# zCable Model for Frequency Dependent Modelling of Cable Transmission Systems

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Abstract – **Although much effort has been devoted in the last decade to solve the problem of synthesizing the frequency dependent transformation matrix in unbalanced multiphase cable system, we believe the best solution is to completely eliminate the necessity of the frequency dependent transformation matrix. This paper presents advances in the development of a new cable model (zCable) which splits the representation of the cable model into two parts: a constant ideal line section and a frequency dependent loss section. This approach permits the representation of the frequency dependent part of the cable parameters directly in phase coordinates thus avoiding the need for frequency dependent transformation matrices. In this paper, we present a number of simulations comparing the zCable model with the full frequency dependent cable model in the EMTP, the LMARTI (FDQ cable) model.**

## I. INTRODUCTION

A number of models can be found in the literature [1,2,3,4,7] for the electromagnetic transient simulations of overhead lines and cables. Most of them use modal decomposition theory [5,6] to decouple the physical system (phase domain) into mathematically equivalent decoupled systems (modal domain) in order to solve a multiphase line as if it consisted of a number of separated single-phase lines.

One of the most widely used transmission line models is the FD line model proposed by J. Martí [1]. This model includes the frequency dependence of the line parameters and their distributed nature, and assumes a real and constant transformation matrix to decouple the propagation modes. The FD line model has been very reliable and accurate for most of the overhead line cases, but not for underground cables, which transformation matrix depends strongly on frequency. The full frequency dependent FDQ cable model developed by L. Martí [4] solve the problem of a strongly frequency dependent transformation matrix by synthesizing this matrix with rational functions in the frequency domain. This model gives very good results for cable simulations in both low and high frequency phenomena but it is computationally expensive. A new vector-fitting technique has been recently introduced by Gustavsen and Semlyen [7] to increase the computational efficiency of the simulations. Despite the very accurate results reported with this technique, the procedure still cannot guarantee the absolute numerical stability of the frequency dependent modal domain functions.

In order to avoid the need for a frequency dependent transformation matrix, Castellanos and Martí proposed the zLine model of [10,11,12] for overhead lines. This model can be formulated directly in phase coordinates and completely avoids the use of modal decomposition matrices.

The basic principles of the zCable model were proposed as early as [8,9,10,11,12]. The present paper, however, is the result of a number of years of experiences and refinements that make the zCable model a valid production-model alternative to existing EMTP frequency dependent line and cable models. The approach of the zCable model is to split the representation of the wave propagation phenomena into two parts: a) the ideal line section with constant parameters, and b) the loss section with frequency dependent parameters. A difference from [10,11,12] is that for the cable case the ideal line section in the model is not solved directly in the phase domain but in the modal domain to account for the different modal velocities in cable propagation. Nonetheless, since the parameters of the ideal line section are not frequency dependent, the transformation matrix used in this part of the model is still frequency independent. An improved fitting procedure is developed here to more accurately synthesize the frequency dependent loss impedance matrix $[Z_{loss}(\omega)]$ with rational functions in phase coordinates. The results presented in this paper show that the zCable model gives very accurate results for all components of cable transients, including the multiphase sheath voltages which are reported in [9] as being particularly difficult to reproduce accurately with the sectionalized line concept.

## II. DEVELOPMENT OF THE ZCABLE MODEL

The wave propagation equations in the frequency domain can be expressed as

$$\frac{d^2V}{dx^2} = [ZY]\cdot V \quad \text{and} \quad \frac{d^2I}{dx^2} = [YZ]\cdot I \tag{1}$$

where $[ZY]$ and $[YZ]$ are full matrices that couple the wave propagation of voltage and current in every phase. The elements of the frequency dependent series impedance matrix $[Z]$ can be described as

