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Analysis of Very Fast Transients in Transformer

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Abstract - **A practical calculation method is developed to calculate the possible voltage oscillation in transformer subjected to very fast transient overvoltages (VFTOs) generated by operating a disconnector in gas insulated switchgear (GIS). It is based on the multiconductor transmission-line (MTL) theory for a shell-form transformer. In the frequency domain analysis, the high frequency characteristics of a transformer are calculated using the distributed constants obtained from the winding geometry.**

The calculated frequency characteristic of interturn voltage agrees with the experimental up to several megahertz. The interturn voltage waveforms for a given VFTO pulse are calculated using FFT. The time-domain results are found to be particularly in good agreement with the experimental. The feature of this method is that the high frequency response of transformer can be calculated directly from the winding geometry, so that the effects of VFTO on transformers should be assessed during designing.

Keywords: Very fast transient overvoltage, Transformer, Multiconductor transmission line, Transient Analysis, Modelling.

I. INTRODUCTION

Very fast transient overvoltages (VFTOs) generated by operating a disconnector in gas insulated switchgear (GIS) could cause a voltage oscillation inside the transformer connected. The situation is severer in the system that the main transformer is directly connected to GIS since the high frequency surges travel more easily along the coaxial gas insulated bus. Particularly, it is important to assess the interturn induced voltage level since it is the most vulnerable point in this situation.

Lumped circuit models have been used to analyse the transformer transients associated with the lightning or switching surges [1-4]. For the analysis involving the VFTO of much higher frequencies, the multiconductor transmission-line (MTL) theory is applied [5]. The method is the turn-to turn modelling which requires a large computation capacity. To avoid this, the authors have proposed a hybrid method combining the single transmission-line analysis of whole winding and the MTL analysis of the highest-voltage section [6].

The main purpose of present paper is to provide a practical MTL method to calculate the high frequency characteristics of shell-form transformers developing a method to reduce the number of unknowns by combining multiple turns under common parameters. The interturn voltage is calculated and compared with the experimental results obtained in the actual transformers both in frequency and time domains. The effects of VFTO on the transformer in actual systems are briefly discussed.

II. MTL MODEL

A. Assumptions

Fig. 1 shows the shell-form transformer in which HV winding is composed of coils and two static plates (SPs) indicated as SP_o, SP_e. It is assumed that the electromagnetic waves propagate along conductors inside the core window just as in a waveguide. Regarding the fields in the vicinity of N+2 conductors (total turns N plus two SPs) to be transverse electromagnetic (TEM), the following relation is obtained for the distributed capacitance and inductance matrices $[C]$, $[L]$ of N+2 dimensions [7].

$$[L]=[C]^{-1}/v_s^{\,2} \quad (1)$$

Where, v_s is the electromagnetic velocity:

$$v_s = 1/\sqrt{\varepsilon_r \varepsilon_0 \mu_0} \quad (2)$$

Here, ε_r is the dielectric constant of insulation.

B. Application of MTL theory

The model shown in Fig. 2 is derived considering the transformer subjected to a VFTO. Here, the sinusoidal voltage E_o of frequency ω represents the VFTO. The thick parallel lines are the inter-linked transmission lines of average turn length a, and the dashed lines are fictitious zero-length leads. The current and voltage of lines are denoted by the vectors $(I^{(m)}(x))$ and $(V^{(m)}(x))$, with m=1 or 2 indicating region (1) or (2). Although the core rejects high frequency magnetic fluxes but still some flux may form a common flux there. The common flux in-

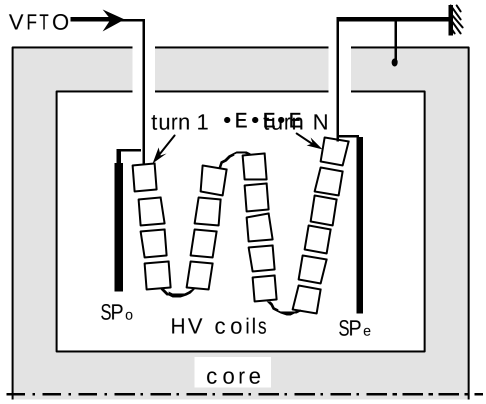


Fig. 1. Shell-form transformer winding.

