COMPUTATIONAL METHODS FOR EMTP STEADY-STATE INITIALIZATION

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Abstract - **The calculation of the non-sinusoidal periodic steady-state of a power system with nonlinear and variable-topology components becomes an important problem in some EMTP applications. A dedicated procedure for obtaining the initial steady-state with harmonics is therefore an important feature. This paper presents a summary of the main techniques developed for emtp steady-state initialization. Their advantages and limitations are discussed.**

Keywords : EMTP, Transients Analysis, Steady-State Initialization, Frequency-domain methods, Time-domain methods, Hybrid methods.

1. INTRODUCTION

EMTP capabilities have been significantly increased since its development was started, and some other tools based on the same solution method have been developed. One capability implemented in some emtps is a steady-state initialization algorithm. There are many applications for which this feature is very useful, i.e. subsynchronous resonance, harmonic propagation. Several important aspects are to be considered for developing an emtp initialization method : many nonlinear components (transformers, rotating machines, switched converters) generate harmonics; in many applications (i.e. power electronics converters), the initialization of control systems is also needed; different initial specifications must be dealt with, i.e. voltage and power constraints (load flow data); several types of waveshapes can result in an initial steady state (periodic with harmonics, quasi- or non-periodic).

Obviously, the development and implementation of a unique steady-state initialization procedure which could cover all emtp potential applications is a very difficult and complex task. A very intense activity has been performed in the development of procedures for the initial steady-state calculation in networks with nonlinear and variable-topology converters during the last thirty years [1], [2], [3]. Many algorithms with a limited number of applications have been proposed for emtp initialization, and some are already implemented. This paper presents a summary of the most important procedures for emtp steady-state initialization of networks with nonlinear components and control strategies. Only the case of a periodic initial steady-state is considered.

2. EMTP SOLUTION METHODS

2.1 Transient solution of power networks

Electromagnetic transients programs are circuit-oriented tools based on a time-domain solution method, which combines the Bergeron's method and the trapezoidal rule into an algorithm capable of solving transients in single- and multi-phase networks with lumped and distributed parameters. The trapezoidal rule is used to convert the differential equations of the network components into algebraic equations involving voltages, currents and past values. These algebraic equations are assembled using a nodal approach [4]

$$[G]\,[v(t)] = [i(t)] - [I] \tag{1}$$

where [G] is the nodal conductance matrix, [v(t)] is the vector of node voltages, [i(t)] is the vector of current sources, and [I] is the vector of "history" terms. The conductance matrix is symmetrical and remains unchanged as the integration is performed with a fixed time-step size. The solution of the transient process is then obtained using triangular factorization.

Several modifications have been proposed for solving networks nonlinear and time-varying elements [5], [6]. A very efficient one is based on compensation. Nonlinear elements are represented as current injections which are superimposed to the solution of the linear network after this solution has been computed. Once the solution of the network without the nonlinear element has been computed, its contribution is deduced from the characteristic of the nonlinear element, $v_b(i_b(t))$, and the following equation

$$v_b = v_{b(0)} - r_{thev} i_b \tag{2}$$

where $v_{b(0)}$ is the voltage solution without the nonlinear element, and r_{thev} the Thevenin equivalent resistance. An iterative solution, generally based on the Newton's method, must be used to solve this step. This method can be generalized to networks with several nonlinear components. However, its application is limited to only one nonlinear element per node.

2.2 Transient solution of control systems

Control systems can be represented by block diagrams with interconnection between system elements. Control elements can be transfer functions, FORTRAN algebraic functions, logical expressions and some special devices [7]. A linear block can be described in the s-domain by a general relationship X(s) = G(s) U(s), where U(s) and X(s) are respectively the input and the output, and G(s) is a rational transfer function. The solution method used in all emtps is also based on the trapezoidal rule, being transfer functions converted into algebraic equations in the time-domain, with the following general form

$$[A_{xx}][x] + [A_{xu}][u] = [hist] \tag{3}$$

These equations are by nature unsymmetrical. Due to this fact, the electric network and the control system are solved separately. The network solution is first advanced, the network variables are next passed to the control section, then control equations are solved. Finally, the network receives control commands. The whole procedure introduces a time-step delay. Besides, it is simultaneous only for linear blocks, and sequential for nonlinear blocks. When these blocks are present, a true simultaneous solution is not performed; a closed-loop is broken and the system is solved by inserting a time delay. These delays can originate instabilities and inaccuracies.

