

# Quasi-Modes Three-Phase Transmission Line Model - Comparison with Existing Frequency Dependent Models

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**Abstract** - This paper presents a new model to represent multiphase transmission lines in transient studies, including the frequency dependence of longitudinal parameters. The frequency dependence is represented with synthetic circuits, with a cascade of  $\pi$ -circuit for each mode. A real transformation matrix is used for the entire frequency range and it is modeled through ideal transformers in a transient program like ATP, as described.

An application of the methodology is presented for an actual 440 kV single three-phase transmission line where some transients results simulated in ATP are presented. The model is compared to two ATP line models, JMarti and Semlyen.

**Key words** : Transmission line model, frequency dependence, mode domain, transformation matrix, EMTP.

## I. INTRODUCTION

Overhead transmission lines and underground cables are the basic transmission links across a power system. Wave propagation and distortion along lines and cables affects the wave shapes reaching the system equipment and the associated stress upon the equipment.

Despite of their importance in transient studies, the accurate modeling of transmission lines in time domain simulations is still not fully satisfactorily resolved.

One of the main difficulties when dealing with transient simulation studies in a digital simulator program like EMTP [1] is the correct representation of transmission lines. The EMTP works in the time domain and the line is generally represented by its phase quantities. Nevertheless, the transmission line parameters, namely the longitudinal parameters, vary with distance and frequency.

This article presents a new model to represent three-phase transmission lines including the frequency dependence of longitudinal parameters. The model uses exact modes, for ideally transposed lines, and quasi-modes for non-transposed lines.

The longitudinal impedance is represented in the modal domain through synthetic circuits, and the frequency dependence can be modeled with high accuracy. The line is represented through  $\pi$ -circuits, with one  $\pi$ -circuit for each mode.

As is well known, the longitudinal parameters vary with frequency, and there is one impedance matrix for each frequency. When working in the mode domain it is simple to model the frequency dependence, as there is no coupling between the modes. Nevertheless, the transformation matrix, which makes the link between the phase and the mode domain, varies with frequency, which is hard to simulate in a time domain program like EMTP.

In the present model real transformation matrices are used as the unique transformation matrices for the entire frequency range. As the matrix elements are real ones they can be represented in a time domain program like EMTP using ideal transformers. This is implemented making the proper connections with the transformers, using their ratios and polarities.

This model can be implemented in any digital program that has ideal transformer representation or in an analog simulator (TNA). This means that with any digital program it is possible to have a good representation of three-phase transmission line, even if the program only has single-phase transmission line model or resistor and inductors (used to make the  $\pi$ -sections of the mode), but has ideal transformers elements.

In the proposed model, taking advantage of the line geometry, for three-phase transmission lines, Clarke [2] transformation matrix is used as the unique transformation matrix for the entire frequency range. As Clarke matrix is real it can be implemented in EMTP through ideal transformers, as shown.

The quasi-modes model is compared with two ATP frequency dependent models : Semlyen and JMarti for an actual 440 kV transmission line. These models represent the frequency dependence in mode domain and use a transformation matrix calculated for a single frequency value for the entire frequency range. The models are analyzed through some simulations, such as : mode and frequency response and some transient phenomena.

## II. SINGLE THREE-PHASE TRANSMISSION LINE MODEL

### II.1. Mode Domain

Once the electrical parameters (longitudinal impedance and transversal admittance) have been properly calculated in phase domain, the line can be represented to start the desired simulations.

The transformation matrix is unique for each impedance matrix, which means that for a defined line there is one impedance matrix for each frequency. This seems to make the mode determination very complex. It is usual to make some simplifications as to use a single transformation matrix calculated for a chosen frequency for the entire frequency range.

In the proposed model, for non transposed three-phase transmission lines with a vertical symmetry plane, which are the most common UHV lines, there is a real and frequency independent matrix which separates exactly two groups of modes, the Clarke transformation matrix [3, 4]. In one group there is the exact mode and in the other the two quasi-modes, which may be treated as a good approximation of the exact modes. For an ideally transposed line the quasi modes are exact modes. If the line has no symmetry plane, which is not usual in transmission systems and most sub-transmission systems, the Clarke transformation matrix will still be suitable.

The transformation matrix is real and constant, frequency independent. It is a consequence of the line geometry and has no simplifications implied. As the transformation is a real one it can be modeled in a program like EMTP by using ideal transformers.

After obtaining the transformation matrix the modal impedance is calculated and the frequency dependence can easily be synthesized with series and parallel resistors and inductors.