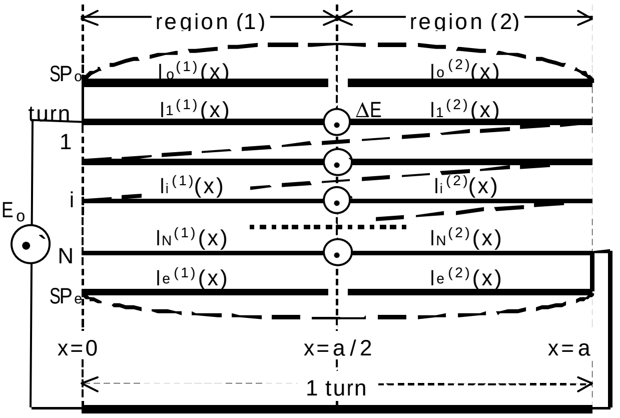


Fig. 2. MTL model of shell-form transformer.

duces the voltage ΔE at the mid point of each turn where SPs have a break.

The following MTL equation holds [7].

$$\begin{cases} \dfrac{d}{dx}(I^{(m)}(x)) = -j\omega\,[C]\,(V^{(m)}(x)) \\ \dfrac{d}{dx}(V^{(m)}(x)) = -j\omega\,[L]\,(I^{(m)}(x)) \end{cases} \tag{3}$$

Using (1) in (3) leads to the wave equation:

$$\frac{d^2}{dx^2}(I^{(m)}(x)) = \Gamma^2\,(I^{(m)}(x)) \tag{4}$$

Here, Γ (=$j\omega/v_s$) is the propagation constant. The general solution of (4) or (3) is written as

$$\begin{cases} (I^{(m)}(x)) = (A^{(m)})\exp(-\Gamma x) + (B^{(m)})\exp(\Gamma x) \\ (V^{(m)}(x)) = v_s[L]\{(A^{(m)}\exp(-\Gamma x) - (B^{(m)})\exp(\Gamma x)\} \end{cases} \tag{5}$$

The vectors ($A^{(m)}$) and ($B^{(m)}$) are unknown constants representing the surge currents. The dissipation can be incorporated if the following Γ is used [6]:

$$\Gamma = \frac{j\omega}{v_s} + \frac{\omega\tan\delta}{2v_s} + \frac{1}{v_s\,d}\sqrt{\frac{\omega}{2\sigma\mu}} \tag{6}$$

This is derived assuming a constant insulation loss tangent of $\tan\delta$ and a proximity effect in the conductors of conductivity σ separated by turn insulation d.

To the 4(N+2) unknowns, ($A^{(m)}$) and ($B^{(m)}$), the same number of linear equations can be constructed by considering the connecting conditions in Fig. 2. In the equations, ΔE can be evaluated by

$$\Delta E \cong j\omega L_c \sum_{i=1}^{N} I_i^{(1)}\left(\frac{a}{2}\right) \tag{7}$$

where, L_c is the inductance of the common flux.

C. Reduction of MTL equation

It is possible to eliminate 8 unknowns in (5): $A_o^{(m)}$, $B_o^{(m)}$, $A_e^{(m)}$, $B_e^{(m)}$ (m=1, 2) – the surge components in all sections of SPs. The result can be written as:

$$\begin{cases} (I^{(m)}(x)) = (A^{(m)})\exp(-\Gamma x) + (B^{(m)})\exp(\Gamma x) \\ (V^{(m)}(x)) = v_s[L]\{(A^{(m)})\exp(-\Gamma x) - (B^{(m)})\exp(\Gamma x)\} \\ \qquad + E_o(k_o) - v_s[M]\{\gamma^{k-1}(A^{(m)}) - \gamma^{-k+1}(B^{(m)})\} \end{cases} \tag{8}$$

Where,

$$\gamma = \exp(-\Gamma a) \tag{9}$$

In (8), the dimension of matrices and vectors is reduced from original N +2 to N. The last two terms in the second relation of (8) express the electrostatic and electromagnetic inductions from SPs. The vector (k_o) and matrix [M] can be defined from the original inductance matrix of N+2 dimensions [8]. The vector (k_o) is so-called initial voltage distribution or capacitive voltage distribution. The connecting conditions at x=a/2 in (8) can be expressed in the form:

$$\begin{cases} (I^{(1)}(a/2)) = (I^{(2)}(a/2)) \\ (V^{(1)}(a/2)) = (V^{(2)}(a/2)) + \Delta E\,(\mathbf{1}) \end{cases} \tag{10}$$

