# OVERVIEW OF OVERHEAD LINE MODELS AND THEIR REPRESENTATION IN DIGITAL SIMULATIONS

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*Abstract* – **Several solutions have been proposed to solve the equations of multiconductor overhead lines in the time domain taking into account the frequency dependence of their parameters. These solutions can be classified into two categories : modal domain and phase domain. This paper presents a summary of the main works in this field with emphasis on those approaches which have been implemented in an EMTP-like tool.**

**Keywords:** Modelling, Transient Analysis, Overhead Lines, EMTP.

## I. INTRODUCTION

Two types of time domain models have been developed for representing overhead lines :

- Lumped-parameter models, that represent transmission systems by lumped elements whose values are calculated at a single frequency
- Distributed-parameter models, for which two categories can be distinguished, constant parameter and frequency-dependent parameter models.

The first type of models is adequate for steady-state calculations. The second type are the most accurate models for transient calculations as they take into account the distributed nature of parameters and consider their frequency-dependence.

A significant number of papers dedicated to analyze the frequency-dependence behaviour of overhead lines for digital simulation has been presented during the last 40 years. The first papers were published during the late 1960's and early 1970's. Most approaches were aimed at solving transmission-line equations using a time domain solution and were based on the modal theory [1] : multi-conductor line equations are decoupled through modal transformation matrices, so that each mode can be separately studied as a single-conductor line [2] - [11]. However, the solution of line equations can be also based on a phase domain formulation, or a combination of modal and phase domain solutions [12] –[24].

Some parts or effects in an overhead line can have an important influence on its transient performance. The concept of nonuniform line includes the corona effect, which is a source of attenuation and distortion of surges and overvoltages in overhead lines.

This paper presents a summary of the solution methods proposed up to date for digital calculations of electromagnetic transients in multiconductor overhead lines with uniform and frequency–dependent parameter. Due to space limitation the emphasis is put on the most recent work and those solutions which have been implemented in an EMTP-like tool [25].

## II. OVERHEAD LINE EQUATIONS

Fig. 1 shows the reference frame and the equivalent circuit of a differential section of a single-conductor overhead line.

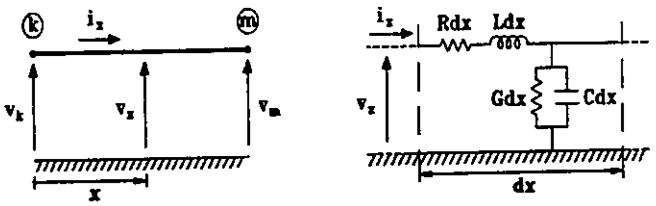


Fig. 1. Single-conductor overhead line.

The time domain equations of this line can be expressed as follows

$$-\frac{\partial v(x,t)}{\partial x} = Ri(x,t) + L\frac{\partial i(x,t)}{\partial t} \quad (1)$$

$$-\frac{\partial i(x,t)}{\partial x} = Gv(x,t) + C\frac{\partial v(x,t)}{\partial t} \quad (2)$$

where v(x,t) and i(x,t) are the voltage and the current of the line. R, L, G and C are the line parameters expressed in per unit length. These parameters are frequency-dependent, although C can be assumed constant, and G can be neglected.

Given the frequency dependence of the series parameters, the approach to the solution of the line equations, even in transient calculations, is performed in the frequency domain. The behaviour of a multiconductor overhead line is described in the frequency domain by the two following matrix equations

$$-\frac{d\mathbf{V}_x(\omega)}{dx} = \mathbf{Z}(\omega)\,\mathbf{I}_x(\omega) \quad (3)$$

$$-\frac{d\mathbf{I}_x(\omega)}{dx} = \mathbf{Y}(\omega)\,\mathbf{V}_x(\omega) \quad (4)$$

where $\mathbf{Z}(\omega)$ and $\mathbf{Y}(\omega)$ are respectively the series impedance and the shunt admittance matrices per unit length. The general solution of these equations can be expressed as follows