$$Z_{ij}(w) = R_{ij}(w) + jw(L_{ij}^{\text{int}}(w) + L_{ij}^{ext}) \tag{2}$$

where $R_{ij}(w)$ includes the resistance of the conductor and ground return; $L_{ij}^{\text{int}}(w)$ is the internal inductance associated with the flux inside the conductor and ground return; and $L_{ij}^{ext}$ is the external inductance due to the flux outside the conductor. Therefore, the series impedance matrix $[Z]$ can also be expressed as

$$[Z(w)] = ([R(w)] + jw[L^{\text{int}}(w)]) + jw[L^{ext}]$$
$$= [Z_{loss}(w)] + jw[L^{ext}]$$
$$= [Z_{loss}(w)] + [Z_{ideal}] \quad (3)$$

The shunt admittance matrix $[Y]$ can be described as

$$[Y] = [G] + jw[C] \quad (4)$$

where $[G]$ is the shunt conductance matrix representing dielectric losses, and $[C]$ is the shunt capacitance matrix that permits the conductor to retain potential across the insulation. The elements of $[C]$ are dependent on the permittivity of the insulator and the diameter of the conductors and insulators. In this paper, the elements of $[G]$ and $[C]$ are assumed to be constant.

The zCable model is based on first subdividing the total cable length into a number of shorter segments to simulate the distributed nature of the losses. Then each segment is modelled as consisting of two sections: an ideal line section and a loss section (Fig. 1).

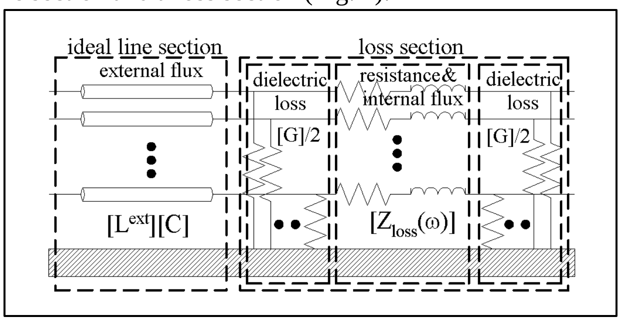



Fig. 1 Separation of basic effects in zCable model

The ideal line section includes the external magnetic and electric fields $[L^{ext}]$ and $[C]$, which parameters depend only on the cable geometry and are frequency independent. The loss section consists of two subsections. The first subsection includes the resistance $[R(\omega)]$ and the internal inductance $[L^{int}(\omega)]$, which parameters are frequency dependent due to skin effect. This subsection can be grouped into one lumped series impedance matrix $[Z_{loss}(\omega)]$. The other subsection is the constant dielectric losses [G].

## III. MODELLING OF THE IDEAL LINE SECTION

Because of the different permittivities of inner and outer insulators in underground cables, the propagation velocities for each mode of the ideal line section are different. This property results in different travelling times and time delays for each mode. This complicates the formulation when trying to solve for the propagation directly in phase coordinates as is done in the case of overhead lines [10,11,12]. This difficulty can be overcome if the phase variables are decoupled into mode variables. Each of the independent equations in the modal domain can then be solved as single-phase lines using the modal travelling time and modal surge impedances.

From circuit theory [13], the modal domain end-node voltage vectors $[v_k^{\text{mode}}(t)]$ and $[v_m^{\text{mode}}(t)]$ can be expressed as functions of the characteristic impedance $[Z_c^{\text{mode}}]$, the current into the end-node vectors $[i_k^{\text{mode}}(t)]$ and $[i_m^{\text{mode}}(t)]$, and the history voltage-source vectors $[e_{kh}^{\text{mode}}(t)]$ and $[e_{mh}^{\text{mode}}(t)]$ (Fig. 2).