Where, ($\mathbf{1}$) means the vector of all the elements being unity. The first equation can be satisfied if the following equation is introduced with an arbitrary vector (ΔA):

$$\begin{cases} (A^{(2)}) = (A^{(1)}) - (\Delta A) \\ (B^{(2)}) = (B^{(1)}) + \gamma\,(\Delta A) \end{cases} \tag{11}$$

This means that the forward and backward travelling waves are modified or scattered by (ΔA) at the mid-turn. The second relation of (11) demands the following condition:

$$v_s\{[L]-[M]\}(\Delta A) = v_s\sinh\frac{\Gamma a}{2}[M]\{(A^{(1)}) + \gamma^{-1}(B^{(1)})\} + \frac{\Delta E}{2\,\gamma^{1/2}}(\mathbf{1}) \tag{12}$$

Here, the approximation $\cosh(\Gamma a/2) \cong 1$ is used:

Taking $A_i^{(1)}$, $B_i^{(1)}$ and ΔA_i as new unknowns, the number of unknowns is reduced to 3N. A set of 3N linear equations can be constructed considering both (12) and the connection equations:

$$\begin{cases} V_1^{(1)}(0) = E_0 \qquad V_N^{(2)}(a) = E_e \\ I_i^{(2)}(a) = I_{i+1}^{(1)}(0),\ V_i^{(2)}(a) = V_{i+1}^{(1)}(0) \quad 1 \quad i \quad N-1 \end{cases} \tag{13}$$

D. Combining turns for further reduction

Preliminary numerical calculation has shown that $A_i^{(1)}$, $B_i^{(1)}$ change with i, but ΔA_i scarcely changes except at the coil-ends [8]. It is suggested to set a division comprised of multiple turns in the inner coil region as shown in Fig. 3. Here, the combined turns are drawn as if they are of a single transmission line, in which the same scattering parameter ΔA_k is assumed. The following relations should hold among the components of i–1 and i-th turns:

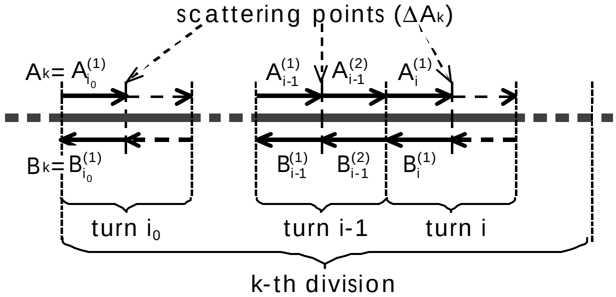


Fig. 3. Travelling waves in a multiple-turn division.

$$\begin{cases} A_i^{(1)} = \gamma\, A_{i-1}^{(2)}, & B_i^{(1)} = \gamma^{-1}\, B_{i-1}^{(2)} \\ A_i^{(2)} = A_{i-1}^{(1)} - \Delta A_k, & B_i^{(2)} = B_{i-1}^{(1)}\, \gamma^{-1} + \gamma\, \Delta A_k \end{cases} \tag{14}$$

The first two are from the continuity of surge components, and the last two from (11). From (14), the following iterative relations are derived.

$$A_i^{(1)} = \gamma\, A_{i-1}^{(1)} - \gamma \Delta A_k,\ \ B_i^{(1)} = \gamma^{-1} B_{i-1}^{(1)} + \Delta A_k \tag{15}$$

Let i_0 the first turn of division k, then these give:

$$\begin{cases} A_i^{(1)} = \gamma^{i-i_0}\, A_k - \dfrac{\gamma - \gamma^{i-i_0+1}}{1-\gamma} \Delta A_k \\ B_i^{(1)} = \gamma^{-i+i_0}\, B_k + \dfrac{\gamma - \gamma^{-i+i_0}}{1-\gamma^{-1}} \Delta A_k \end{cases} \tag{16}$$

Here, $A_{i_0}^{(1)}$, $B_{i_0}^{(1)}$ are renamed as A_k, B_k.