### II.2. Three-Phase Transmission Line Model

Suppose a three-phase transmission line with the ground wire already reduced, as shown in Fig. 1.

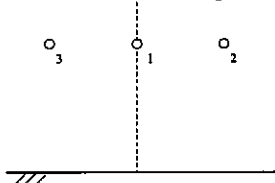


Figure 1 - Schematic representation of a single three phase line

In phase domain

$$-\frac{\partial u}{\partial x} = Z \cdot i \quad (1) \quad \text{and} \quad -\frac{\partial i}{\partial x} = Y \cdot u \quad (2)$$

The impedance matrix, in phase components, is :

$$Z = \begin{bmatrix} A & D & D \\ D & B & F \\ D & F & B \end{bmatrix} \quad (3)$$

Due to the vertical symmetry axis, Clarke's transformation can be applied and the currents in the

conductors are divided as shown in Fig. 2, for each component [5] :

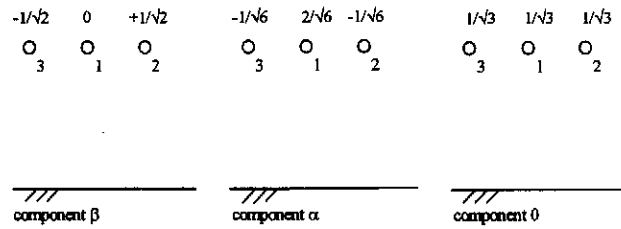


Figure 2- Current in the conductors, for Clarke's components, in rationalized form

The line can be modeled through a cascade of  $\pi$ -circuits, one for each mode. The frequency dependence of longitudinal parameters can be synthesized with series and parallel resistors and inductors, as shown in Fig. 3.

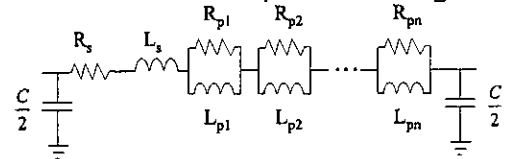


Figure 3 -  $\pi$ -circuit for a mode

In Figs. 4-5 the synthetic circuit response is compared with the exact longitudinal parameters.

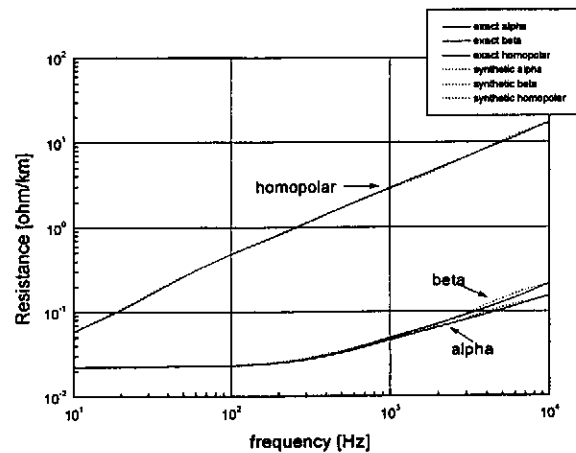


Figure 4 - Per unit resistance - Non transposed line - Exact x synthetic results

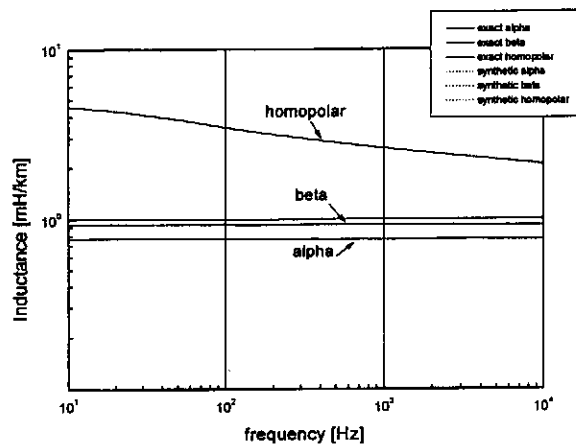


Figure 5 - Per unit inductance - Non transposed line - Exact x synthetic results

### III. TRANSFORMATION MATRIX MODELING

As Clarke transformation matrix is a real one it was represented in ATP-EMTP through ideal transformers. With this it was possible to represent part of the electrical circuit in phase components and part in mode components, as shown in Fig. 6. This methodology can be applied to any digital program that has an ideal transformer or in an analog simulator (TNA).