The history voltage sources are updated at each time step as functions of past voltage and current values, and have the form:

$$[e_{kh}^{\text{mode}}(t)] = [Z_c^{\text{mode}}][i_m^{\text{mode}}(t - t_i)] + [v_m^{\text{mode}}(t - t_i)] \quad (5)$$
$$[e_{mh}^{\text{mode}}(t)] = [Z_c^{\text{mode}}][i_k^{\text{mode}}(t - t_i)] + [v_k^{\text{mode}}(t - t_i)] \quad (6)$$

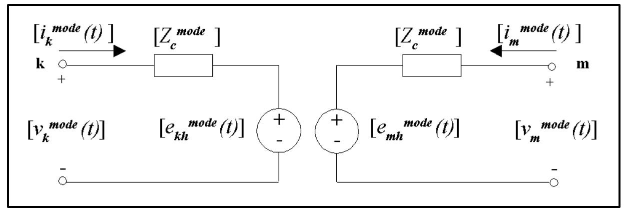



Fig. 2 Ideal line section model of multiphase cables in modal domain

After solving each equation in the modal domain, the modal quantities have to be transformed back to the phase quantities in order to combine the ideal line section with the rest of the model, which is defined in phase domain to combine with the rest of the network. Although a transformation matrix is used for solving the equation of the ideal line section, its elements are real and constant owing to the property of constant inductance and capacitance for the ideal line section parameters.

## IV. MODELLING OF THE LOSS SECTION

In general, the elements of the series loss impedance matrix $[Z_{loss}(\omega)]$ are frequency dependent functions and should be synthesized with rational function approximation for discrete-time EMTP solution. In network theory, the stable finite poles of the system functions correspond to the natural frequencies of the system, which are a property of the network. To guarantee the stability of the $[Z_{loss}(\omega)]$ matrix in the zCable model, all elements of this matrix are approximated using the same set of stable poles. These poles can then be factored out of the matrix giving a single scalar transfer function with stable poles.

Since for all elements of $[Z_{loss}(\omega)]$ the poles must be the same, each element of the matrix must be synthesized in coordination with each other. A modification of the curve-fitting procedure originally proposed in [11] is presented here to assure the same poles for all elements of $[Z_{loss}(\omega)]$ and thus guarantee the stability of the model for all possible system simulation cases.

In the synthesized functions, each element of $[Z_{loss}(\omega)]$ is expressed as a sum of the same number of parallel RL blocks having the same basic form $sK/(s+P)$ ($s=j\omega$), as follows

$$Z_{fii}^{loss}(w) = R_{iiDC} + \sum_{l=1}^{m} \frac{sK_{ii(l)}}{s + P_{ii(l)}} \text{ for diagonal elements} \quad (7)$$

$$Z_{fij}^{loss}(w) = \sum_{l=1}^{m} \frac{sK_{ij(l)}}{s + P_{ij(l)}} \quad \text{for off-diagonal elements} \quad (8)$$

where subscript "*f*" indicates "fitted" function

A. Fitting Procedure

The developed fitting procedure is described in Fig. 3. After the procedure, the set of $m$ RL parallel blocks for each element of the series loss impedance $[Z_f^{loss}(w)]$ become the complete expressions of (7) and (8), and represent a series of RL parallel circuits. In the new procedure developed, the evaluation of the current RL block is dependent on the influence of the previous blocks.