If HV winding is divided into N_d divisions, the number of unknowns is $3N_d$. Choosing adequately small N_d, the number of unknowns can be reduced substantially.

III. PRELIMINARY CALCULATION

A. Transformers analysed

The two transformer windings in Table 1 are analysed. One is a 2-coil model constructed for experimental purposes using the plastic-film insulated coils extracted from a gas insulated transformer. The other is a complete set of HV winding of a 500 kV autotransformer with oil/paper insulation.

For the 2-coil model of N =84, turn-to-turn calculation is possible i.e. without multiple-turn divisions. However, for 500 kV transformer of N =363, it is difficult to obtain the solution without reducing the number of unknowns by setting multiple-turn divisions in the authors' computer environment.

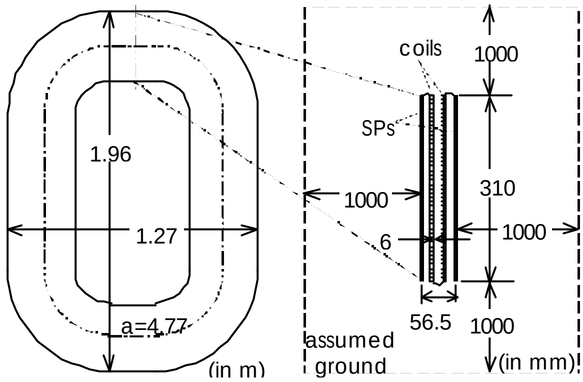
B. Constants of Transformers

The inductance $[L]$ necessary for the present analysis is calculated from the winding geometry in the following steps. Firstly, $[C]$ is calculated by the 2-dimensional charge simulation. In the calculation, ground surfaces are assumed at some distance from the winding as shown in Fig. 4 (the case of 2-coil transformer). Then, $[L]$ is obtained using (1). Since core was not used in the experiment to secure the necessary measuring space, some distances are correspondingly taken between the winding and the grounded plane in calculating $[L]$.

Fig. 5 shows selected line elements in $[L]$ of the 2-coil model. The common inductance L_c is set zero

Table 1. Transformers analysed.

	constants	2-coil model	500 kV autotransformer
winding dimensions	number of coils	2	10
	total no. of turns, N	84	363
	av. turn length, a [m]	4.77	7.6
insulation	interturn thickness, d [mm]	1.5	1.6 - 3.0
	dielectric constant, ε_r	1.6	2.9
	dissipation factor, $\tan\delta$	0.05	0.05
conductor conductivity, σ [S/m]		5×10^7	5×10^7



(a) side view (b) cross-section

Fig. 4. Configuration of 2-coil model.

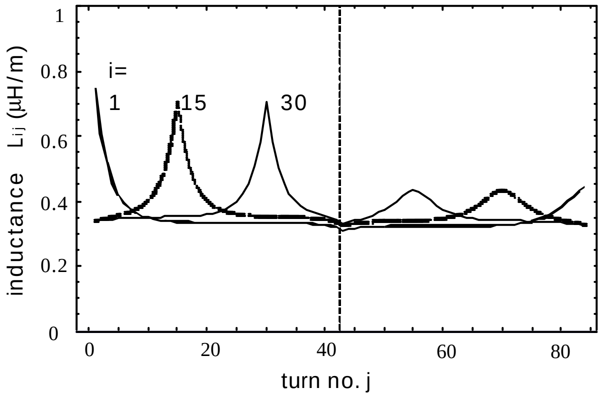


Fig. 5. Inductance L_{ij} calculated for 2-coil model.

considering the experimental condition of without core.

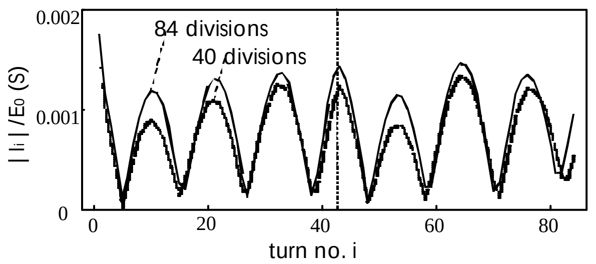
C. Validation of calculation

The surge components A_k and B_k can be numerically determined solving the $3N_d$-dimension linear equation if the frequency of input voltage E_0 is given (the amplitude is set as E_0=1 in calculation). Then, current or voltage at any point is to be calculated for the continuous sinusoidal input voltage.