3. INITIALIZATION METHODS

The solution of a transient phenomenon is dependent on the initial conditions with which it is started. Although some simulations can be performed with zero initial conditions, there are many cases for which the simulation must be started from power-frequency steady-state conditions. In addition, an initialization procedure can be a useful tool on its own, for instance to calculate resonant voltages due to coupling effects between parallel transmission lines. The steady-state solution of linear networks at a single frequency is a simple task, and can be obtained using nodal admittance equations [5]

$$[Y]\,[V] = [I] \qquad (4)$$

where [Y] is the nodal complex admittance matrix, [V] is the vector of node voltages, [I] is the vector of current sources. This task can be very complex in the presence of nonlinearities which can produce steady-state harmonics. The initial solution with harmonics can be obtained using some simple approaches. The simplest one is known as "brute force" approach : the simulation is started without performing any initial calculation and carried out long enough to let the transients settle down to steady-state conditions. This approach can have a very slow convergence if the network has components with light damping. A more efficient method is to perform an approximate linear ac steady-state solution with nonlinear branches disconnected or represented by linearized models. Some emtps have a "snapshot" feature. Using a "brute force" initialization, the system is started from standstill, once it reaches the steady-state, a snapshot is taken and saved, so later runs can be started at this point.

One of the first methods, known as Initialization with Harmonics (IwH), was presented in [8]. It is an iterative procedure to obtain the magnitude of harmonics generated by saturable reactors. In this method, nonlinear inductances are modelled as voltage dependent harmonic current sources. The model of the power network is linear, therefore the voltages at any harmonic frequency can be found by solving the nodal equations. Other harmonic sources whose magnitude is known are included in the solution as additional current or voltage sources. The periodic steady-state solution is computed using two iterative loops :

a) A preliminary step is used to calculate the voltage phasors of nonlinear inductances taking into account their V_{RMS} - I_{RMS} characteristic.

b) These voltages are used to obtain the current harmonics from the actual flux-current characteristic. These harmonics are reinjected back into the network to calculate a new set of voltage phasors that are superposed to find the new fluxes. The procedure continues with the calculation of the new harmonic currents until convergence is achieved. It is a very simple procedure. However, it can fail to convergence and cannot be applied to ferroresonance studies [8], [11].

Several computational methods for emtp initialization have been developed after IwH was presented. They can be classified into three groups : frequency-domain, time-domain, and hybrid methods. The procedure above described can be considered as a hybrid method as it mixes frequency-domain calculations using nodal admittance equations with a time-domain calculation for obtaining the values of the harmonic current sources used to represent nonlinear reactances. A summary of the most relevant methods proposed up to date, and presented after Iwh, is given in the subsequent sections.

Initialization of control systems is important in many emtp applications, i.e. power electronic systems. Although some improvements have been implemented since the first TACS version was released, the initialization feature is still very limited.

3.1 Frequency-domain methods

They are the most efficient when the network is linear, and the natural approach when the initial operating conditions include power (load flow) constraints. Most procedures use one the two techniques summarized below.

a) Fixed-point iterative methods : The latest values of the distorted terminal voltages are used to drive updated information of the harmonic current injections. The nodal equations of the linear network are then invoked to update the voltage harmonics for the next iteration.

b) Newton type methods : The equations of the linear and the nonlinear parts are solved simultaneously at any harmonic frequency.

The two methods which are described below were developed to improve the IwH. Both use a Norton equivalent (NE) to represent nonlinear components.

1 - Multiphase Harmonic Load Flow (MHPF) [9], [10]

A nonlinear component is represented by means a Norton equivalent at each harmonic frequency. Although only Thyristor Controlled Reactor (TRC) applications were analyzed in the original references, the methodology can be extended to other nonlinear components. Fig. 1 shows the TCR equivalent model. The inductance at fundamental frequency is

$$L_{eq} = \pi\ L\ (\sigma\ -\ \sin\sigma)^{-1} \qquad (5)$$

where σ is the conduction angle. This inductance can also be used to represent the TCR at higher frequencies, therefore

$$Y_{eqh} = (jh\omega L_{eq})^{-1} \qquad (6)$$

The value of the equivalent harmonic current source is derived from the calculated voltage and current harmonics using the following expression

$$I_{eqh} = (jh\omega L_{eq})^{-1} V_h\ -\ I_h \qquad (7)$$

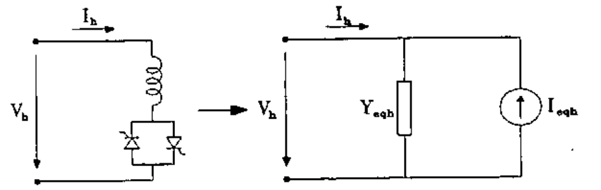

Fig. 1. Norton equivalent model for a TCR [9].

