Dynamic Phasors in Modeling of Arcing Faults on Overhead Lines

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Abstract - **This paper describes a novel approach to dynamical modeling of arcing faults on overhead lines. The proposed technique is a polyphase generalization of the dynamic phasor approach from electric drives and power electronics. This technique is applicable to nonlinear models, and offers distinct advantages in simulation and control with respect to standard time-domain models. In a steady-state, the dynamic phasors reduce to standard symmetrical components from AC circuit theory. We present dynamical models and numerical experiments that illustrate the capabilities of dynamic phasors in analysis and simulation of arcing faults in power systems.**

Keywords: dynamic phasors, arcing faults, symmetrical components, analytical modeling, numerical simulation.

I. INTRODUCTION

Many elements in power networks have been undergoing profound changes in recent years. This process is primarily driven by a desire to increase efficiency, with concomitant reduction in energy costs, in power losses (and cooling requirements) and in component size. Power systems have a large number of components connected in a hierarchical, multilayered structure. The components exhibit various types of nonlinearities due to properties of materials, geometries of associated electromagnetic fields, and switching modes of operation. The continuous operation of these systems is made possible by feedback control. The control performance becomes critical for overall reliability when sudden and potentially detrimental transients are triggered by abrupt changes in the system environment and by load variations. Thus a precise characterization of such transients is of primary interest, particularly for emerging protection strategies that are based on signal processing and microcontrollers.

The voltages and currents in power networks, electric drives and power electronic converters are generally periodic, but often non-sinusoidal. Dynamics of interest for analysis and control are often those of deviations from periodic behavior. The standard analysis of power systems relies on simplifications gained by approximating certain quantities (e.g., voltages and currents) as sinusoids ("phasors"), possibly with "slowly varying" magnitude and phase (often defined in an implicit, heuristic way). This "sinusoidal quasi-steady-state" approximation is widely used to study electromechanical dynamics, and it is almost invariably included in software tools for power systems. There are, however, cases reported in the power system literature when this type of approximation leads to incorrect conclusions about stability [1, 2]. For faster phenomena, the primary tools are time-domain simulations that take no advantage of the particular nominal analytical form of the variables of interest. Time-domain simulations are not only a tremendous computational burden, but they also offer little insight into problem sensitivities to design quantities and no basis for design of protection schemes. This type of fundamental analytical problem is not mitigated by improvements in computational technology.

The modeling methodology presented in this paper builds on an existing, but non-systematic knowledge base in the field of power networks. Various forms of frequency selective analysis have deep roots in power engineering in the form of phasor-based dynamical models, and the main advantage of the analytical approach proposed here is its systematic derivation of phasor dynamics. The idea of deriving dynamical models for Fourier coefficients goes back to classical averaging theory. The dynamical equations for Fourier coefficients or *dynamic phasors* are *time-invariant* and often nonlinear, and their analytical usefulness stems from the availability of families of approximations that are based on physical insights offered by the underlying frequency decomposition.

Phasor dynamic models are typically developed from time-domain descriptions (differential equations) using the procedure that is denoted here as generalized averaging. In the case of nonlinear equations, a key element in the modeling process is the development of approximations to the right-hand side of the time-domain equations at a particular frequency. These approximations are based on the method of describing functions [3]. This paper presents analysis and simulations based on standard time-domain and on dynamic phasor models; the simulations are performed in Matlab.

Due to their time-invariance, dynamic phasor models hold promise in speeding-up simulations as they allow large time steps; our hope is that these models can be combined with time-domain models in simulation environments like EMTP/ATP to achieve improved overall performance.

The rest of the paper is organized as follows: Section II introduces the dynamic phasors for single phase and polyphase variables; Section III presents two models of an arcing fault; Section IV contains simulation results, while brief conclusions are stated in Section V.

