disconnect faulted line after 0.12 s and then after t_{AR} turn on the line again. If this line still faulted, the switches would turn it of after 0.12s definitely. From the torque of disturbance appearance, masses of each discs (turbine rotors) start to oscillate. These oscillations continue after fault elimination. Line reclosure after t_{AR} imposes new conditions to whole oscillatory system. What effect of this commutation will be, depends, first of all, is this fault temporary (successful AR) or not (unsuccessful AR). The other case is certainly more difficult. Effect of faulted line reclosure depends on reclosure torque and it could happen that shaft system oscillations may increase. Analysis of time t_{AR} impact is implemented at successful and unsuccessful AR. Transient process is observed during 1s, and time t_{AR} is varied in borders (0.24-0.40)s. Maximal recorded values in aforementioned interval t_{AR}, for each types of faults, are shown on Fig. 7.

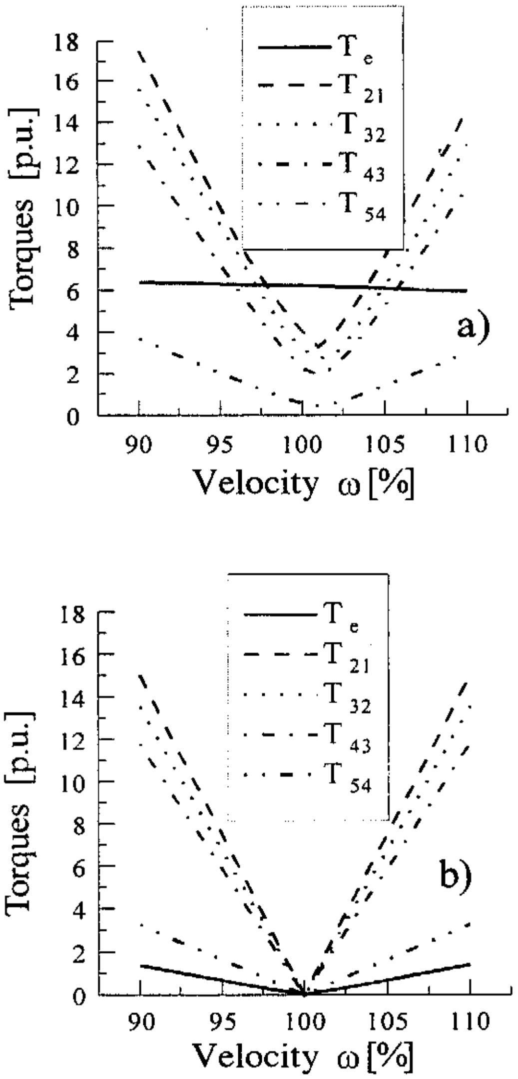

Fig. 6 Dependence of maximal torques on rotor velocity ω at malsynchronization for a) $\theta=128^0$ and b) $\theta=0^0$

The instant of reclosure essentially affects the maximal values of the torsional torques of each shafts. These values depend on fault duration. The extreme values of the maximal torques of shafts appear at various values of t_{AR}, so that no unique critical value of t_{AR} for all shafts can be established. Their values for discussed disturbances are given in Table T1.

Electromagnetic torque has maximal value at malsynchronization of the generator and minimal at three-phase fault. Torsional torques are not in accordance with exciting electromagnetic one, so maximal value of torsional torques is just at three-phase fault. During three-phase fault the generator is practically downloaded, so it's rotor rapidly accelerates. Rotor velocity mismatch in a torque of reclosure is, according Fig. 6, additional factor of torsional torques increasing. For this configuration of electromechanical system, the shaft 4-3 is most sensitive to network faults, so it could be considered as a critical.