NE circuits are harmonically decoupled and the load flow solution is performed sequentially for one harmonic at a time. However, the coupling effects are included as the harmonic current sources are iteratively adjusted, see Discussion of [9]. The general procedure is based on a fixed-point iterative technique, as the IwH. In addition to an initialization stage, which is particularly important in load flow solutions of multiphase unbalanced networks, the procedure consists of two basic steps :

a) The solution of the network at the fundamental and the harmonic frequencies is solved using a Newton-Raphson method. The general form is expressed as

$$F([X]) = 0 \tag{8}$$

where [X] is the vector of state variables.

b) The harmonic NE circuits of nonlinear components are derived, and convergence is checked by comparing I_{eqh} from two successive iterations.

2 - Newton harmonic method [11]

The Norton equivalent of a nonlinear branch at a harmonic frequency is derived as follows, see Fig. 2,

* the admittance G_{bo} is calculated at each iteration as the mean value of

$$\frac{dg_b}{dv_b}\Big|_{v_b^{(k)}} \tag{9}$$

* the current source value is then calculated using

$$I_{Nbh}^{(k+1)} = I_{bh}^{(k)} - G_{bo}^{(k)} V_{bh}^{(k)} \tag{10}$$

being I_{bh} the harmonic current source component found from v_b and the characteristic of the nonlinear branch.

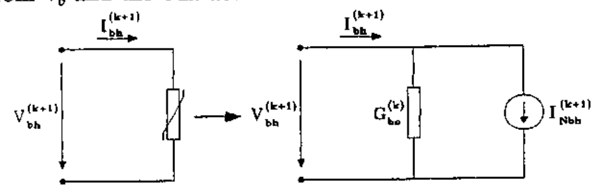

Fig. 2. Norton equivalent of a nonlinear branch [11].

This approach accounts for harmonic coupling. Equivalent circuits of nonlinear branches are included at each iteration in the nodal equations of the linear network. The general formulation for each harmonic h becomes

$$[Y_{nh}^{(k)}]\ [V_{nh}^{(k+1)}] = [I_{nh}^{(k+1)}] \tag{11}$$

where

$$[Y_{nh}^{(k)}] = [A]\ [Y_h^{(k)}]\ [A]^t \tag{12}$$

$$[I_{nh}^{(k+1)}] = [A]\ ([I_{Sh}] - [I_{Nh}^{(k+1)}]) \tag{13}$$

being $[Y_{nh}]$ the nodal admittance matrix, $[Y_h]$ the branch admittance matrix, $[V_{nh}]$ the vector of node voltages, $[I_{nh}]$ the vector of injected currents, $[I_{Sh}]$ the vector of independent current sources, $[I_{Nh}]$ the vector of equivalent current sources, [A] the incidence matrix.

The procedure is initialized without nonlinear branches, unless an approximated model at fundamental frequency was available. After computing $[V_n]$ at each harmonic frequency, the Norton equivalent of all nonlinear branches is determined, and Y_{nh} and I_{nh} are updated. The procedure is continued until convergence. This is not a true Newton-Raphson method, as discussed in [11]. However, it shows a quadratic convergence and can be applied to ferroresonance cases.

Other frequency-domain methods were presented in [12] and [13].

3.2 Time-domain methods

Steady-state initialization algorithms based on a time-domain solution are formulated as a two-point boundary value problem, and use an iterative Newton type approach. If a solution of period T is assumed for all network variables, and [X] denotes the vector of state variables, an iterative procedure is set up to meet two general constraints at the steady state solution

* the periodicity condition

$$[X_0] = [X_T] \tag{14}$$

being $[X_0]$ and $[X_T]$ the vectors of state variables at time 0 and T, respectively

* the relationship between the network solution at two different time steps, usually elapsed one period T,

$$[X_T] = F([X_0]) \qquad ([X_T] = [X(T,\ X_0)]) \tag{15}$$

A system of equations can be defined then as follows

$$H([X_0]) = F([X_0]) - [X_0] = 0 \tag{16}$$

Using a Newton-Raphson approach, the following algorithm is obtained

$$[X_0^{(k+1)}] = [X_0^{(k)}] - [DF([X_0^{(k)}]) - U]^{-1}[X_T^{(k)} - X_0^{(k)}] \tag{17}$$

being [U] the identity matrix.