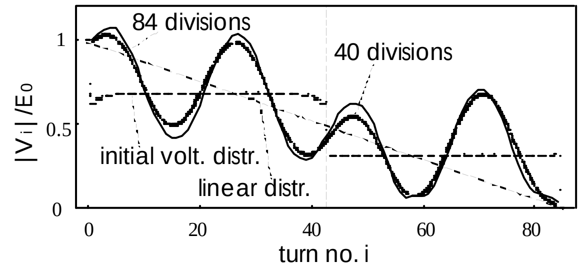
Effects of number of divisions are examined in the case of 2-coil model transformer. Fig. 6 shows the current and voltage distributions at 1.5 MHz calculated in two kinds of division conditions: 84 divisions (turn-to-turn case) and the case of 40 divisions (2 or 3 turns are combined except at coil end). Their good correspondence indicates the applicability of the present method.

The spatial distribution of current of Fig. 6a shows a fluctuation as typically seen in the waves propagating in a transmission line. This may be seen as a standing wave phenomenon. A similar fluctuation is seen in the voltage distributions of Fig. 6b. It should be noted that only the absolute values are shown in those results.

The two broken lines in Fig. 6b are the initial voltage distribution defined by (k_0) which is expected at an extremely high frequency, and the linear distribution expected at a very low frequency. It is justifiable that the calculated distribution for 1.5 MHz is situated between



(a) current distribution



(b) voltage distribution

Fig. 6. Current and voltage distributions in 2-coil model at 1.5 MHz.

those two curves of extreme frequencies, with some fluctuations due to the standing wave phenomenon.

IV. COMPARISON WITH EXPERIMENT

A. Frequency characteristics

The interturn voltage induced by VFTO is of the main concern in the present analysis. Particularly, its frequency characteristic is important since VFTO may have a variety of high frequency components [9]. The frequency characteristic of interturn voltage can be obtained in the present analysis by calculating

$$\delta V_i = V_i - V_{i+1} \tag{17}$$

Figs. 7 and 8 are the frequency characteristics of the first interturn voltage δV_1 for the two transformers.

In the case of 2-coil model (Fig. 7), curves a1 and a2 are calculated by the present MTL method with 84 and 40 divisions, respectively. Curve b is calculated using the turn-to-turn lumped circuit model based on the inductance and capacitance matrices corresponding to those used in the MTL model. The experimental curve c is obtained observing the interturn voltage through an optoelectronic isolation technique applying a continuous sinusoidal voltage of varous frequency [10, 11].

All the characteristics in Fig. 7 have a series of peaks due to internal resonances. Among the calculated results, the resonance peaks by MTL are higher than those by the lumped circuit result demonstrating the feature of transmission line modelling. Although the calculated resonance frequencies are a bit shifted from the experimental, their similarity confirms the applicability of the present method.

In the case of 500 kV autotransformer (Fig. 8), curve a is calculated by the MTL method with 90 divisions, and curve b is obtained experimentally. Their correspondence

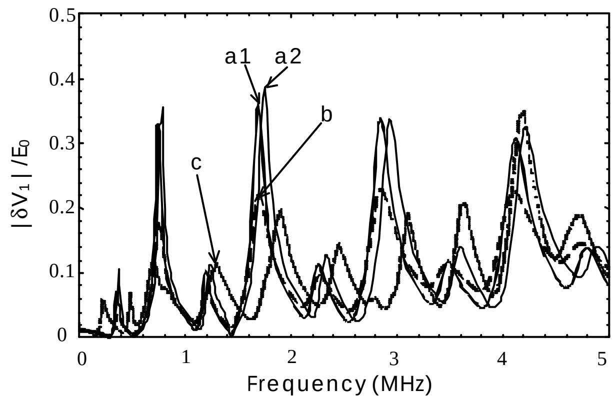


Fig. 7. Frequency characteristics of first interturn voltage for 2-coil model.