$$\mathbf{I}_x(\omega) = e^{-\Gamma(\omega)x}\mathbf{I}_f(\omega) + e^{+\Gamma(\omega)x}\mathbf{I}_b(\omega) \quad (5)$$

$$\mathbf{V}_x(\omega) = \mathbf{Y}_C^{-1}(\omega)\left[e^{-\Gamma(\omega)x}\mathbf{I}_f(\omega) - e^{+\Gamma(\omega)x}\mathbf{I}_b(\omega)\right] \quad (6)$$

being $\mathbf{I}_f(\omega)$ and $\mathbf{I}_b(\omega)$ the vectors of forward and backward travelling wave currents at x=0, $\Gamma(\omega)$ the propagation constant matrix and $\mathbf{Y}_c(\omega)$ the characteristic admittance matrix

$$\Gamma(\omega) = \sqrt{\mathbf{YZ}} \quad (7)$$

$$\mathbf{Y}_c(\omega) = \sqrt{(\mathbf{YZ})^{-1}}\,\mathbf{Y} \quad (8)$$

$I_f(\omega)$ and $I_b(\omega)$ can be deduced from the boundary conditions of the line. Considering the frame shown in Fig. 2, the solution at line ends can be formulated as follows

Two approaches have been used to cope with this problem : constant and frequency-dependent transformation matrices.

a) The modal decomposition is made by using a constant real transformation matrix **T** calculated at a user-specified frequency, being the imaginary part usually discarded. This has been the traditional approach for many years. It has an obvious advantage, as it simplifies the problem of passing from modal quantities to phase quantities and reduces the number of convolutions to be calculated in the time domain, since $\mathbf{T}_V$ and $\mathbf{T}_I$ are real and constant. Differences between methods in the time domain implementation based on this approach differ from the way in which the characteristic admittance function $Y_C$ and the propagation function $\exp(-\gamma l)$ of each mode are represented. The characteristic admittance function is in general very smooth and can be easily synthesized with RC networks. To evaluate the convolution that involves the propagation function, several alternatives have been proposed : weighting functions [2], exponential recursive convolution [3], [4], linear recursive convolution [5], modified recursive convolution [6], [7]. A recent work uses the constant Clarke's transformation matrix for passing from model domain to phase domain, and represents the frequency dependence of uncoupled line modes by a cascade of synthetic π-circuits [8].

b) The frequency dependence of the modal transformation matrix can be very significant for some untransposed multicircuit lines. An accurate time domain solution using a modal domain technique requires then frequency-dependent transformation matrices. This can in principle be achieved by carrying out the transformation between modal and phase domain quantities as a time domain convolution; with modal parameters and transformation matrix elements fitted with rational functions [9], [10], [11]. Although working for cables [9], it has been found that for overhead lines, the elements of the transformation matrix cannot be always accurately fitted with stable poles only [11]. This problem is overcome by the phase domain approaches.

### 3.2 Phase domain techniques

Some problems associated with frequency-dependent transformation matrices could be avoided by performing the transient calculation of an overhead line directly with phase quantities. A summary of the main approaches developed with this purpose is presented below.

a) Numerical convolution. Initial phase domain techniques [12], [13] were based on a direct numerical convolution in the time domain. These approaches are, however, time consuming in simulations involving many time steps. This drawback was partially solved in [14] by applying Ametani's linear recursive convolution to the tail portion of the impulse responses.

b) z-domain approaches. An efficient approach is based on the use of two-sided recursions (TSR), as presented in [15]. The basic input-output in the frequency domain is usually expressed as follows

$$\mathbf{y}(s) = \mathbf{H}(s)\mathbf{u}(s) \qquad (33)$$

Taking into account the rational approximation of $\mathbf{H}(s)$, equation (33) becomes

$$\mathbf{y}(s) = \mathbf{D}^{-1}(s)\mathbf{N}(s)\mathbf{u}(s) \qquad (34)$$

being D(s) and N(s) polynomial matrices. From this equation one can derive

$$\mathbf{D}(s)\mathbf{y}(s) = \mathbf{N}(s)\mathbf{u}(s) \qquad (35)$$

This relation can be solved in the time domain using two convolutions

$$\sum_{k=0}^{n} \mathbf{D}_k \mathbf{y}_{r-k} = \sum_{k=0}^{n} \mathbf{N}_k \mathbf{u}_{r-k} \qquad (36)$$

The identification of both sides coefficients can be made using a frequency domain fitting, see [15] for more details.