II. Introduction to Dynamic Phasors

The generalized averaging that we perform to obtain our models is based on the property [4, 5] that a (possibly complex) time–domain waveform $x(\tau)$ can be represented on the interval $\tau \in (t-T, t]$ using a Fourier series of the form

$$x(\tau) = \sum_{k=-\infty}^{\infty} X_k(t) e^{jk\omega_s \tau} \qquad (1)$$

where $\omega_s = 2\pi/T$ and $X_k(t)$ are the complex Fourier coefficients, which we shall also refer to as *phasors*. These Fourier coefficients are functions of time since the interval under consideration slides as a function of time. We are interested in cases when only a few coefficients provide a good approximations of the original waveform, and those coefficients vary slowly with time. The k-th coefficient (or k-phasor) at time t is determined by the following *averaging* operation:

$$X_k(t) = \frac{1}{T}\int_{t-T}^{t} x(\tau) e^{-jk\omega_s \tau} d\tau = \langle x \rangle_k (t). \qquad (2)$$

Our analysis provides a dynamic model for the dominant Fourier series coefficients as the window of length T slides over the waveforms of interest. More specifically, we obtain a state-space model in which the coefficients in (2) are the state variables. Note that when our original waveforms $x(\cdot)$ are complex-valued, the phasor $\langle x \rangle_{-k}$ equals $\langle x^* \rangle_k^*$ (where x^* is the complex conjugate of x). However, in the general case there are no other relationships among the $+k$-th dynamical phasor of the waveform $\langle x \rangle_k$, the $-k$-th phasor $\langle x \rangle_{-k}$, and the $+k$-th phasor of the conjugate waveform $\langle x^* \rangle_k$. Our interest in complex-valued waveforms stems mostly from their use in applications - for example, complex space vectors [6] are widely employed in dynamical descriptions of electrical drives. In the case of real–valued time domain quantities $x(t) = x^*(t)$ and $X_{-k} = X_k^*$, so (1) can be rewritten as a one–sided summation involving twice the real parts $\Re$ of $X_k(t)e^{jk\omega_s t}$ for positive k. If in addition X_k time–invariant, the standard definition of phasors from circuit theory is recovered.

A key fact for our development is that the *derivative* of the k-th Fourier coefficient is given by the following expression:

$$\frac{dX_k}{dt} = \left\langle \frac{d}{dt} x \right\rangle_k - j\,k\,\omega_s X_k \qquad (3)$$

This formula is easily verified using (1) and (2), and integration by parts. Another straightforward, but very important result is that the phasor set of a product of two time–domain variables is obtained by a *convolution* of corresponding phasor sets of each variable.

The definitions given in (1) and (2) can be adjusted for the analysis of polyphase systems [7]. Let us consider the three phase ($a-b-c$) case, as the general polyphase case follows similarly. Following the standard notation, we introduce $\alpha = e^{j\frac{2\pi}{3}}$; then $\alpha^2 = \alpha^*$. Then a time-domain waveform can be written as :

$$\begin{bmatrix} x_a \\ x_b \\ x_c \end{bmatrix}(\tau) = \sum_{\ell=-\infty}^{\infty} e^{j\ell\omega_s \tau} \underbrace{\frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1 & 1 \\ \alpha^* & \alpha & 1 \\ \alpha & \alpha^* & 1 \end{bmatrix}}_{A} \begin{bmatrix} X_{p,\ell} \\ X_{n,\ell} \\ X_{z,\ell} \end{bmatrix}(t) \qquad (4)$$

and we denote the square transformation matrix with A. It can be checked that the A is unitary, as $A^{-1} = A^H$, where H denotes complex conjugate transpose (Hermitian). As commonly encountered in transforms, scaling factors other than $1/\sqrt{3}$ are possible in the definition of matrix A, but they require adjustments in the inverse transform. The coefficients in (4) are

$$\begin{bmatrix} X_{p,\ell} \\ X_{n,\ell} \\ X_{z,\ell} \end{bmatrix}(t) = \frac{1}{T}\int_{t-T}^{t} e^{-j\ell\omega_s \tau} A^H \begin{bmatrix} x_a \\ x_b \\ x_c \end{bmatrix}(\tau) d\tau \quad (5)$$