a1: 84 divs (1div/turn) by MTL model

a2: 40 divs (20divs/coil) by MTL model

b: by turn-to-turn lumped circuit model

c: experimental

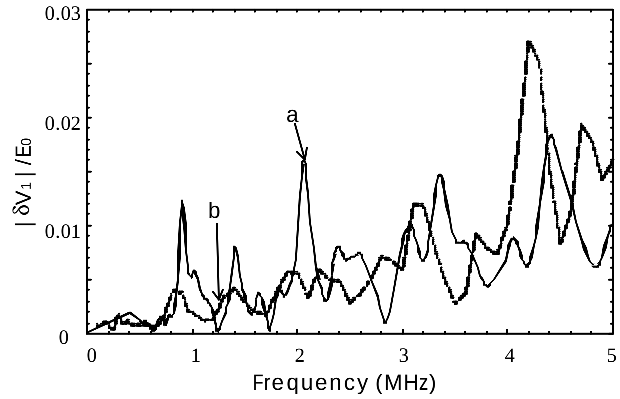


Fig. 8 Frequency characteristics of first interturn voltage of 500kV autotransformer..

a: 90 divs (9divs/coil) MTL

b: experimental

here is less remarkable, probably due to the interferences from many resonances. At least, their resemblance indicates that the present method can be used for a rough estimation

It should be beard in mind that the assessment using the frequency characteristic tends to overestimate the interturn voltage level. This is because the actual VFTO is not a continuous sinusoidal but of a damped oscillation.

B. Interturn voltage waveforms

The VFTO comprises not only oscillating components but also a pulse of higher magnitude [9]. It is important to know how the transient interturn voltage develops by VFTO. The interturn voltage waveform can be calculated from the frequency domain data using FFT.

For convenience, representing the VFTO pulse by one-cycle sinusoidal pulse of 2 MHz, responses of the two transformer models are calculated and compared with experimental. Fig. 9 shows the input pulse used in experiment. The induced interturn voltage is recorded at selected positions using the optoelectronic technique as already mentioned [10, 11].

In the case of 2-coil model, Fig. 10 shows the interturn voltages, comparing the calculated and experimental results at 3 points (first, middle and end positions in the first coil). The calculated (Fig. 10a) and experimental (Fig. 10b) waveforms correspond very well. There is a distinct delay time in the waveshape at the middle part of coil in both results, indicating that travelling waves start from the both of coil ends and propagate toward inner region [10, 11]. As pointed out previously, their velocity is interpreted as the electromagnetic wave along the conductors depending on the dielectric constant of insulation space ε_r as described by (2).

Fig. 11 shows the first interturn voltage of 500 kV autotransformer subjected to the same sinusoidal pulse of 2 MHz. The correspondence of the calculated and experimental is satisfactory. The induced voltage level is smaller than that of 2-coil model. This is because the increased number of coils decrease the voltage entering into a coil, therefore, the interturn voltage level.