A more powerful implementation of the TSR, known as ARMA (Auto-Regressive Moving Average) model, was presented in [16], [17] by explicitly introducing modal time delays in (36).

c) s-domain approaches. A third approach is based on s-domain fitting with rational functions and recursive convolutions in the time domain. Two main aspects are issued : how to obtain the symmetric admittance matrix, **Y**, and how to update the current source vectors. These tasks imply the fitting of $\mathbf{Y}_c(\omega)$ and $\mathbf{H}(\omega)$.

The elements of $\mathbf{Y}_c(\omega)$ are smooth functions and can be easily fitted. However, the fitting of $\mathbf{H}(\omega)$ is more difficult since its elements may contain widely different time delays from individual modal contributions; in particular, the time delay of the ground mode differs from those of the aerial modes. Some recent works have considered a single time delay for each element of $\mathbf{H}(\omega)$ [18], [19]. However, a very high order fitting can be necessary for the propagation matrix in the case of lines with a high ground resistivity, as an oscillating behaviour can result in the frequency domain due to the uncompensated parts of the time delays. This problem can be solved by including modal time delays in the phase domain. Several line models have been recently developed on this basis, using polar decomposition [20]; expanding $\mathbf{H}(\omega)$ as a linear combination of the natural propagation modes with idempotent coefficient matrices [21]; or calculating unknown residues once the poles and time delays have been precalculated from the modes, in the so-called universal line model (ULM) [22]. For a discussion on the advantages and limitations of these models see [23].

d) Non-homogeneous models. The previous approaches were based on equations (13)-(16) and the equivalent circuit shown in Fig. 3, the goal in all cases was to derive a multiterminal decoupled Norton equivalent which would be interfaced to the nodal admittance equations of the full system. Overhead line parameters associated with external electromagnetic fields are frequency independent. That is, the series impedance matrix **Z** is a full matrix which can be split up as follows

$$\mathbf{Z}(\omega) = \mathbf{Z}_{loss}(\omega) + j\omega\mathbf{L}_{ext} \qquad (37)$$

where

$$\mathbf{Z}_{loss} = \mathbf{R} + j\omega\Delta\mathbf{L} \qquad (38)$$

Elements of $\mathbf{L}_{ext}$ are related to the external flux and frequency independent, while elements of **R** and $\Delta\mathbf{L}$ are related to the internal flux and frequency-dependent. Finally, the elements of the shunt admittance matrix

$$\mathbf{Y}(\omega) = j\omega\mathbf{C} \qquad (39)$$

$$\mathbf{I}_k(\omega) = \mathbf{Y}_c(\omega)\mathbf{V}_k(\omega) - \mathbf{B}_k(\omega) \quad (9)$$

$$\mathbf{B}_k(\omega) = \mathbf{H}(\omega)[\mathbf{Y}_c(\omega)\mathbf{V}_m(\omega) + \mathbf{I}_m(\omega)] \quad (10)$$

$$\mathbf{I}_m(\omega) = \mathbf{Y}_c(\omega)\mathbf{V}_m(\omega) - \mathbf{B}_m(\omega) \quad (11)$$

$$\mathbf{B}_m(\omega) = \mathbf{H}(\omega)[\mathbf{Y}_c(\omega)\mathbf{V}_k(\omega) + \mathbf{I}_k(\omega)] \quad (12)$$

being $\mathbf{H} = \exp(-\Gamma l)$.

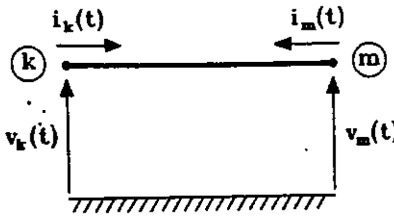


Fig. 2. Reference frame.