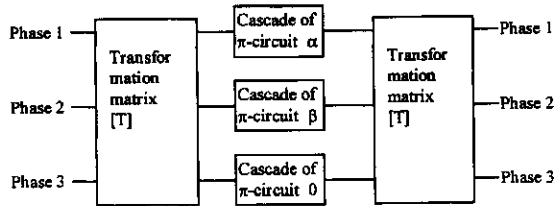


Figure 6 - Schematic representation of three-phase line in EMTP

Clarke transformation matrix representation is performed connecting the ideal transformers to model the relation between phase and mode voltage and current. To do so the transformers are connected with the coils in the positive polarity if the matrix element is positive or in negative polarity if the element is negative. The transformation ratio reproduce the matrix value. There are two basic models [6]:

- three transformers with one coil for phase and one for mode.
- one coil in the primary side (phase) and three coils in the mode side, composing a four-coils transformer;

The first model is described below and the two models lead to the same result. The difference between them is the number of transformers used and the way of connecting them.

#### III.1. One Transformer per Phase Model

In Fig. 7 the phase to phase diagram of the transformation matrix model with three transformers per phase is presented.

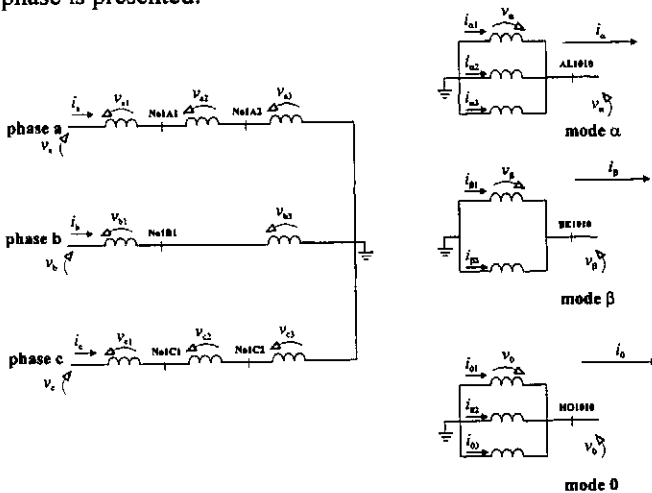


Figure 7 - Link between phases and modes - three transformers per phase

In this model there should be three independent transformers for each phase, with one primary coil (in the phase side) and one secondary coil (in the mode side), and each transformer should be associated to one mode. The primary coils should be connected in series and the

secondary coil of three different phases should be connected in parallel. In Fig. 8 the three phase a transformers are shown in detail.

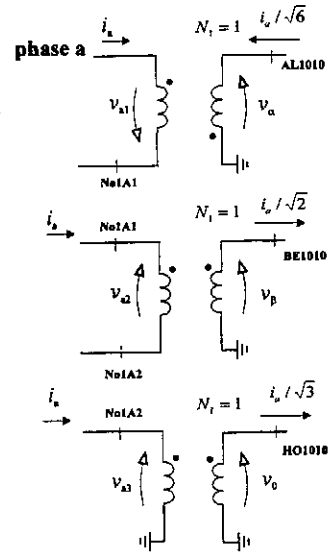


Figure 8 - Phase a transformers associated to the modes

An EMTP file is presented to help to describe the matrix representation (one transformer per phase).

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C transformer between phase A and modes
TRANSFORMER AV1 1.00+6
9999
1JAGA .001 1.0000
2 AL1P1 .001 .40825
3BE1P1 .001 .70711
4HO1P1 .001 .57735
C
C transformer between phase B and modes
TRANSFORMER BV1 1.00+6
9999
1JAGB .001 1.0000
2AL1P2 AL1P1 .001 .81650
3HO1P2 HO1P1 .001 .57735
C
C transformer between phase C and modes
TRANSFORMER CV1 1.00+6
1JAGC
2AL1P2 AL1010
3BE1P1 BE1010
4HO1010HO1P2
C
    