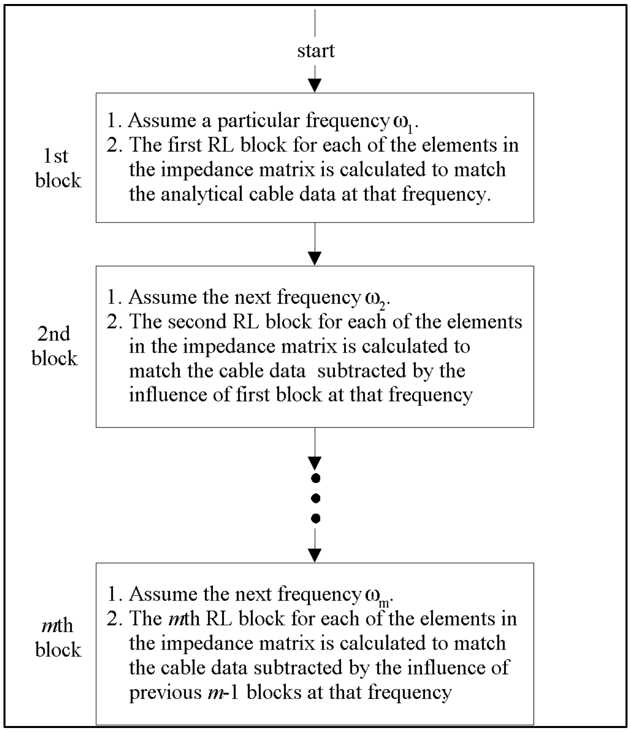


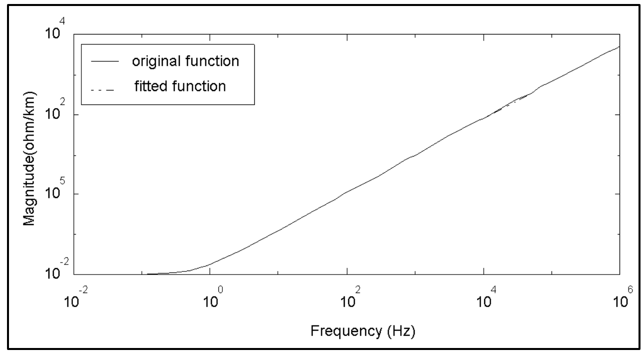

Fig. 3 Curve-fitting procedure

This procedure is different from the curve-fitting procedure of [11] where each block was calculated to match the line data independently of the other blocks.

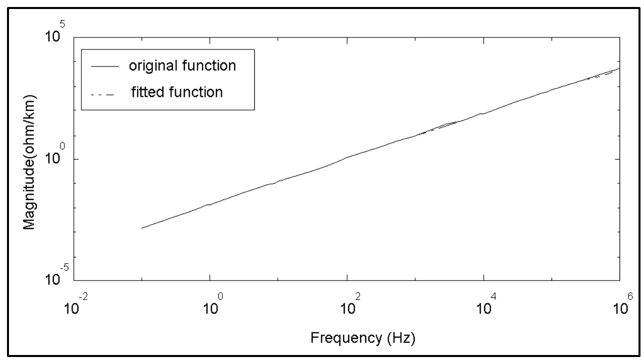
B. Optimization Procedure

After obtaining the synthesized function in the previous procedure, some apparent errors between the original and fitted functions are still found. An optimization procedure based on Gauss-Seidel iteration and the work of [8] is applied to minimize these errors. This procedure is similar to the previous fitting procedure except that each RL parallel block for each element of $[Z_{loss}(\omega)]$ is calculated to match the analytical cable data subtracted by the influence of all other "*m*-1" blocks at that frequency. After obtaining $m$ matching RL blocks for each element of $[Z_f^{loss}(w)]$, the accuracy of the fitted functions is checked. If the errors are within an acceptable specific level, the optimization procedure can be ceased and the final fitted functions are obtained. Otherwise, more iterations are needed until the errors are within the specified limit. The optimization procedure used here is also different from that of [11] where a least square technique was used. The number of iterations used in this procedure is usually dependent on the number of RL blocks used. Generally, acceptable errors are obtained with ten iterations or less. Fig. 4 shows a comparison of fitted curve calculated with eight RL blocks and the original curve for $[Z_{loss}(\omega)]$.

From Fig. 4, we could apparently tell that the fitted function matches very closely the original function based on the maximum error of 3%. In our experience, the average number of blocks used for an accurate synthesis and curve-fitting is about one block per decade.




(a) Diagonal element




(b) Off-diagonal element

Fig. 4 Comparison of original and fitted functions

## V. PHASE DOMAIN CABLE SOLUTION

The Solution of each short cable segment is obtained by combining the solutions of the ideal line section and loss section. In order to better represent the distributed nature of the losses, the loss section is divided into two halves, and each half is placed at both ends of the ideal line section. Then the solution for the complete cable length is accomplished by combining the solutions of all segments in cascade.