Transforming equations (9) – (12) into the time domain gives

$$\mathbf{i}_k(t) = \mathbf{y}_c(t) * \mathbf{v}_k(t) - \mathbf{b}_k(t) \quad (13)$$

$$\mathbf{b}_k(t) = \mathbf{h}(t) * \{\mathbf{y}_c(t) * \mathbf{v}_m(t) + \mathbf{i}_m(t)\} \quad (14)$$

$$\mathbf{i}_m(t) = \mathbf{y}_c(t) * \mathbf{v}_m(t) - \mathbf{b}_m(t) \quad (15)$$

$$\mathbf{b}_m(t) = \mathbf{h}(t) * \{\mathbf{y}_c(t) * \mathbf{v}_k(t) + \mathbf{i}_k(t)\} \quad (16)$$

where symbol * indicates convolution and $\mathbf{x}(t) = F^{-1}\{\mathbf{X}(\omega)\}$ is the inverse Fourier transform.

These equations suggest that an overhead line can be represented at each terminal by a multiterminal admittance paralleled by a multiterminal current source, as shown in Fig. 3.

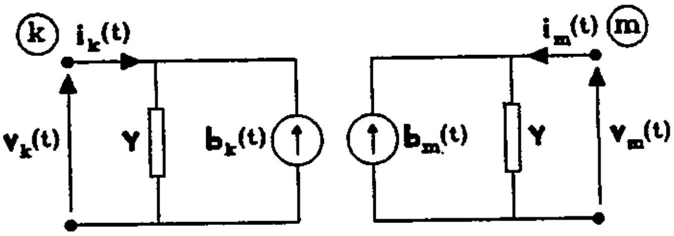


Fig. 3. Overhead line equivalent circuit for time domain calculations.

The implementation of this equivalent circuit requires the synthesis of an electrical network to represent the multi-terminal admittance, while the current source values have to be updated at every time step during the time domain calculation. Both tasks are not straightforward, and many approaches have been developed to cope with this problem, as described in the next section.

## III. REVIEW OF LINE MODELS

The techniques developed to solve the equations of a multiconductor frequency-dependent overhead line can be classified into two main categories : modal domain techniques and phase domain techniques. An overview of the main approaches is presented below.

### 3.1 Modal domain techniques

Overhead line equations can be solved by introducing a new reference frame

$$\mathbf{V}_{(ph)} = \mathbf{T}_v\, \mathbf{V}_{(m)} \quad (17)$$

$$\mathbf{I}_{(ph)} = \mathbf{T}_i\, \mathbf{I}_{(m)} \quad (18)$$

where the subscript "ph" and "m" refer to the original phase quantities and the new modal quantities. Matrices $T_V$ and $T_I$ are chosen such that they diagonalize the products $\mathbf{Y}(\omega)\mathbf{Z}(\omega)$ and $\mathbf{Z}(\omega)\mathbf{Y}(\omega)$

$$\mathbf{T}_v^{-1}\mathbf{ZYT}_v = \Lambda \quad (19)$$

$$\mathbf{T}_i^{-1}\mathbf{YZT}_i = \Lambda \quad (20)$$

being $\Lambda$ a diagonal matrix. To find $T_V$ or $T_I$ is the eigenvalue/eigenvector problem.

Overhead line equations in modal quantities become

$$-\frac{d\mathbf{V}_{(m)}}{dx} = \mathbf{T}_v^{-1}\mathbf{ZT}_i\mathbf{I}_{(m)} \quad (21)$$

$$-\frac{d\mathbf{I}_{(m)}}{dx} = \mathbf{T}_i^{-1}\mathbf{YT}_v\mathbf{V}_{(m)} \quad (22)$$

It can be proved that $[T_V]^{-1} = [T_I]^t$, and that the products $T_V^{-1}ZT_I$ ($= Z_{(m)}$) and $T_I^{-1}YT_V$ ($= Y_{(m)}$) are diagonal [25].

