Transient Analysis of Electrical Machines by Differential Taylor Transform

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Abstract - **Modeling, solution, control and even design of many electrical systems involve dealing with nonlinear partial differential equations of which analytic solutions are rarely available. In this paper, the possibility of analyzing transient processes in electrical machines by using a fairly new method known as Differential Taylor (DT) Transform and its advantages are proved. It has been shown that the method is efficient for analyzing the electrical machine transients.**

Keywords: Differential Taylor Transform, Transient Analysis, Electrical Machines.

I. INTRODUCTION

In the theory of electrical machines, at low frequencies, the electromagnetic processes generated in the windings of transformers and machines are analyzed by using lumped parameter circuits. But, at high frequencies these systems have to be considered as distributed parameter circuits. The steady-state and transient behaviors of such circuits are calculated by solving Telegrapher's equations by using D'Alembert time domain method or Fourier transform techniques [1]. In the first method, although it is easy to explain the physical processes occurred in the system by using the superposition of incident and reflected waves, it is difficult to include deformation and attenuation of waves. Since standing waves are used in the second method, the system has a simple solution, but in this case it is difficult to describe the physical process taking place in the system. Thus, the traditional methods have some general drawbacks for analyzing distributed parameter circuits in some respects. For this reason, the development of new and efficient mathematical methods for the simple and accurate analysis of electrical transients is a current problem. In this paper, transients appearing in electrical machines are investigated by using DT Transform as a possible alternate of the traditional methods of analysis.

II. DIFFERENTIAL TAYLOR (DT) TRANSFORM

It is well known that linear differential equations with constant coefficients can be transformed into algebraic equations and then easily solved in complex frequency domain by using Laplace and/or Fourier transformations [1]. For time varying systems, although the application of these techniques is possible by some modifications, it is not as easy and simple as the former case. For nonlinear differential systems, the problem gets more complex due to the frequency domain convolution integrals which result from the time domain products of dependent variables or their derivatives. Therefore, the use of ordinary transform techniques is impractical for nonlinear systems. Fortunately, the use of DT Transform overrides most of the mentioned difficulties and the convolution integrals are replaced by simple sums of algebraic terms.

DT Transform method converts the differential form mathematical model of a system in to its spectral form on which algebraic operations can be carried to derive and understand the system performance. For an analytic function *x(t)* described by its Maclaurian series

$$x(t)=\sum_{k=0}^{\infty}\frac{1}{k!}\left.\frac{d^k x(t)}{dt^k}\right|_{t=0} t^k, \tag{1a}$$

in the interval t∈[0,T], the spectral model (or transform) is defined to be the discrete function

$$X(k)=\frac{T^k}{k!}\left.\frac{d^k x(t)}{dt^k}\right|_{t=0} \tag{1b}$$

which is known to be the differential transform. Using this transform the Taylor series (1a) can be written as

$$x(t)=\sum_{k=0}^{\infty}X(k)\left(\frac{t}{T}\right)^k \quad (2)$$

which is now named as the Taylor Transform [2,3]. In this equation the interval length T is known to be the scale factor of the transform which should be chosen properly.

An important property of the DT Transform is its applicability to systems involving two independent variables and naturally partial differential equations. The DT Transform expressions of a function of two variables (generally space and time) are given by

$$U(p,k)=\frac{X^pT^k}{p!k!}\left.\frac{\partial^{p+k}u(x,t)}{\partial x^p\partial t^k}\right|_{\substack{x=0\\t=0}}, \quad (3a)$$

$$u(x,t)=\sum_{p=0}^{\infty}\sum_{k=0}^{\infty}U(p,k)\left(\frac{x}{X}\right)^p\left(\frac{t}{T}\right)^k \quad (3b)$$

where p,k=0, 1, 2, ...; X and T are the scale factors with respect to x and t, respectively. The use of DT Transform will be shown in the following sections as applied to the transient analysis of electrical machines. For this reason, we will give the mathematical modelling of electrical machines for transient analysis at first.