The work in [14] presents an algorithm that chooses the maximum short segment length for desired model accuracy. An exact solution is compared with a solution based on the zCable model at several frequencies to obtain a relationship between the maximum segment length and the maximum frequency of interest. The curve in Fig. 5 is derived from [14] and corresponds to a maximum error of 5% and a ground resistivity of 100 Ω-m.

## VI. TRANSIENT SIMULATION AND VALIDATION

In this paper, the results of the zCable model are compared with those of the JMARTI line model (FD line)

[1] and the LMARTI cable model (FDQ cable) [4]. The 230KV underground cable arrangement and physical data of a test cable are shown in Fig. 6 and Table 1 [4]. The cables are modelled with the zCable model using 50 segments of 0.1km each to simulate accurately up to 20kHz (Fig. 5).

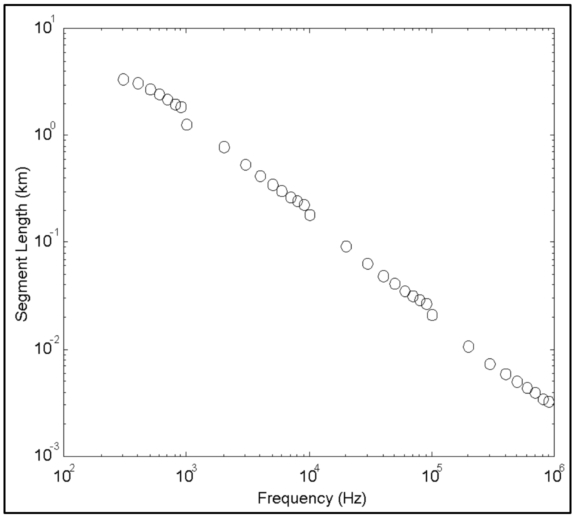



Fig. 5 Relationship between segment length and frequency

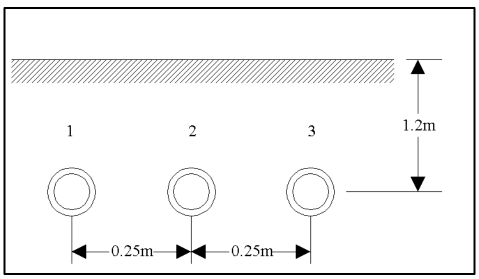



Fig. 6 Underground cable arrangement for the test case

Table 1 Physical data of 230KV cable for the test case

| Name | Value |
|---|---|
| Inner radius of core (cm) | 0.0 |
| Outer radius of core (cm) | 2.34 |
| Inner radius of sheath (cm) | 3.85 |
| Outer radius of sheath (cm) | 4.13 |
| Outer insulator radius (cm) | 4.84 |
| Core resistivity (Ω-m) | $0.0170\times10^{-6}$ |
| Sheath resistivity (Ω-m) | $0.2100\times10^{-6}$ |
| Inner insulator tan*d* | 0.001 |
| Outer insulator tan*d* | 0.001 |
| Inner insulator $e_r$ | 3.5 |
| Outer insulator $e_r$ | 8.0 |
| All relative permeability $m_r$ | 1.0 |
| Earth resistivity (Ω-m) | 100 |

A. Single-phase line-to-ground fault test

The system configuration for a single-phase line-to-ground fault test is shown in Fig. 7. A three-phase balanced sinusoidal voltage source with peak magnitude 1.0 p.u. is connected and switched on at time zero ($t = 0$), energizing the sending end of each core. The receiving end of each core is connected to a resistive load of 500 Ω. The sending and receiving ends of the sheaths are directly connected to ground. A single-phase short circuit fault is applied at the receiving end of core 1 through a resistance of 0.05 Ω at time zero.