The solution of a line in the phase domain and modal quantities can be then expressed as follows

$$\mathbf{I}_{k(m)}(\omega) = \mathbf{Y}_{c(m)}(\omega)\mathbf{V}_{k(m)}(\omega) - \mathbf{B}_{k(m)}(\omega) \quad (23)$$

$$\mathbf{B}_{k(m)}(\omega) = \mathbf{H}_{(m)}(\omega)[\mathbf{Y}_{c(m)}(\omega)\mathbf{V}_{m(m)}(\omega) + \mathbf{I}_{m(m)}(\omega)] \quad (24)$$

$$\mathbf{I}_{m(m)}(\omega) = \mathbf{Y}_{c(m)}(\omega)\mathbf{V}_{m(m)}(\omega) - \mathbf{B}_{m(m)}(\omega) \quad (25)$$

$$\mathbf{B}_{m(m)}(\omega) = \mathbf{H}_{(m)}(\omega)[\mathbf{Y}_{c(m)}(\omega)\mathbf{V}_{k(m)}(\omega) + \mathbf{I}_{k(m)}(\omega)] \quad (26)$$

The solution in time domain is obtained again by using convolution

$$\mathbf{i}_{k(m)}(t) = \mathbf{y}_{c(m)}(t) * \mathbf{v}_{k(m)}(t) - \mathbf{b}_{k(m)}(t) \quad (27)$$

$$\mathbf{b}_{k(m)}(t) = \mathbf{h}_{(m)}(t) * \{\mathbf{y}_{c(m)}(t) * \mathbf{v}_{m(m)}(t) + \mathbf{i}_{m(m)}(t)\} \quad (28)$$

$$\mathbf{i}_{m(m)}(t) = \mathbf{y}_{c(m)}(t) * \mathbf{v}_{m(m)}(t) - \mathbf{b}_{m(m)}(t) \quad (29)$$

$$\mathbf{b}_{m(m)}(t) = \mathbf{h}_{(m)}(t) * \{\mathbf{y}_{c(m)}(t) * \mathbf{v}_{k(m)}(t) + \mathbf{i}_{k(m)}(t)\} \quad (30)$$

However, since both $T_V$ and $T_I$ are frequency-dependent, a new convolution is now needed to obtain line variables in phase quantities

$$\mathbf{v}_{(ph)}(t) = \mathbf{t}_v(t) * \mathbf{v}_{(m)}(t) \quad (31)$$

$$\mathbf{i}_{(ph)}(t) = \mathbf{t}_i(t) * \mathbf{i}_{(m)}(t) \quad (32)$$

The procedure to solve the equations of a multi-conductor frequency-dependent overhead line in the time domain involves in each time step the following :

1) Transformation from phase domain terminal voltages to modal domain.
2) Solution of the line equations using modal quantities, and calculation of (past history) current sources.
3) Transformation of current sources to phase domain quantities.

Fig. 4 shows a schematic diagram of the solution of overhead line equations in the modal domain.

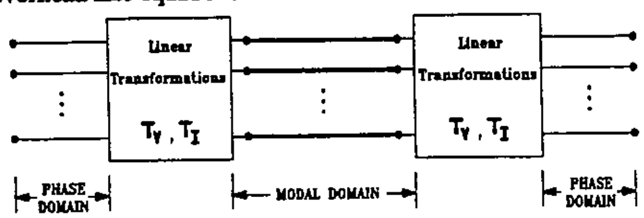


Fig. 4. Transformations between phase domain and modal domain quantities.

depend on the capacitances, which can be assumed frequency independent. Taking into account this behaviour, frequency-dependent effects can be separated, and a line section can be represented as shown in Fig. 5 [21], [24].