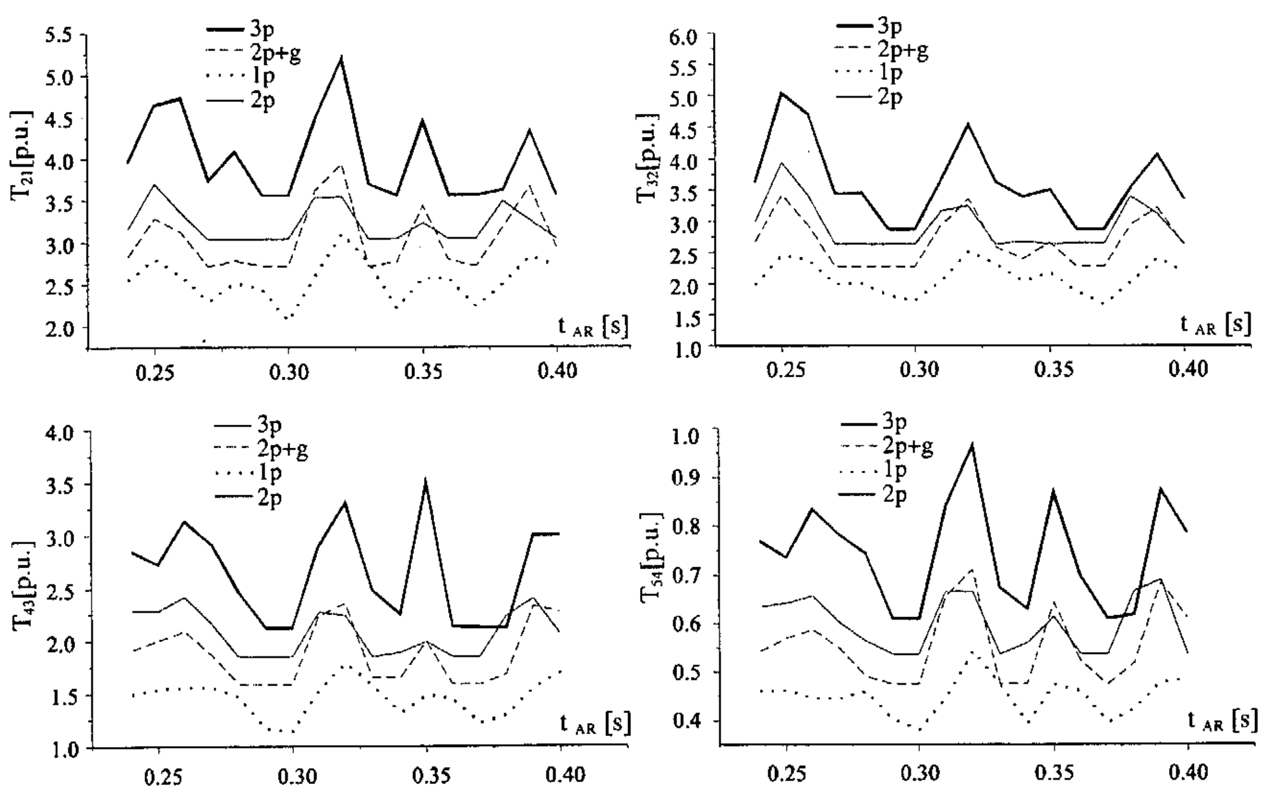

Fig. 7 Maximal values of torsional torques

Table T1 Extreme values of torques

Type of disturbance	El. mag. torque	Torsional torques			
	$T_1 = T_e$	T_{21}	T_{32}	T_{43}	T_{54}
Malsynchronization ω=1 and θ=128^0	6.21763	4.11239	3.33989	2.31758	0.59149
Tree-phase fault - (3p)	3.10246	3.5745	2.86204	2.12817	0.6096
Single-phase fault - (1p)	2.24203	1.51714	1.24885	0.67466	0.2425
Double-phase fault - (2p)	3.45253	3.04916	2.63176	1.61037	0.48075
Double-phase to ground fault (2p+g)	3.58085	2.72266	2.26663	1.358	0.4458
Repeated 3p	3.10246	5.2122	5.03701	3.31009	0.8729
Repeated 1p	3.53877	3.11191	2.51358	1.79123	0.54387
Repeated 2p	3.45253	3.71068	3.94147	2.42371	0.69106
Repeated 2p+g	3.58085	3.94897	3.42376	2.36123	0.71002
Rated values	0.8517	0.8517	0.5792	0.2066	0.13627
Overload coefficient	4.204	6.12	8.696	10.796	6.406

It is well known that fatigue and life expectancy of the materials for the dynamic system depend not only on maximal torques but also on the frequency of the torque oscillations. This is the reason for performing the spectral analysis of certain shaft torques. For that purpose the fast Fourier transform (FFT) is used. The spectar of torque amplitudes from Fig. 5 are shown on Fig. 8.

Since there are five rotation masses, five basic frequencies are clearly observable: $f_1 = 1.4010\,Hz$, $f_2 = 13.4096\,Hz$, $f_3 = 23.0165\,Hz$, $f_4 = 29.6213\,Hz$ and $f_5 = 39.9951\,Hz$. The first frequency of the electromechanical oscillations is determined primarily by the parameters of turbine-generator and parameters of transmission network, i.e. by impedance of connection of generator to powerful network. Other frequencies represents eigen frequencies of the mechanical part of shafts and rotation masses system.