The voltages at the receiving end of core2 for the zCable, FD line, and FDQ cable models are shown in Fig. 8.

The results of Fig. 8(a) show a noticeable difference between the result with the FD line model versus the results with the FDQ cable and zCable models. The accurate behavior of the FD line model within the first 300μs (Fig. 8(b)) is probably related to the fact that the constant transformation matrix used in this model was calculated at the high frequency of 1kHz. The results with the zCable model present a very good agreement with those of the FDQ cable model over the entire simulation range.

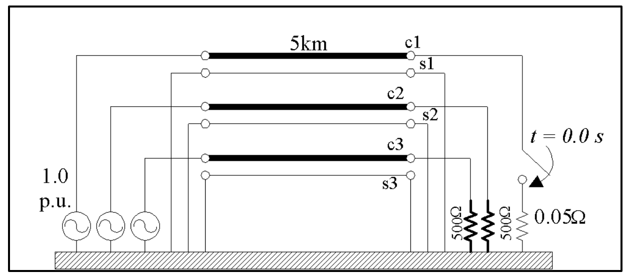
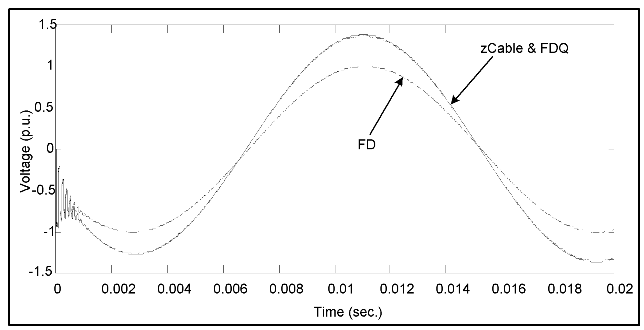
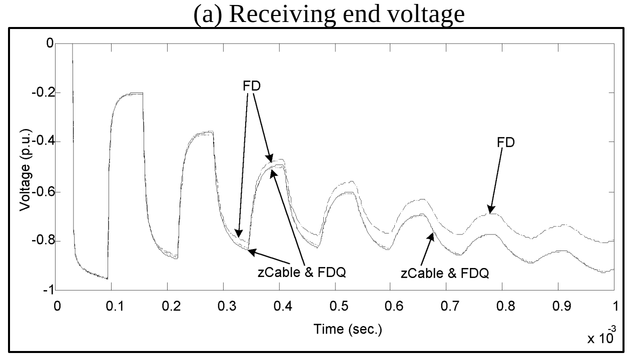



Fig. 7 System configuration of single-phase line-to-ground fault test




(a) Receiving end voltage




(b) Initial transient of the receiving end voltage

Fig. 8 Voltage at the receiving end of core 2

B. Open circuit test

The test system for an open circuit test is shown in Fig. 9. A three-phase balanced sinusoidal voltage source with peak magnitude 1.0 p.u. is connected and switched on at time zero ($t$ = 0), energizing the sending end of each core. The receiving ends of all cores and sheaths are left open. The sending ends of all sheaths are directly connected to ground. The voltages at the receiving end of core1 and sheath1 are shown in Fig. 10 and Fig. 11, respectively.