The equation (5) defines *dynamical* positive $X_{p,\ell}$, negative $X_{n,\ell}$, and zero–sequence $X_{z,\ell}$ symmetric components at frequency $\ell\,\omega_s$, as

$$\frac{d}{dt}\begin{bmatrix} X_{p,\ell} \\ X_{n,\ell} \\ X_{z,\ell} \end{bmatrix}(t) = A^H \begin{bmatrix} \langle \frac{d}{dt} x_a(t) \rangle_\ell \\ \langle \frac{d}{dt} x_b(t) \rangle_\ell \\ \langle \frac{d}{dt} x_c(t) \rangle_\ell \end{bmatrix} - j\ell\omega_s \begin{bmatrix} X_{p,\ell} \\ X_{n,\ell} \\ X_{z,\ell} \end{bmatrix}(t) \qquad (6)$$

Among the advantages of the proposed definitions are the compatibility with conventional symmetric components in a periodic steady-state, and a similarity to the single-phase case. Observe that (6) is a vector generalization of (3).

In applications, we are interested in cases when a finite (and preferably small) number of dynamic phasors is used in (4). From the presented definitions, it follows that the dynamical symmetrical components of complex-valued polyphase quantities are related as: $\langle x \rangle_{p,\ell} = \langle x^* \rangle_{n,-\ell}^*$, $\langle x \rangle_{n,\ell} = \langle x^* \rangle_{p,-\ell}^*$, and

$< x >_{z,\ell} = < x^* >^*_{z,-\ell}$. In particular, in the case of real-valued waveforms, ℓ in (4) ranges over the same positive and negative harmonics and $X_{p,\ell} = X^*_{n,-\ell}$, $X_{n,\ell} = X^*_{p,-\ell}$ and $X_{z,\ell} = X^*_{z,-\ell}$. Thus again the two sided summation in (4) can be replaced by a one-sided version, so that for example

$$x_a(\tau) = \frac{2}{\sqrt{3}}\Re[\sum_{\ell=0}^{\infty}(X_{p,\ell}(t)+X_{n,\ell}(t)+X_{z,\ell}(t))e^{j\ell\omega_s\tau}]-X_{z,0}$$

The last term takes care of accounting at $\ell = 0$, when $X_{p,0} = X^*_{n,0}$ and $X_{z,0}$ is real. Note that in the case of time-independent symmetric components, the standard definition from polyphase circuit theory is again recovered.

III. Model of the Arcing Fault

A. *Single Phase Model*

We first consider a single-phase description proposed in [8] as a model for a arcing fault on an overhead line. The only nonlinearity comes from the voltage drop on

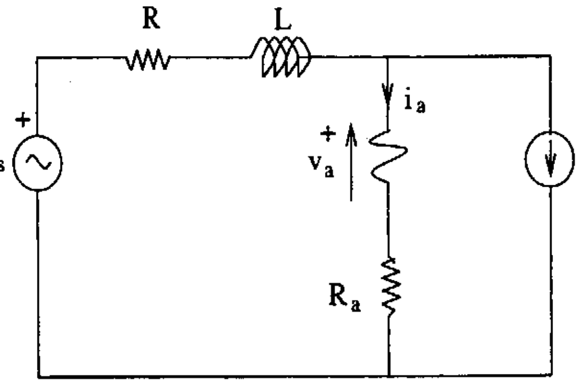

Figure 1: Equivalent circuit for the faulted phase.

the arc which is modeled as a DC voltage whose polarity reverses as the fault current changes sign:

$$v_a(t) = V_a\mathbf{sign}(i_a(t)). \quad (7)$$

Then we can write the following equation for the equivalent circuit in Fig.1 (with the explicit time-dependence suppressed to streamline the notation):

$$L\frac{di_a}{dt} = -(R+R_a)i_a - V_a\mathbf{sign}(i_a) + \underbrace{v_s - L\frac{di_L}{dt} - Ri_L}_{v} \quad (8)$$

where $v(\cdot)$ is a known function of time (evaluated from the pre-disturbance operating point).