```

### IV. SINGLE THREE-PHASE LINE APPLICATION

In Fig. 9 it is presented the data of the three-phase line used to illustrate the model.

The line parameters were calculated in the range of 10 Hz to 10 kHz. As it is a single line, to represent its modes (exact ones for transposed line and quasi-modes for non-transposed line) it was applied Clarke's transformation matrix, as explained. With the longitudinal and transversal impedance in mode domain, the synthetic circuits were calculated, composing one cascade of  $\pi$ -circuits for each mode, each representing 10 km length. The line was modeled in ATP, with the real and frequency independent transformation matrix represented through ideal transformers.

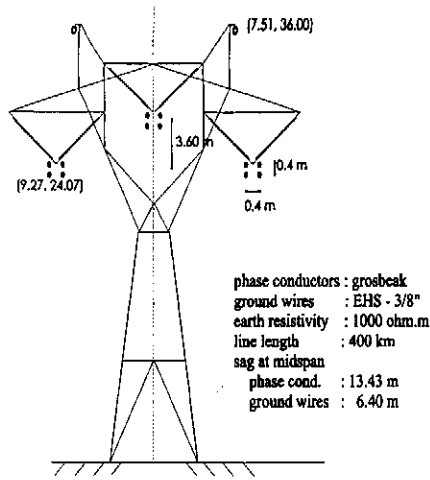


Figure 9 - Schematic representation of the 440 kV three-phase line

The line was treated as transposed and non-transposed. It was also represented using Semlyen [7] and JMarti [8] internal ATP models. Some transient results are presented.

#### IV.1. Mode Analysis

The first test performed with the proposed model was to verify the natural mode behavior for a single three-phase transmission line, supposing it ideally transposed and non-transposed. To do so some simulations were performed with the three models in ATP.

The simulation consisted of applying a 1V step of 1 ms to verify the model behavior to transients in the frequency range of the normal switching phenomenon. In Fig. 10 the diagram of the studied system is shown.

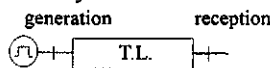


Figure 10 - Simulated system for mode analysis

To represent the modes the step voltages were inputted as described in Tab. 1. The reception end was opened. The simulated cases were : transposed and non-transposed lines; quasi-mode, Semlyen and JMarti models.

Table 1 - Steps to represent the modes

mode	phase	voltage (V)
alpha	a	-0.5
	b (central)	1.0
	c	-0.5
beta	a	1.0
	b	0
	c	-1.0
zero	a	1.0
	b	1.0
	c	1.0

The mode behavior should be coherent with the analysis performed before, where the intrinsic simplification of quasi-mode model of using Clarke transformation as the unique transformation for the entire frequency range was shown.

In Figs. 11 to 13 some results of the mode analysis are presented.

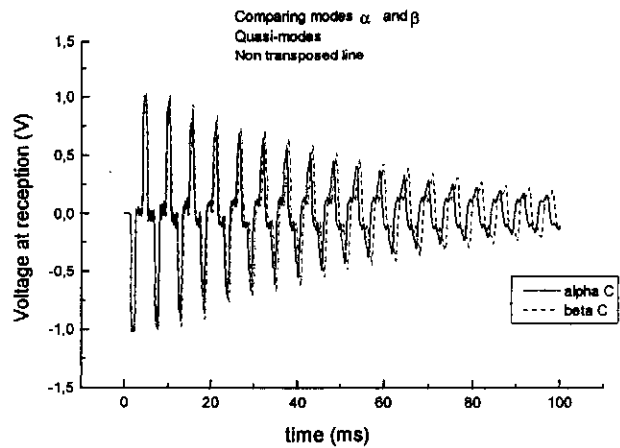


Figure 11 - Step response for mode  $\alpha$  and  $\beta$  - Quasi-mode model - Non-transposed line - Reception end

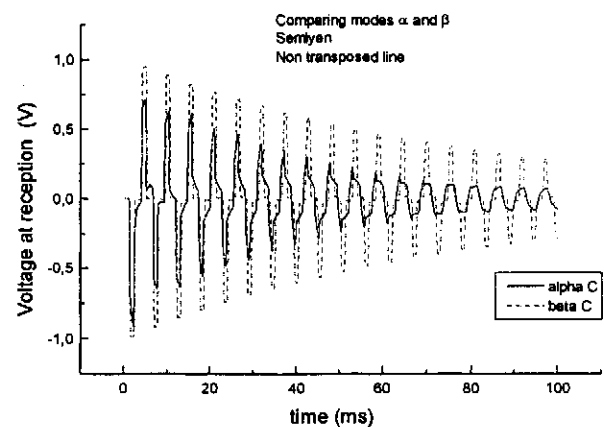


Figure 12 - Step response for mode  $\alpha$  and  $\beta$  - Semlyen model - Non-transposed line - Reception end

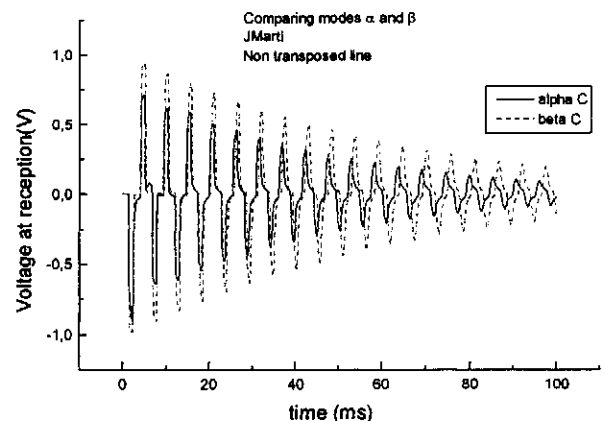


Figure 13 - Step response for mode  $\alpha$  and  $\beta$  - JMarti model - Non-transposed line - Reception end

Analyzing the results it can be seen that the proposed model had a very coherent result, as detailed below :