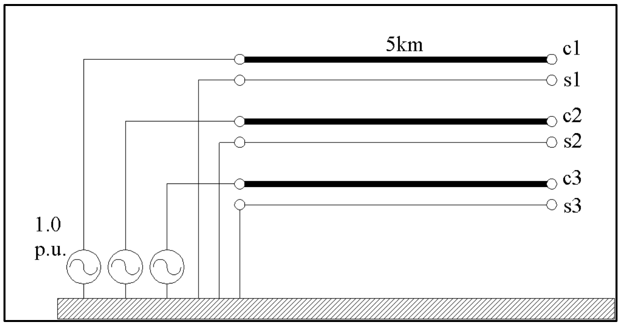
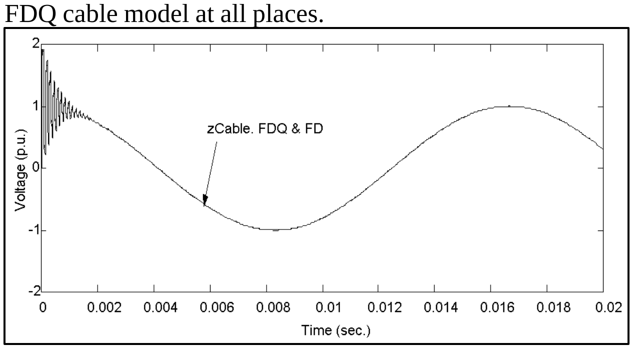
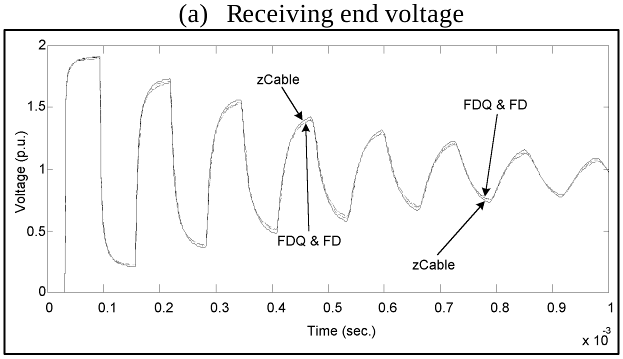



Fig. 9 System configuration of open circuit test

From Figs. 10 and 11, we can observe that the FD line model matches closely the other two models at core1, but strongly deviates from the other two models at sheath1. The zCable model presents a very good agreement with the FDQ cable model at all places.

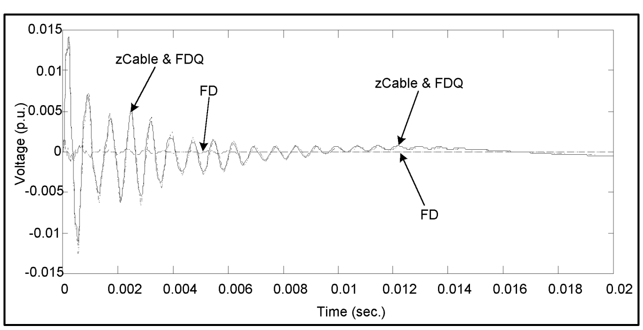



(a) Receiving end voltage

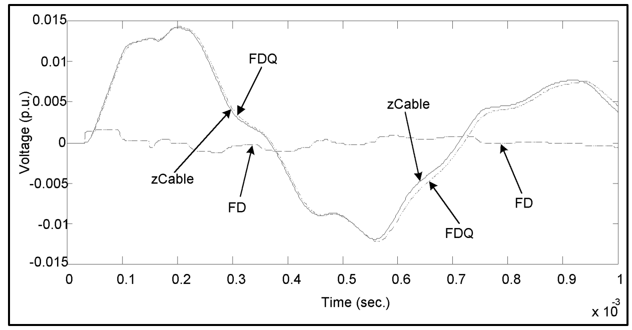



(b) Initial transient of the receiving end voltage

Fig. 10 Voltages at the receiving end of core 1




(a) Receiving end voltage




(b) Initial transient of the receiving end voltage

Fig. 11 Voltages at the receiving end of sheath 1

C. Impulse response test

The system configuration for an impulse response test is shown in Fig. 12. A 1.2 x 5.0μs voltage impulse is connected to the sending end of core1, and switched on at time zero (t = 0). The receiving ends of all cores and sheaths are left open. Except for the sending end of core1, all other sending ends of all other cores and sheaths are directly connected to ground. The voltages at the receiving ends of core1 and sheath1 are shown in Fig. 13 and Fig. 14, respectively.