Let us assume that i_a is well approximated by its first (fundamental) harmonic; then from (3) we can write a differential equation for the *complex dynamical phasor* $I_{a,1}$ as:

$$L\frac{dI_{a,1}}{dt} = \underbrace{-(R+R_a)I_{a,1} - V_a\frac{2}{\pi}e^{j\arg I_{a,1}} + < v >_1}_{< di_a/dt >_1} - jL\omega_s I_{a,1}. \quad (9)$$

The only challenge in evaluating $< di_a/dt >_1$ is in calculation of the first harmonic of the **sign** function; this can be accomplished either from the definition (2), or from the tables of *describing functions* which are available for most nonlinearities encountered in control engineering practice [3]. Note that there is no need to write the equation for $I_{a,-1}$, since $i_a(\cdot)$ is real valued and $I_{a,-1}(t) = I^*_{a,1}(t)$, so the time-domain estimate of i_a can be extracted from $I_{a,1}$ only. We can rewrite complex equation (9) as two coupled real equations – let $R_T = R + R_a$, $I_{a,1} = \alpha + j\beta$ and $< v >_1 = V_1 = < v >^R_1 + j < v >^I_1$:

$$\begin{aligned} L\frac{d\alpha}{dt} &= -R_T\alpha + L\omega_s\beta - \frac{2V_a}{\pi}\frac{\alpha}{\sqrt{\alpha^2+\beta^2}} + < v >^R_1 \\ L\frac{d\beta}{dt} &= -R_T\beta - L\omega_s\alpha - \frac{2V_a}{\pi}\frac{\beta}{\sqrt{\alpha^2+\beta^2}} \\ &\quad + < v >^I_1 . \end{aligned} \quad (10)$$

Equation (10), together with the "output" equation (compare with (1))

$$i_a(t) = 2\Re[(\alpha + j\beta)e^{j\omega_s t}] \quad (11)$$

can be used as an approximation of (9). In the next section we show that this is indeed a very good approximation. Its advantages include: **1)** additional analytical insight - for example, a standard steady–state phasor diagram can be derived from (9) by setting the time derivative to zero (see Fig. 2), revealing a constant term aligned with the fault current phasor; **2)** slow temporal variation of $I_{a,1}$, which has a potential to translate into efficient numerical simulations.

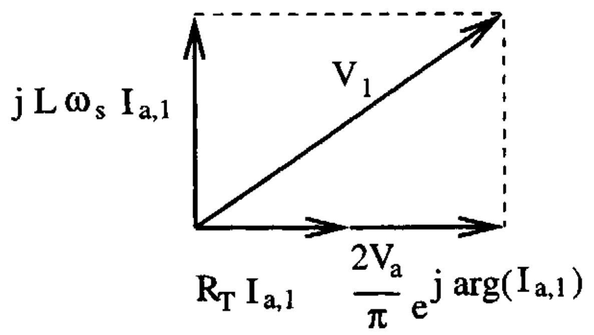

Figure 2: Steady-state (standard) phasor diagram.

B. *Symmetrical Component Model*

The symmetrical component model presented on [8] introduces two additional parameters - zero-sequence resistance R_0 and inductance L_0, while positive and negative sequence resistance remain R and L. Let $\mathcal{L}$ denote the diagonal (3×3) matrix $\mathbf{diag}([L\ L\ L_0])$ and $\mathcal{R}$ denote the diagonal matrix $\mathbf{diag}([R+R_a\ \ R+R_a\ \ R_0+R_a])$. Then the time-domain symmetric component model from [8] can be written as

$$\mathcal{L}\frac{d}{dt}\begin{bmatrix} i_p \\ i_n \\ i_z \end{bmatrix} = -\mathcal{R}\begin{bmatrix} i_p \\ i_n \\ i_z \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ V_a \mathbf{sign}(i_z) \end{bmatrix} + \begin{bmatrix} v_p \\ v_n \\ v_z \end{bmatrix} \tag{12}$$

Note that in (12) the sign of voltage drop due to the arc is made dependent on the sign of the zero component current i_z, while in (7) the sign depends on the total fault current $i_p+i_n+i_z$. For typical parameter values, the approximation is quite small [8]; it also decouples the component equations in (12).