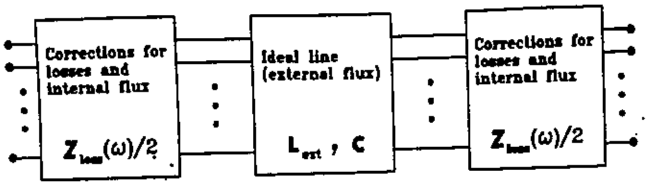


Fig. 5. Section of a non-homogeneous model.

The fact that $Z_{loss}$ is modelled as lumped has advantages, i.e. their elements can be synthesized in phase quantities, and limitations, since a line has to be divided into sections to reproduce the distributed nature of parameters. For more details see the main body and the discussion of [24].
The phase domain approach seems to be the most efficient one; some models based on these procedures have already been implemented in some electromagnetic transients programs [16], [22], [23].

## IV. SUMMARY OF LINE MODELS

Fig. 6 shows a scheme of line models considering the approaches above discussed.

## V. EXAMPLES

Simulation results from the JMARTI line model, as implemented in ATP-EMTP [7], and the Universal Line model (ULM), as implemented in EMTDC V3 [22], [23], are compared. The constant transformation matrix used by JMARTI is in all examples calculated at 5000 Hz. The results are validated in example B) against a phase-domain Fourier solution.

A) Single circuit line

Figures 7 and 8 show respectively the conductor configuration and the test case scheme of a 50 km, shielded untransposed overhead line. Figure 9 compares the receiving end voltages derived from the JMARTI and the ULM models. The calculated voltages are seen to be in close agreement.

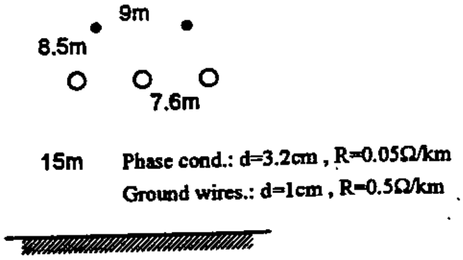


Fig. 7. Single circuit line.

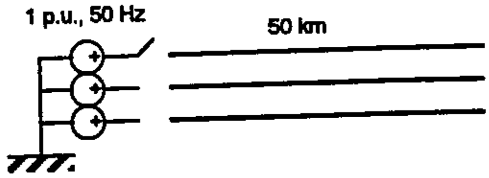


Fig. 8. Single-phase energization.

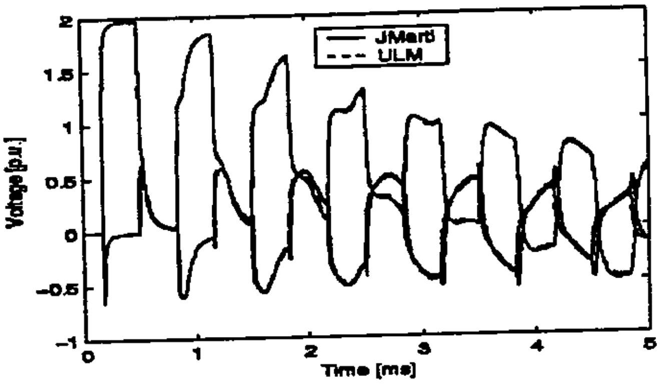


Fig. 9. Receiving end voltages.

B) Parallel overhead lines

A new overhead line is placed in parallel with the previous one; the goal is now to calculate the inducing effect from one line to the other. Figure 10 shows the configuration of the new test system.
Two different situations will be considered to compare the performance of these two models. Both situations correspond to fault conditions of the energized line, as presented in the following paragraphs.