From Figs. 13 and 14, we can see that the results of the zCable model closely agree with those of the FDQ cable model. Like the results of the open circuit test, the results of the FD line model match closely those of the FDQ cable and zCable models at the receiving end of core1, but deviate at the receiving end of sheath1. Since underground cables have strongly asymmetrical configurations, we believe that the differences with the FD line model are due to its use of a real and constant transformation matrix. The imaginary part of this matrix is important in strongly asymmetrical configurations.

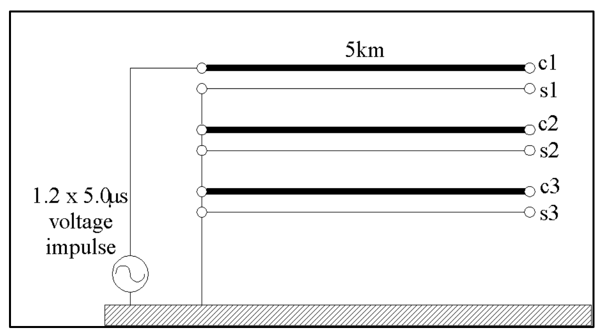



Fig. 12 System configuration of Impulse response test

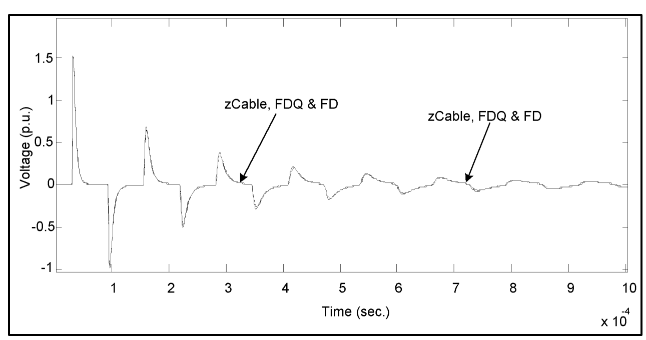



Fig. 13 Voltage at the receiving end of core 1

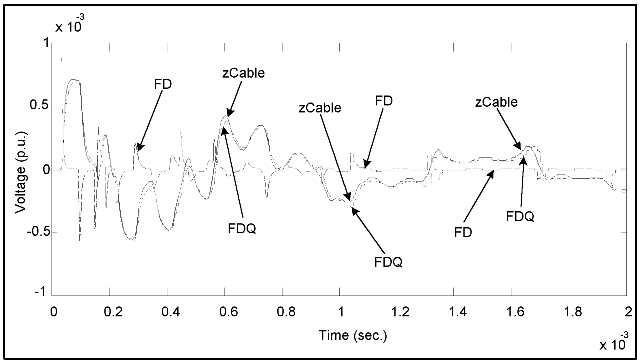



Fig. 14 Voltage at the receiving end of Sheath 1

## VII. CONCLUSION

The zCable model presented in this paper is based on the approach of Castellanos and Marti's work of [10,11,12], which divides the representation of the cable model into two parts: an ideal line section associated with the external magnetic field $[L^{ext}]$ and the electric field $[C]$, and a loss section which includes the resistance $[R(\omega)]$, the internal flux $[L^{int}(\omega)]$, and the dielectric losses $[G]$. This concept has been extended to the case of different propagation velocities for different modes due to the cable insulation layers.

A new fitting procedure is also presented here to synthesis and fit all elements of the frequency dependent loss impedance matrix $[Z_{loss}(?)]$ simultaneously to guarantee equal poles for all elements of this matrix and avoid the numerical stability issues of traditional frequency dependent transformation matrix line and cable models.

The results of the zCable model show a very good agreement with the FDQ cable model and are more accurate than the constant transformation matrix FD line model. The main advantages of the zCable model over the FDQ model are: 1) zCable avoids the difficulties of synthesizing the frequency dependent transformation matrix, and 2) the fitting procedure of zCable is very robust and efficient, and leads to absolutely stable synthesis functions.

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## BIOGRAPHIES


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