If we assume that all symmetric components are well approximated by their fundamental dynamic phasors, we obtain the following set of three decoupled complex equations:

$$\begin{aligned} \mathcal{L}\frac{d}{dt}\begin{bmatrix} I_{p,1} \\ I_{n,1} \\ I_{z,1} \end{bmatrix} &= -(\mathcal{R}+j\omega_s\mathcal{L})\begin{bmatrix} I_{p,1} \\ I_{n,1} \\ I_{z,1} \end{bmatrix} \\ &\quad - \begin{bmatrix} 0 \\ 0 \\ \frac{2V_a}{\pi} e^{j \arg I_{z,1}} \end{bmatrix} \\ &\quad + \begin{bmatrix} V_{p,1} \\ V_{n,1} \\ V_{z,1} \end{bmatrix} \end{aligned} \tag{13}$$

The time-domain waveforms for phase quantities corresponding to (13) are obtained from (4).

IV. Simulation Results

In our simulations we use data for a 110 kV line taken and inferred from [8]: $V_a = 1.5\text{kV}$, $R+R_a = R_0+R_a = 20\Omega$, $L\omega_s = 10\Omega$, $L_0\omega_s = 30\Omega$, $i_L = 700A$, $|V_1| = 60.5\text{kV}$, $v_p = 44\text{kV}$, $v_n = 0$, $v_z = 28\text{kV}$. We consider the single-phase arcing fault that develops in phase a at $t=0$ at which point the phase current equals i_L.

In Fig. 3 we display the transient responses for the total current in the faulted phase from (8) (top panel) and (13) (bottom panel); the discrepancy is below 1% at any point in time (and thus hardly visible), demonstrating the quality of the dynamic phasor approximation. In Fig. 4 we compare the actual arc voltage (solid line) with the fundamental harmonic approximation (dashed line) which is used in the dynamic phasor model (9). Note that while the approximation is not accurate point-wise in time, the effects of the arc voltage on *current* are accounted very well, as seen in Fig. 3.

For the symmetric components model, we first compare in Fig. 5 the transient responses for the total current in the faulted phase from (12) (top panel) and (9) (bottom panel); the discrepancy is well below 0.5% (and thus barely visible in any reasonable magnification), establishing high quality of the dynamic phasor approximation. The zero component of the fault current is calculated with a similar accuracy when the phasor model (13) is used instead of the time-domain model (12). In Fig. 6 we quantify the approximations introduced in (12) by using $\mathbf{sign}(i_z)$ instead of $\mathbf{sign}(i_p+i_n+i_z)$; while there are discrepancies, their effects on the overall transient are quite small.

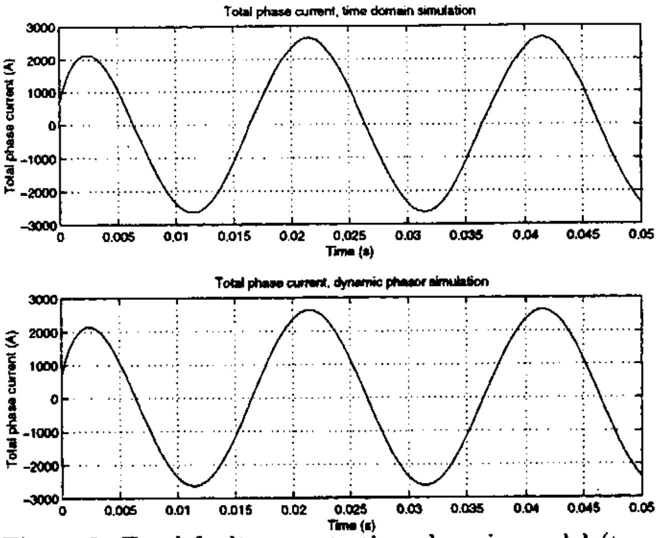

Figure 3: Total fault current: time-domain model (top panel) and dynamic phasor model (bottom panel).