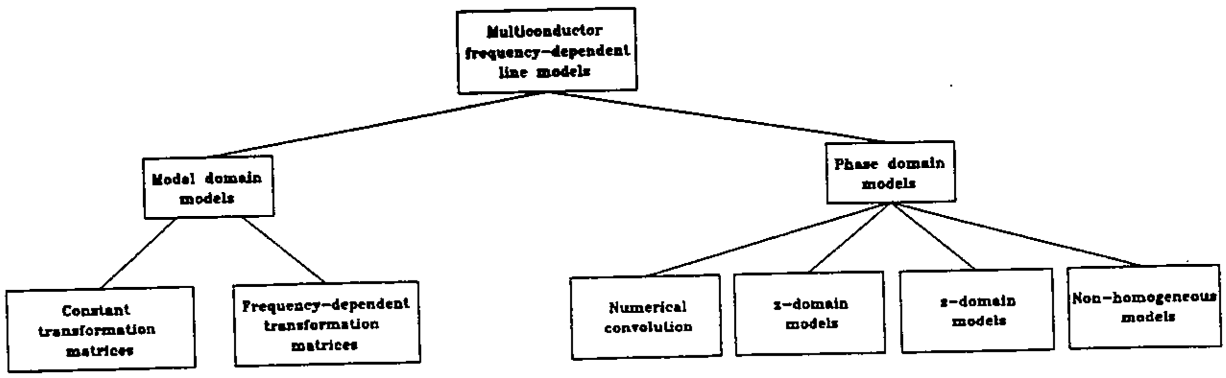


Fig. 6. Classification of multiconductor frequency-dependent line models.

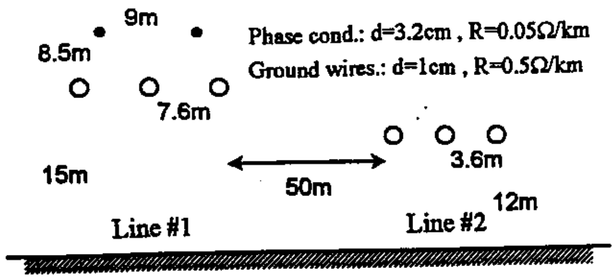


Fig. 10. Parallel overhead lines

*B.1. Single-phase short-circuit*

Figures 12 and 13 show the receiving voltages on conductor 6 (Line #2) resulting from the energization of Line #1 with a single phase short circuit at the receiving end, see Fig. 11. The overvoltages are depicted for time spans of 5 ms and 50 ms., respectively. It can be seen that the relative deviation is now significantly larger than in the previous case. The ULM shows a close agreement with the Fourier solution, what validates the accuracy of this model.

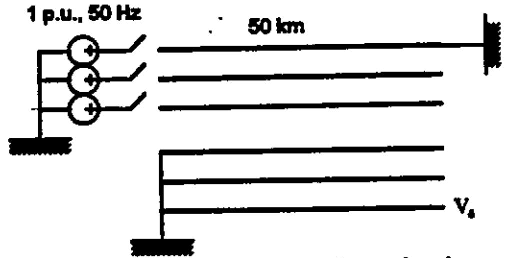


Fig. 11. Single-phase short-circuit.

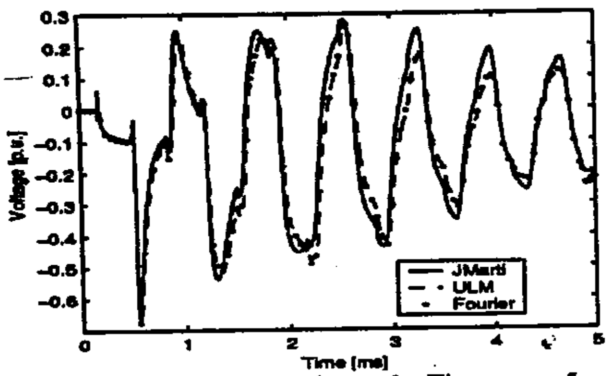


Fig. 12. Voltages on conductor 6 – Time span =5 ms.

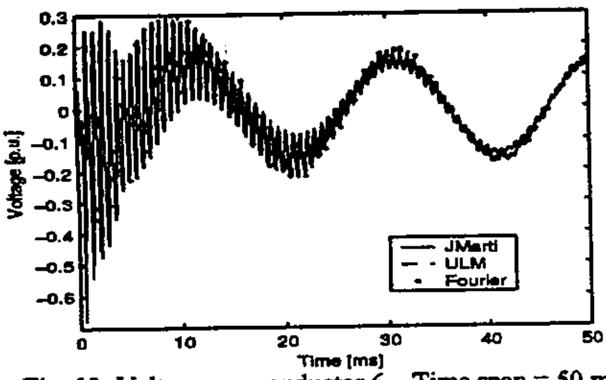


Fig. 13. Voltages on conductor 6 – Time span = 50 ms.