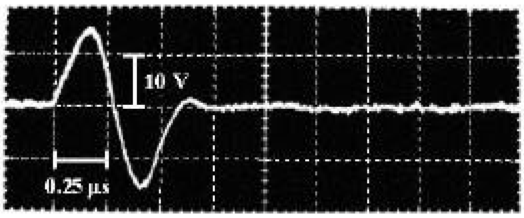
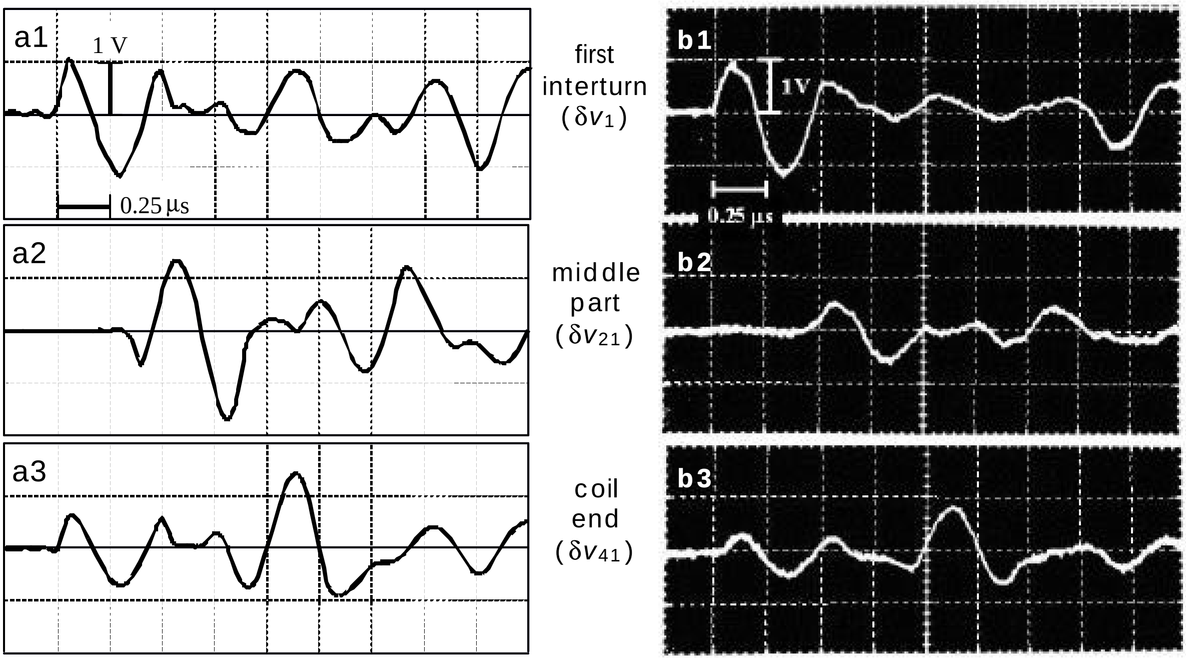


Fig. 9 Sinusoidal pulse used as input voltage in the experiment.

V. CONCLUDING REMARKS

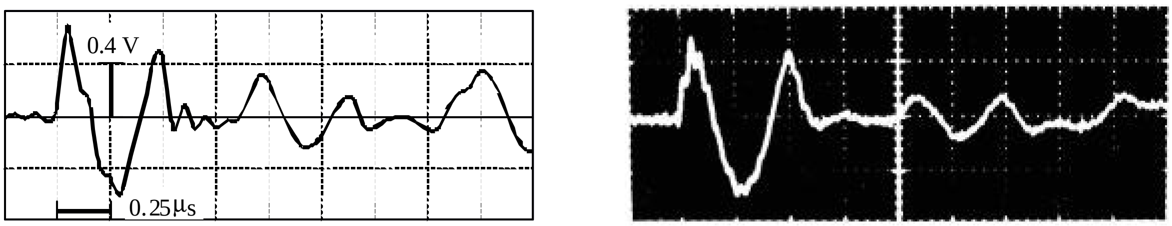
A practical method to analyse the high frequency transients in the power transformer is developed by reducing the number of unknowns in applying the MTL theory. Voltage or current at any point in the winding is calculable both in the frequency and time domains. The constants necessary in the calculation can be estimated from the transformer geometry or design parameters. The experiments including an actual 500 kV transformer have confirmed the applicability of the present method to the analysis of high frequency transients of several megahertz.

Comparing the calculated and experimental waveforms in Figs. 10 and 11, they seem to differ at the later time of the figure. This shows the dissipation represented by (6) is not accurate enough. However, their correspondence is better than the frequency domain (Figs. 7, 8). It might be that the some dissipation mechanism is not well



(a) calculated (b) experimental

Fig. 10. Interturn voltage waveforms of 2-coil model at application of sinusoidal pulse.



(a) calculated (b) experimental

Fig. 11. Voltage induced at first interturn δv_1 of 500 kV autotransformer subjected to sinusoidal pulse.

represented in the present model, but it does not play an important role in the time domain.

From Fig.7, one may think traditional lumped circuit model can be used for high frequency analysis equally well. There have been proposed other ways of applying MTL theory. Cornick et al. used MTL in the highest voltage section only and replaced the rest of winding by an impedance [5]. And there is a hybrid method combining the single transmission-line analysis of whole winding and the MTL analysis of the highest-voltage section [6]. Those techniques have following problems comparing with the present method.

Lumped circuit model: The turn-to-turn modelling requires a large computation capacity.

Cornick's MTL method: The rest of winding is difficult to be represented by a simple impedance.

Hybrid MTL method: Resonances tend to be in excess due to the use of single transmission line.