The general procedure can be summarized as follows. Starting from an initial guess of the vector of state variables $[X_0]$, a simulation is performed over one cycle to obtain $[X_T] = F([X_0])$. The periodicity constraint is checked. If it is not satisfied, the Jacobian matrix is computed and the N-R algorithm applied. Once the vector of state variables has been updated, a time-domain simulation over a new period is performed. The procedure is continued until periodicity is achieved.

Different techniques have been developed to obtain periodic initial solution using this general procedure. They differ in the way in which the network solution (16) is formulated. Three of these techniques are summarized in this paper. The role of the state variables in all of them is played by the history current sources associated to dynamic and nonlinear branches.

1 - Fast steady-state initialization [14]

The equations of the network are assembled using branch formulation and nodal impedance matrix. A relationship between two solution vectors (the history current sources) at different time steps can be obtained. If it is formulated to obtain the values at t = T from those at t = 0, the relationship can be written as follows

$$[I_T] = [\Phi][I_0] + [B] \qquad (18)$$

being [I] the vector of history current sources, [Φ] a matrix which depends on dynamic elements, and [B] a vector related to independent sources only.

Periodicity is achieved when $[I_T] = [I_0]$, then (18) can be expressed in the following form

$$([\Phi][I_0] + [B]) - [I_0] = 0 \qquad (19)$$

which is similar to (16). Using the N-R approach it yields

$$[I_0^{(k+1)}] = [I_0^{(k)}] - [\Phi^{(k)} - U]^{-1}[I_T^{(k)} - I_0^{(k)}] \qquad (20)$$

This procedure is simple and handles unbalanced networks with nonlinear components and converters. However, [Φ] is a full matrix which is updated at every time step. These are important drawbacks for very large scale systems.

2 - The shooting method [15]

The equations of a network with nonlinear components can be expressed, using a modified nodal approach, in the following form

$$\begin{bmatrix} G & B \\ C & 0 \end{bmatrix} \begin{bmatrix} V(t) \\ I_b(t) \end{bmatrix} = \begin{bmatrix} I(t) \\ V_b(I_b(t)) \end{bmatrix} \qquad (21)$$

where **G** is the matrix of nodal conductances of the linear part, **B** and **C** are the branch incidence matrices. **V**(t) is the node voltage vector, $\mathbf{V}_b(t)$ and $\mathbf{I}_b(t)$ are the vectors of voltages and currents of the nonlinear branches. The vector $\mathbf{V}_b(\mathbf{I}_b(t))$ denotes the functional relationship between voltages and currents in the nonlinear branches. **I**(t) is the vector of current sources, including both external current sources and history current sources, see equation (1). If the nonlinear branches are represented by means of piecewise linear segments, the vector of branch voltages $\mathbf{V}_b(t)$ can be expressed as

$$[V_b(I_b(t))] = [R(t)][I_b(t)] + [E_k(t)] + [E_h(t)\ 0]^t \qquad (22)$$

where **R**(t) is a diagonal matrix of time-varying resistances, $\mathbf{E}_k(t)$ is the vector of time-varying voltages, and $\mathbf{E}_h(t)$ is the vector of Thevenin history sources. An equivalent of the linear part of the network can be obtained using the following expression

$$[V_{th}(t)] - [T][I_b(t)] = [V_b(I_b(t))] \qquad (23)$$

where $\mathbf{V}_{th}(t)$ is the vector of open circuit voltages at the nonlinear branch nodes. The following form is derived

$$[V_0(t)] - [E_k(t)] = [T + R(t)]\,[I_b(t)] \qquad (24)$$

being $\mathbf{V}_0(t) = \mathbf{V}_{th}(t) - [\mathbf{E}_h(t)\ \mathbf{0}]^t$. Therefore a piecewise-linear N-R method can be used to solve the equations of nonlinear branches at each time step

$$[I_b^{(k+1)}(t)] = [T + R^{(k)}(t)]^{-1}\,[V_0(t) - E_k^{(k)}(t)] \qquad (25)$$

This solution is completely decoupled from the calculation of the unknown node voltages of the linear part of network which can be found by solving

$$[G_{AA}][V_A(t)] = [I_A(t)] - [G_{AB}][V_B(t)] - [B_B][I_b^*(t)] \qquad (26)$$

where $\mathbf{I}_b^*(t)$ is the solution vector of the nonlinear branch currents. Equations (25) and (26) are used to perform the time-domain calculation from t=0 to t=T. The periodic steady-state solution is obtained using the general iterative algorithm given in (17). The vector of state variables is again the vector of history current sources, which now includes sources from both linear and nonlinear elements, $[\mathbf{X}(t)] = [\mathbf{I}_h(t)\ \ \mathbf{E}_h(t)]^t$.