- modes alpha and beta had very similar results for transposed and non-transposed lines.

The Semlyen model presented the following characteristics :

- the mode alpha had similar behavior for both lines.
- the mode beta presents some differences for higher frequencies, between both lines.
- comparing modes alpha and beta between the same lines, for the transposed line the response is identical, as should be. However, for non transposed line the response is very different, which indicates a model inaccuracy.

The JMarti model presented the following characteristics

- the mode alpha had similar behavior for both lines.
- the mode beta had almost equal response for both lines.
- comparing modes alpha and beta between the same lines, for the transposed line the response is identical, as should be. However, for non transposed line the response is very different, which also indicates a model inaccuracy.

#### IV.2. Frequency Analysis

A frequency scan analysis was performed for both models where the sending terminal had a 1 V source and the receiving end was opened. The relations between the line ends were analyzed in the range of 10 Hz to 10 kHz. An exact calculation, in what concerns the aspects compared in the paper, was also realized so the models could be more properly confronted, as shown in Fig. 14. This so called exact calculation was performed by computing the line quadripole for each frequency in mode domain and obtaining the exact eigenvectors to transform the mode quantities into phase.

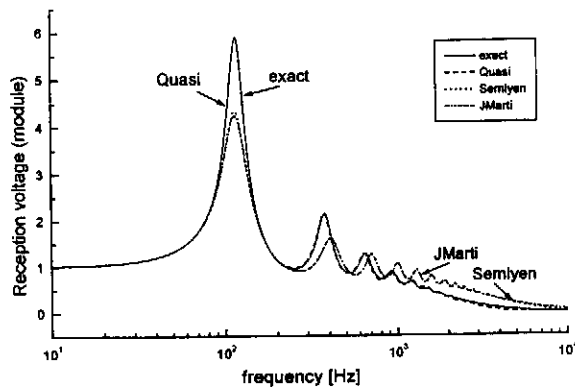


Figure 14 - Zero sequence - Transposed line

The results for both lines were similar, and the main characteristics can be summarized :

- the positive sequence response was very similar for all three models and the exact case.
- the zero sequence response for quasi-mode model was quite equal to the exact response, while the Semlyen and JMarti models have too much damper.

#### IV.3. Statistical Energization

To verify model performance for a transient phenomenon a statistical energization was simulated, with 500 shots and the same seed. The system configuration was :

- reception end opened; reception voltage : 0.95 pu;
- base power : 170 MVA; X (generator + transformer) :

0.3618 pu; X/R : 11.4;  $X_1/X_0$  : 4.41; pre-insertion resistor : 300  $\Omega$

The configuration of the simulated system is presented in Fig. 15.

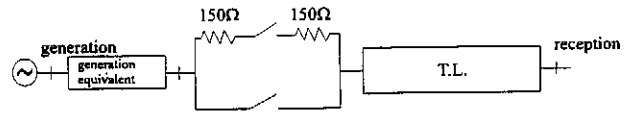


Figure 15 - Studied system diagram

In Fig. 16 the results for non-transposed lines are shown.

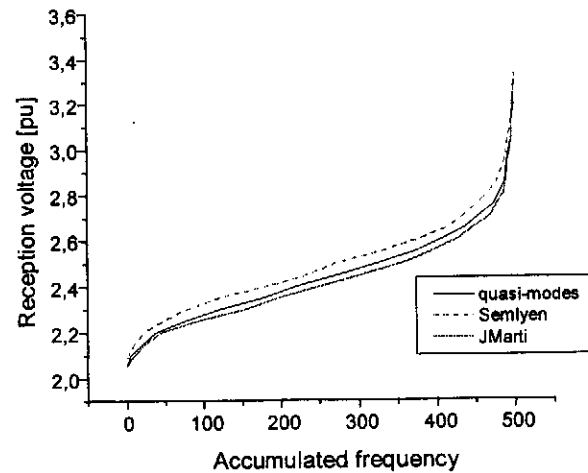


Figure 16 - Statistical energization of non transposed line - Reception voltage - Accumulated frequency - 500 shots

For both lines there are differences between the models. For the transposed line the difference is around 0.1 pu and 0.2 pu for the non-transposed line. For the latter, this is a notable difference which can affect an optimized line project and its costs.