III. MATHEMATICAL MODELING OF WAVE PROPAGATION IN THE WINDINGS OF ELECTRICAL MACHINES

Electromagnetic fields resulted from transient conditions of electrical machines cause wave propagation on its windings. An electrical machine under the wave propagation can be considered as a magnetic network having many sections with different magnetic reluctances. This network has a longitudinal and transversal wave propagation. The wave propagated through the windings effects both conductors and laminations. For such conditions, transients of electromagnetic phenomenon in the windings and energy transform processes have to be analyzed as in distributed parameter systems. For such systems, the general form of Telegrapher's equations for interval $0\le x\le l$, where x is distance, is well known [1,4]. Thus, parameters of windings generally depend on *x*; on the other hand voltage *u(x,t)* and current *i(x,t)* are continuous functions which depend both space *x* and time *t*.

It is considered that each distributed parameter component is composed of lumped sections. In this respect, windings of electrical machines are composed of a number of coil sections in practice; hence, the transformation $x=(n/N)l$ is used for the functions *u(x,t)* and *i(x,t)*, where *n* is the number of sections counted starting from the one end of the winding, *N* is the total number of sections in the equivalent circuit of one phase and *l* is the length of a single winding. With this assumption, determination of the values of voltages and currents at the connecting points of the coil sections is needed.

Equivalent circuit for infinitely small section of a machine winding is shown in Fig. 1 [5].

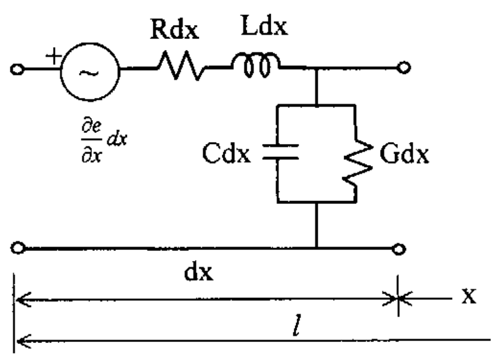

Fig.1. A differential length of machine winding.

The equation describing this circuit is a second order partial differential equation given by

$$\frac{\partial^2u(x,t)}{\partial x^2}-CL\frac{\partial^2u(x,t)}{\partial t^2}-$$
$$(GL+CR)\frac{\partial u(x,t)}{\partial x}-GRu(x,t)=\frac{\partial^2e(x,t)}{\partial x^2}, \quad (4)$$

with the initial and boundary conditions

$$u(x,0)=\sqrt{2}E_a\frac{x}{l}\sin\psi+u_f(0), \quad (5a)$$

$$u(l,t)=\sqrt{2}E_a\sin(\omega t+\psi)+U(e^{-\alpha t}-e^{-\beta t}) \quad (5b)$$

for the isolated phase windings. In these equations L is the winding inductance, R simulates Foucault currents, iron and copper losses, C is the shunt capacitance, and G characterizes the dielectric losses in the machine assembly; the values of these parameters are determined by dividing the total values by the length of winding. E_a is the effective value of emf of phase a, ψ is the phase angle, ω=2πf and f is frequency in Hz.

$e(x,t)=\sqrt{2}E_a\frac{x}{l}\sin(\omega t+\psi)$ is the instantaneous value of induced emf on windings; u_f is the potential effecting the machine windings and it is defined as

$$u_f(t)=U(e^{-\alpha t}-e^{-\beta t}),\ t\geq 0, \tag{5c}$$

where U, α and β are machine constants [5]. By using the expression for e(x,t), Eq. 4 changes to

$$\frac{\partial^2 u(x,t)}{\partial x^2}-CL\frac{\partial^2 u(x,t)}{\partial t^2}-(GL+CR)\frac{\partial u(x,t)}{\partial x}-GRu(x,t)=0. \tag{6}$$

It is obvious that the analytical solution of (6) with the given initial and boundary conditions is difficult. DT transform solution simplifies the analysis procedure and it can easily be applied to solve (6).