In this sense, the present MTL method is more advantageous in analysing VFTO's effects on large transformers.

Another salient feature of the present MTL method is that the high frequency characteristics of a transformer can be calculated from the configuration of HV winding, or design data. Fig.12 depicts this process. The first step is the calculation of capacitances by a charge simulation, then inductances. Next, the surge components (unknowns) are calculated at a series of Fourier frequencies, which gives frequency characteristics. And finally, the FFT calculation turns out time domain waveforms.

The VFTO generated in GIS comprises various high frequency oscillations extending to several megahertz [9]. By the use of present MTL method, it is possible to estimate the interturn induced voltage in frequency or time domains based on the constants determined from the winding geometry. This is particularly useful in designing phase since the most vulnerable insulation is interturn in VFTO's point of view.

The time domain calculation using FFT in the present method seems to give more accurate than the frequency domain. Therefore, it is recommended to run the time domain calculation for the predicted VFTO waveform in the case high levels of interturn voltage are anticipated in the frequency characteristics.

The authors hope this technique contributes to the better design of transformer and to the improved power system reliability.

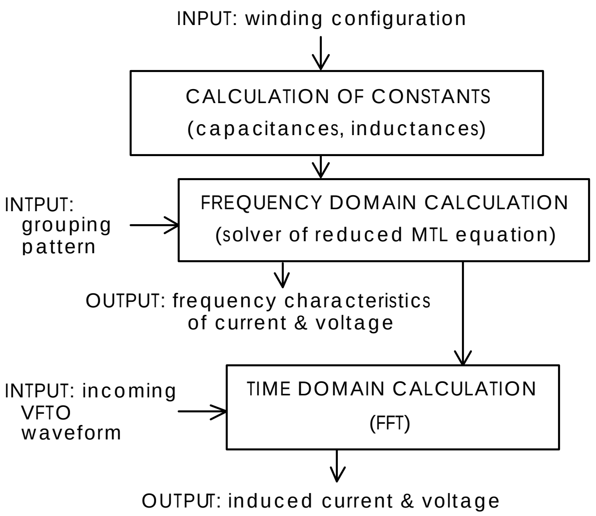


Fig. 12. Flow to calculate high frequency characteristics of transformer using MTL model.

VI. REFERENCES

[1] J. H. McWhirter, et al., "Determination of impulse stresses within transformer windings by computers", *Transactions of AIEE*, vol.76 Pt.III, 1957, pp.1267-1274.

[2] W. J. McNutt, et al., "Response of transformer windings to system transient voltages", *IEEE Transactions on Power Apparatus and Systems*, vol.PAS-93 (2), 1974, pp. 475-467.

[3] R. C. Degeneff, "A general method for determining resonances in transformer windings", *IEEE Transactions on Power Apparatus and Systems*, vol.PAS-96, 1977, pp. 423-430.

[4] D. J. Wilcox, et al., "Application of modified modal theory in the modelling of practical transformers", *Proceedings of IEE*, vol.139 Pt.C (6), 1992, pp. 513-520.

[5] K. Cornick, et al., "Distribution of very fast transient overvoltages in transformer windings", *CIGRE Report*, 12-204, 1992.

[6] Y. Shibuya, et al., "Analysis of very fast transient overvoltage in transformer winding", *IEE Proceedings-Generation Transmission and Distribution*, vol.144, No.5, 1997, pp.461-468.

[7] R. P. Clayton, *Analysis of multiconductor transmission lines*, *Willy*, New York, 1994.

[8] Y. Shibuya, et al., "Analysis of Very Fast Transients in Transformer", to be published in *IEE Proceedings-Generation Transmission and Distribution*.

[9] Y. Shibuya, et al., "Effects of very fast transient overvoltages on transformer", *IEE Proceedings-Generation Transmission and Distribution*, vol.146, No.5, 1999, pp.459-464.

[10] S. Fujita, et al., "Experimental study of very fast transient phenomena in transformer winding", *IEEE Transactions on Power Delivery*, Vol.13, No.4, 1998, pp.1201-1207.

[11] S. Fujita, et al., "Voltage oscillation in transformer windings affected by very fast transient surges", *Transactions of IEE Japan*, Vol.120-B, No.5, 2000, pp.766-772.