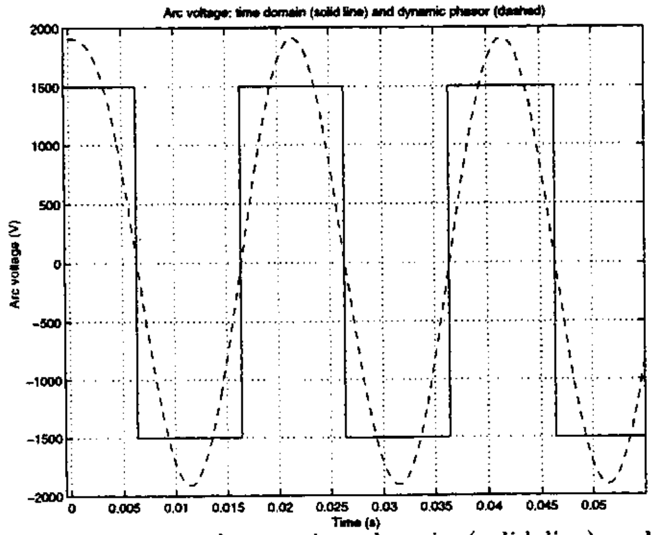

Figure 4: Arc voltage: time domain (solid line) and dynamic phasor (dashed).

V. Conclusions

The paper describes a novel approach to dynamical modeling of arcing faults on overhead lines. The proposed analysis and simulation technique is a polyphase generalization of the dynamic phasor approach, and it is applicable to nonlinear models. The procedure offers distinct advantages in analysis and simulation with respect to standard time-domain models. It also builds on engineering intuition, as dynamic phasors become standard phasors or symmetrical components in a steady-state. Presented dynamical models and numerical experiments illustrate the capabilities of dynamic phasors in analysis and simulation of arcing faults in power sys-

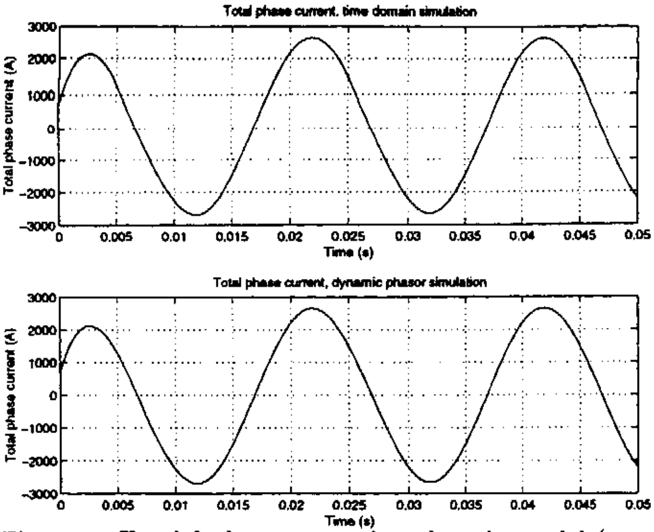

Figure 5: Total fault current: time-domain model (top panel) and dynamic phasor model (bottom panel).

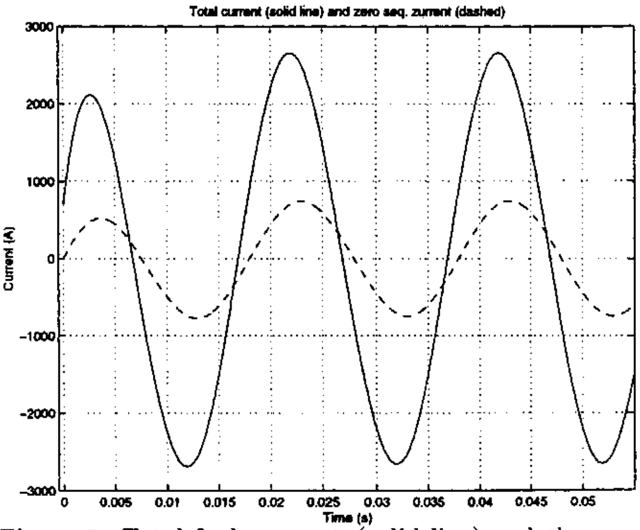

Figure 6: Total fault current (solid line) and the zero sequence current (dashed line).

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