IV. DT TRANSFORM SOLUTION OF ELECTRICAL MACHINE WAVE EQUATIONS

DT transform solution of the above problem can be arrived by two different ways: a) By direct determination of spectrums of (5) and (6), then by obtaining Taylor series expansion of *u(x,t)* from (3), b) By writing *u(x,t)* in general form and determining the unknown coefficients of this expression from differential spectrums of (6). Since the second method is much simpler, we can obtain DT model of (6) as follows:

The general function *u(x,t)* is defined as [5]

$$u(x,t)=U\left(e^{-\alpha t}-e^{-\beta t}\right)+\sqrt{2}E_a\frac{x}{l}\sin(\omega t+\psi)+v(x,t). \tag{7a}$$

Let us assume that the unknown function v(x,t) be formed by the Fourier series as

$$v(x,t)=\sum_{n=1}^{\infty}V_n(t)\cos\xi_n x. \tag{7b}$$

For simplicity, (7) can be written in the following form

$$u(x,t)-U(e^{-\alpha t}-e^{-\beta t})-\sqrt{2}E_a\frac{x}{l}\sin(\omega t+\psi)=\sum_{n=1}^{\infty}V_n(t)\cos\xi_n x. \tag{8}$$

Writing the above equation in a simpler form we have

$$\overline{U}(x,t)=\sum_{n=1}^{\infty}V_n(t)\cos\xi_n x \tag{9}$$

where

$$\overline{U}(x,t)=u(x,t)-U(e^{-\alpha t}-e^{-\beta t})-\sqrt{2}E_a\frac{x}{l}\sin(\omega t+\psi), \tag{10}$$

and $\xi_n=\frac{\pi n}{2l}$ (for *n=1,3,5,...*) is the generalized harmonic.

The Differential Taylor transformation of Eqs. 6, 9 and 10 can be written as

$$\frac{(p+1)(p+2)}{X^2}U(p+2,k)-C_1\frac{(k+1)(k+2)}{T^2}U(p,k+2)-C_2\frac{k+1}{T}U(p,k+1)-C_3U(p,k)=0, \tag{11}$$

$$\overline{U}(p,k)=\sum_{n=1}^{\infty}V_n(k)\frac{(\xi_n X)^p}{p!}\cos\frac{\pi p}{2}, \tag{12}$$

$$\overline{U}(p,k)=U(p,k)-\frac{U}{k!}[(-\alpha T)^k-(\beta T)^k]-\sqrt{2}E_a\frac{X}{l}\delta(k-1)\frac{(\omega T)^k}{k!}\sin(\frac{\pi k}{2}+\psi) \tag{13}$$

where C_1*=CL*, C_2*=CL+CR* and C_3*=GR*.

The differential transforms of initial and boundary conditions given by (5) are obtained as

$$\overline{U}(p,k)=\sqrt{2}E_a\sin\psi\frac{X}{l}\delta(p-1), \tag{14a}$$

$$\sum_{p=0}^{\infty}U(p,k)=\frac{U}{k!}[(-\alpha T)^k-(-\beta T)^k]+\sqrt{2}E_q\frac{(\omega T)^k}{k!}\sin(\frac{\pi k}{2}+\psi), \tag{14b}$$

where $\delta(p-1)=1$ if *p=1*, otherwise it is 0.