#### IV.4. Worst case energization

From the results obtained from the statistical analysis, the worst case was reprocessed, both for transposed and for non-transposed lines, for all three models.

The results are presented in Fig. 17 and 18.

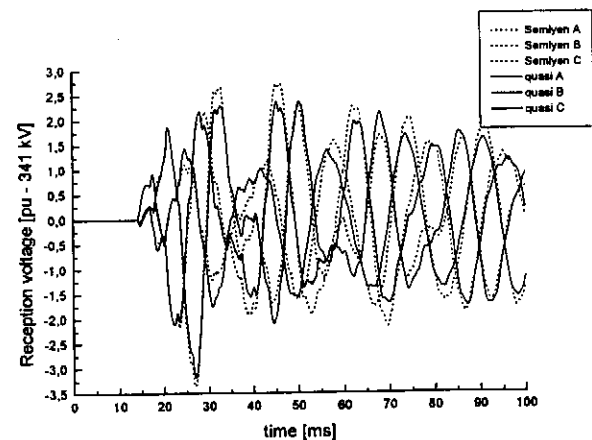


Figure 17 - Worst case energization of non transposed line - reception voltage - Quasi-modes x Semlyen

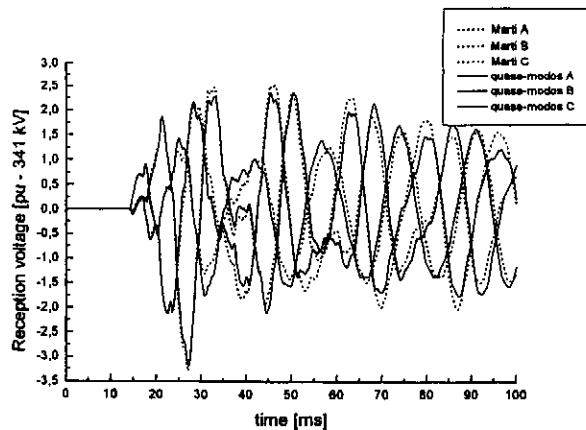


Figure 18 - Worst case energization of non transposed line - reception voltage - Quasi-modos x JMarti

The wave forms are similar, and for non transposed lines the difference are higher.

## V. CONCLUSIONS

This paper presents a new model to represent multiphase transmission lines including the frequency dependence of longitudinal parameters. The model uses the exact modes, for ideally transposed lines, and quasi-modes for non-transposed lines.

The longitudinal impedance is represented in the mode domain through synthetic circuits, and the frequency dependence can be modeled with high accuracy. The line is represented through cascade of  $\pi$ -circuits, one for each mode.

The transformation matrix was modeled in a time domain program like ATP using ideal transformers.

A detailed comparison has been carried out of the quasi-mode model and two constant transformation matrix models, equal to the matrix for a previously chosen frequency (the ATP Semlyen and JMarti models).

The method presented in the paper intends to take full advantage of the basic conceptual frame of time domain based programs, as it is the case of EMTP and ATP, and to enable a flexible accuracy control of the model. The accuracy depends only on dominant frequency range of analyzed phenomena and accepted accuracy of line parameters reproduction. Another feature of the method is to allow direct control of frequency accuracy of line model. In conditions in which resonance or almost resonance occurs, and which may originate very high overvoltages (e. g., ferroresonance), it is very important to have an accurate frequency representation near critical frequencies.

It is shown that, even though the quasi-mode model has some simplifications, it produces more accurate results than the others. It is observed that for typical switching

transients with a large frequency spectrum, in most cases, the use of Clarke transformation matrix has a better result than the use of a single transformation matrix calculated for a single frequency chosen "a priori". This can be explained by the fact that Clarke's transformation matrices lead to good averages, avoiding the amplification of a particular frequency behavior [9].

## VI. ACKNOWLEDGMENTS

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