Besides the above mentioned frequencies which reflect the nature of the mechanical system in the spectrum of electromagnetic torques frequencies, the frequencies that reflect the nature of electromagnetic process also occur. These are the frequencies in the range of 44-46 Hz at three-phase fault and in the ranges of 44-56 Hz and 92-108 Hz at double-phase fault. The later range of frequencies occurs at all asymmetrical faults as a consequence of negative sequence current. These higher frequencies that occur in electromagnetic torques practically are not sensed by the mechanical frame of turbine shafts.

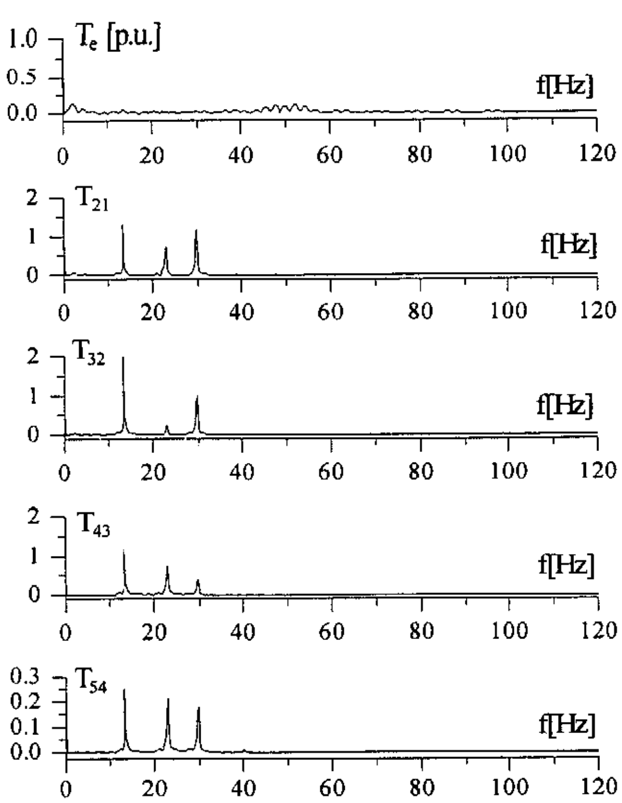

Fig. 8 Spectral composition of torques

The analysis of these frequencies from the aspect of shaft material and life expectancy exceeds the scope of this work and it is the subject of separate study.

IV. CONCLUSION

On the basic of performed calculations and analysis of the results the following conclusions may be derived:

The presented mathematical model offers, with satisfied accuracy, the possibility of studying torsional oscillations and shaft torques of large turbine-generator occurring at various disturbances in the power system. This model also allows the checking of the factors that affects the course of a transient.

Maximal torques and the most violable location depend on the specified parameters of turbine and generator so that the detailed computation must be performed for each configuration.

The biggest torsional torques of the shaft occur at repeated three-phase fault, i.e. at unsuccessful AR. Having in mind the other conveniences that AR offers concerning the overall safety of the system, AR should not be discarded except in special situations when the risk of the shaft is to high.

Extreme values of the torsional torques exceed the rated values several times so that the shaft strains should be checked in those situations as well as their influence on the life expectancy of the shafts.

Spectral analysis of torques may be the basis assessing the fatigue and life expectancy of the shafts. This analysis may also be used for establishing the most favorable parameters of the mechanical system in order to eliminate the critical vibrations.

V. REFERENCES

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VI. APPENDIX

Numerical data:

- generator:

$P_n = 660\,MW$, $U_n = 24\,kV$, $\cos\varphi_n = 0.85$, $n = 3000\,\mathrm{min}^{-1}$, $x_d'' = 0.2232$, $x_d' = 0.3178$, $x_d = 2.253$, $J_1 = 1.586\,s$

- transformer:

$S_n = 780\,MVA$, $u_k = 13.8\%$, $420/24\,kV$, $P_{Cun} = 1550\,kW$

- line:

$l = 250\,km$, $r = 0.031\,\Omega/km$, $x = 0.325\,\Omega/km$,

- turbines and shafts:

$J_2 = 1.788\,s$, $J_3 = 1.804\,s$, $J_4 = 0.502\,s$, $J_5 = 0.360\,s$, $K_{21} = 34.70$, $K_{21} = 34.7$, $K_{32} = 42.8$, $K_{43} = 57.0$, $K_{54} = 69.7$

$D_{21} = D_{32} = D_{43} = D_{54} = 0.005$,

$D_{22} = D_{33} = D_{44} = D_{55} = 0.0$