Eqs. 11-14 are DT model of the problem in concern together with initial and boundary conditions. Taking even values for *p*, we can determine spectrums $V_n(k)$ for different value of *p* and *k* from

$$V_1(0)+V_3(0)+V_5(0)+\cdots=\overline{U}(0,0)$$

$$V_1(0)\frac{(\xi_1 T)^2}{2!}+V_3(0)\frac{(\xi_3 T)^2}{2!}+V_5(0)\frac{(\xi_5 T)^2}{2!}+\cdots=\overline{U}(2,0)$$

$$V_1(0)\frac{(\xi_1 T)^4}{4!}+V_3(0)\frac{(\xi_3 T)^4}{4!}+V_5(0)\frac{(\xi_5 T)^4}{4!}+\cdots=\overline{U}(4,0)$$

...

$$V_1(1)+V_3(1)+V_5(1)+\cdots=\overline{U}(0,1)$$

$$V_1(1)\frac{(\xi_1 T)^2}{2!}+V_3(1)\frac{(\xi_3 T)^2}{2!}+V_5(1)\frac{(\xi_5 T)^2}{2!}+\cdots=\overline{U}(2,1).$$

In order to obtain $u(x,t)$, spectrums $\overline{U}(p,k)$ in the above equations are determined. Then, spectrums $U(p,k)$ in (13) are found from (11) and (14). After the determination of spectrums $V_n(k)$ in (7), we obtain unknown function $v(x,t)$. Finally $u(x,t)$ is calculated from Eq. 8 directly.

The obtained results are shown in Fig. 2 where the variation of the voltage u(x,t) is plotted against time for three different values of x. In the worked example, a 7 kW 380/220 V induction motor with a nominal speed 1440 rpm is considered. The wave parameters of the motor are assumed to be f=200 kHz, $\alpha=10^3$ s^{-1}, $\beta=10^5$ s^{-1}, R=62.82 Ω/m, L=33.3 μH/m, C=5 nF/m, G=9.4×10^{-6} S/m, l=1.56 m. For simplicity U=1 V, E_a=0 V (unexcited case) are assumed. The variations shown in the figure are similar to the typical characteristics computed by other methods and to the transients experimentally verified. It is possible to obtain more accurate results by considering higher order spectrums by increasing the values of p and k in the calculations.

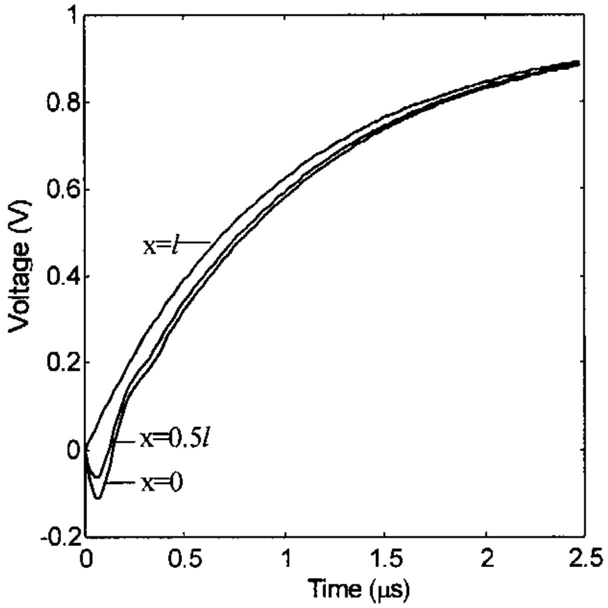

Fig. 2. The variation of the potential at three different locations on the windings.

V. CONCLUSIONS

In this presentation it is shown that the wave propagation in the windings of electrical machines can be calculated by Differential Taylor (DT) Transform.

When it is used for the solution of above and similar problems, DT transform has the following advantages:

i) Both numerical and analytical solutions can be obtained.

ii) The convolution is done by algebraic summation, which makes easy to analyze both linear and nonlinear partial differential equations.

iii) It speeds up the computation when the method is combined with the other numerical tools such as the method of finite elements, polynomial approximations and Fourier transform.

In the problem presented in this paper the machine parameters are assumed constant. But, the method can be used to analyze the systems involving time and/or space dependent parameters. The convolution operations for such systems can be determined as simple algebraic sums. Analysis and modelling of time and/or space dependent systems is also possible by DT transform and this will be the subject of a future work.

VI. REFERENCES

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