The most complex task when applying the iterative solution (17) is the calculation of the Jacobian matrix $\mathbf{DF}(\mathbf{X}_0)$. This method yields the following variational form

$$\begin{bmatrix} G_{AA} & B_A \\ 0 & -T-R(t) \end{bmatrix} \begin{bmatrix} V_A(t) \\ I_b(t) \end{bmatrix} = \begin{bmatrix} I_A(t) \\ E(t) \end{bmatrix} - \begin{bmatrix} G_{AB} \\ C_B \end{bmatrix} V_B(t) \qquad (27)$$

where $[\mathbf{E}(t)] = [\mathbf{E}_k(t)] + [\mathbf{E}_h(t)\ \ \mathbf{0}]^t - [\mathbf{C}_A][\mathbf{G}_{AA}]^{-1}$. As shown in [15], the Jacobian $\mathbf{DF}(\mathbf{X}_0)$ is the solution of the sensitivity network of the variational network at time T when the initial condition is the identity matrix. In general the Jacobian $\mathbf{DF}(\mathbf{X}_0)$ will be a full matrix. Convergence failures have also been reported [16].

3 - The relaxation method [16]

A relationship can be found for calculating the values of the history current sources at two successive time steps

$$[X_i] = [\Lambda][X_{i-1}] - \Gamma([X_{i-1}]) \qquad (28)$$

Assume that the values of the *n* state variables are calculated in a time span T, and *m-1* solutions have been obtained from the initial estimation of the state variable vector (i = 1, t = 0) using (28). The relationship between the history current sources at two successive time steps can be written in the following form

$$F_i([X_i],[X_{i-1}]) = [X_i] - [\Lambda][X_{i-1}] - \Gamma([X_{i-1}]) = 0 \qquad (29)$$

for i = 2, ..., m. From these constraints and from the periodicity condition, $[\mathbf{X}_1] = [\mathbf{X}_m]$, the following N-R iterative procedure is deduced

$$\begin{bmatrix} I & 0 & 0 & \dots & 0 & -I \\ D_2 & I & 0 & \dots & 0 & 0 \\ 0 & D_3 & I & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & D_m & I \end{bmatrix} \begin{bmatrix} \Delta X_1 \\ \Delta X_2 \\ \Delta X_3 \\ \dots \\ \Delta X_m \end{bmatrix} = \begin{bmatrix} 0 \\ F_2 \\ F_3 \\ \dots \\ F_m \end{bmatrix} \qquad (30)$$

being $[\mathbf{D}_i] = [d\mathbf{F}_i/d\mathbf{X}_{i-1}]$. The derivation of these matrices is presented in [16]. From the periodicity condition, $[\mathbf{F}_1] = \mathbf{0}$. The banded form of the relaxation matrix permits the design of a very efficient algorithm.

This procedure does not require any explicit integration of the solution and it can be applied to networks with ideal diodes using a Multiple Area Thevenin Equivalent : when a diode is blocked, the related subnetworks are disconnected and solved separately.

3.3 Hybrid methods

Although most of the methods described above are very efficient, and some of them have been tested with large scale networks, no one can be applied to all potential emtp case studies : harmonic models are not always available, and they can be specially complicated for variable topology and user-defined components; control sections are disconnected and initialized separately in most emtps, and although some initialization features are implemented in some versions, they do not cover all capabilities. A hybrid method may benefit of both frequency and time-domain methods : frequency-domain methods are used to solve the linear part, time-domain methods are used for the nonlinear and variable topology parts, see Fig. 3.

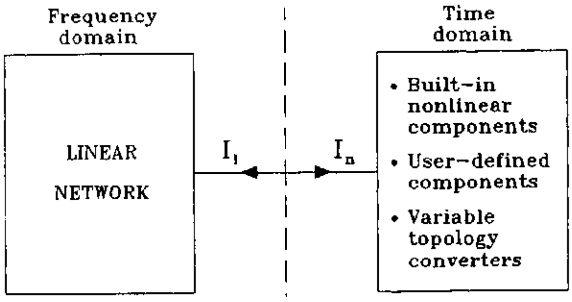

Fig. 3. Interface between linear and nonlinear parts.