*B.2. Three-phase short-circuit*

Figures 15 and 16 show the receiving voltages on conductor 6 resulting from energization of Line #1 with a three-phase short circuit at the receiving end, see Fig. 14. The overvoltages are depicted again for time spans of 5 ms and 50 ms, respectively. The ULM line model shows again a close agreement with the Fourier solution, while the result by the JMARTI line is seen to have a 50Hz error. The relative deviation is now quite large for the initial transient.

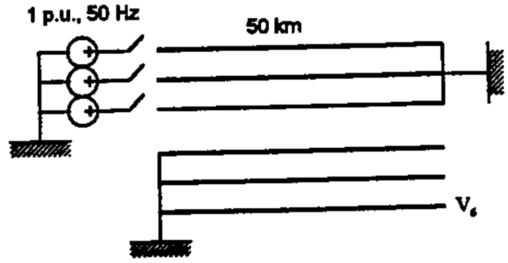


Fig. 14. Three-phase short-circuit.

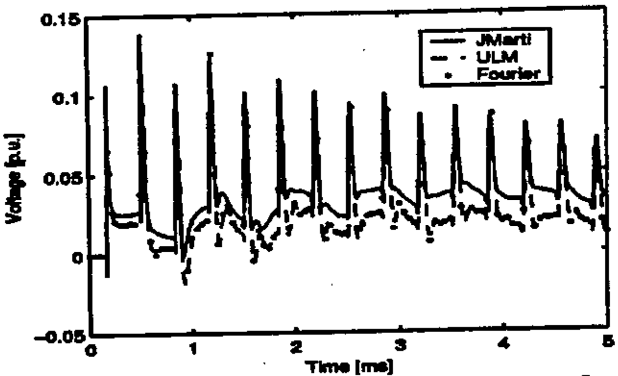


Fig. 15. Voltages on conductor 6 – Time span = 5 ms.

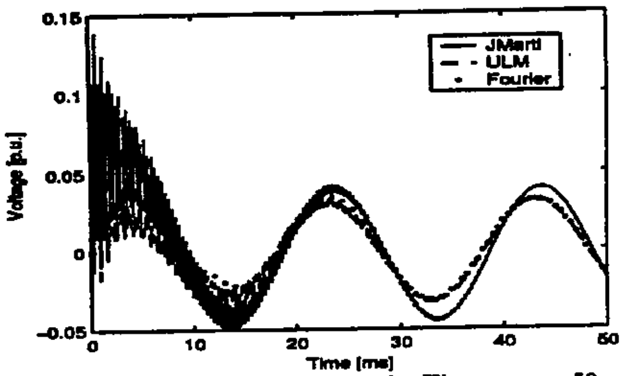


Fig. 16. Voltages on conductor 6 – Time span = 50 ms.

The calculated results have shown that the new ULM of EMTDC gives transient responses which are in good agreement with the JMARTI line of ATP for a single circuit test; the deviation between the responses is quite small, less than 0.03 p.u. However, the relative error becomes quite apparent in Figures 12, 15 and 16, which correspond to results from a parallel line configuration. Since the ULM results are in close agreement with those of a Fourier solution, this suggests that the deviation is being caused by inaccuracies in the JMARTI line model. In particular, the assumption in JMARTI of the constant transformation matrix can be of importance.

## VI. CONCLUSIONS

An important effort has been made during the last 30 years to solve the equations of multiconductor overhead lines accurately represented taking into account the frequency dependence of their parameters. This paper has summarized the main works related to this topic, with emphasis on those approaches which have been implemented into an EMTP-like tool. The paper has shown that, although some refinements are still needed, the solutions developed during the last years have greatly improved the performance of line models and solved problems which have been pending for many years.

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