The Generalized Harmonic Balance method (GHBM) [17] exploits the advantages of nonlinear extrapolation methods to accelerate the computation of the periodic solution of the nonlinear parts using a time-domain calculation, while the linear part is solved using a frequency-domain calculation.

Nonlinear extrapolation methods can be used to obtain the fixed solution of a vector sequence, $[X^{(k+1)}] = F([X^{(k)}])$, generated using either a linear or a nonlinear process. The GHBM is based on the Random Rank Extrapolation method and uses a two loop iterative procedure:

a) An outer loop where the periodic steady-state of the linear part is calculated using the nodal admittance equations $[Y_{nh}]\,[V_{nh}] = [I_{nh}]$.

b) An inner loop where the periodic solution of the nonlinear part is calculated using a three-step procedure :

1 - the waveforms of the voltages at the interface nodes are calculated from the harmonic phasors
2 - these voltages are used to obtain in the time-domain the steady-state waveforms of the currents injected into the nonlinear parts using a nonlinear extrapolation method
3 - the harmonic phasors of these currents are extracted, and the vector of injected currents is then updated.

The whole procedure is a fixed-point iterative method. However, the nonlinear extrapolation method is also used to accelerate the convergence in the outer loop. In this manner, the method is quadratically convergent. Their advantages are obvious : no Jacobian matrix has to be calculated, and it can be applied to any power system case. See [17] for a discussion about the divergence.

Other methodologies also based on a hybrid approach were presented in [18], [19].

4. DISCUSSION

All methods presented in this paper have advantages and limitations which should be taken into account in the development of a more general and powerful procedure :

1) Frequency-domain methods are the most efficient and accurate for the calculation of the periodic steady-state of a linear network. They can handle very efficiently some nonlinearities and are very adequate when the operating conditions are specified as power (load flow) constraints. However, there are many nonlinear components for which a harmonic model cannot be easily developed, and the situation becomes more complicated when a control strategy has to be included in the initialization stage.

2) Time-domain methods have a very important advantage as they are fully consistent with the subsequent time-domain simulation of transients. Although the methods summarized in this paper were developed for lumped element networks, their basic ideas seem to be useful also for distributed parameter networks, but some corrective measures could be needed when many lines with different transition times are to be simulated.

Numerical oscillations could be produced when a time-domain approach based on the trapezoidal rule is used, so additional corrective measures are needed [11]. Time domain procedures have been tested with rather simple cases, and the choice of history current sources as state variables is not enough when the system incorporates some components, for instance rotating machines and control strategies.

3) Hybrid methods benefit from advantages of both frequency-domain and time-domain techniques. Theoretically, they can handle systems incorporating all types of nonlinearities and control systems. In practice, results related to large scale networks with frequency-dependent distributed parameter transmission lines, rotating machines and some variable-topology components have not been yet reported. And some problems remain about the convergence of the proposed techniques.

Table I shows a summary of advantages and limitations of the three classes of initialization techniques.

5. CONCLUSIONS

EMTP-like programs are powerful simulation tools which can be applied in a wide range of case studies. However, their flexibility becomes an important drawback for the development of a procedure for steady-state calculation which could cover all potential cases. Many techniques have been proposed and, although no one has been yet tested in some applications, most of them can be used in many important studies. Particularly interesting are hybrid methods as they can benefit from the advantages of both frequency-domain and time-domain methods.

TABLE I - COMPARISON OF TECHNIQUES FOR EMTP INITIALIZATION

TECHNIQUE	ADVANTAGES	LIMITATIONS
Frequency-domain	* They are the most efficient for steady-state initialization of linear networks * They are also very efficient when accurate harmonic models of nonlinear components can be incorporated	* Harmonic models are not available or cannot be easily developed for some components * They cannot incorporate most control strategies
Time-domain	* The initial steady-state is consistent with the subsequent transient simulation * The choice of history current sources as state variables reduces the computational burden	* They have been applied only to simple cases * The choice of history current sources cannot be easily extended to systems with rotating machines , control strategies, user-defined components
Hybrid	* They benefit from advantages of both frequency and time domain techniques * In theory, some methods can handle all type of case studies	* They have not been tested with large scale systems which incorporate some components, i.e. rotating machines * Some convergence problems have been reported

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