

# Transformer Saturation after a Voltage Sag Recovery

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**Abstract** - The interest in voltage sags is mainly due to the interruption they may cause on several industrial process as some piece of equipment may trip out when the rms voltage drops below certain value for longer than one or two cycles. However, it has been found that transformer may saturate after a voltage sag recovery. This paper presents an analysis on this phenomenon.

**Keywords:** Transformer saturation; voltage sag; voltage recovery; magnetising current.

## I. INTRODUCTION

Transformer saturation is always a matter of concern to power system operation as it can generate transformer magnetising current of high magnitudes which are very rich in harmonic components. Under most practical system conditions, this current is of little consequence. In some cases, however, a combination of circumstances may be obtained as to impair momentarily the proper operation of the system [1]. If, for example, the harmonics of the magnetising current coincide with resonance points in the system, harmonic voltages and currents of high magnitude will build up, which may cause the operation surge arrests and capacitor bank over-current relays [2-6].

The saturation phenomenon that occurs in power transformer during energising is very well known and has been extensively documented in the relevant literature. However, saturation in transformers that are already in operation has only been drawn attention to power system engineers and researchers during the last decade after the evidence that this phenomenon is very likely to occur when a transformer is energised onto a system in which there are others transformers already in operation (sympathetic interaction) [5, 6].

Recently, a new case of transformer saturation was observed in the Swedish transmission system during a monitoring program. Measurements took in that system at different voltage levels did present high distortion suggesting that transformers had saturated after the voltage sag recovery [7]. This new type of power system event, is discussed in this paper. Emphasis is given to explain quantitatively the phenomenon. Simulation results is also shown.

## II. TRANSFORMER SATURATION

It is very well known from the general theory that, neglecting resistance, when a voltage is applied onto the terminals of a transformer, the instantaneous value of the

magnetic flux within the transformer will be determined by the residual flux in the iron-core added to the volt-second area swept by the applied voltage wave between the instant  $t_0$  of switching and the time  $t$  under consideration, i. e.,:

$$\varphi(t) = \varphi_{res} + \frac{1}{N} \int_{t_0}^t v(t) dt \quad (1)$$

where  $\varphi_{res}$  is the residual flux in the core limb wound by the winding energised,  $v$  is the applied voltage and  $N$  is the winding number of turns.

Assuming that the voltage is:

$$v = V_m \cos \omega t \quad (2)$$

Then,

$$\varphi(t) = \varphi_{res} + \Phi_m (\sin \omega t - \sin \omega t_0) \quad (3)$$

or

$$\varphi(t) = \Delta \varphi + \Phi_m \sin \omega t \quad (4)$$

where

$$\Phi_m = \frac{V_m}{N\omega} \quad (5)$$

and

$$\Delta \varphi = \varphi_{res} - \Phi_m \sin \omega t_0 \quad (6)$$

Transformers saturate when the magnetic flux density in the core exceeds the core saturation density level. Under this condition, the core presents a very low permeability and the transformer behaves as having just air-core windings, with the magnetising current rising abruptly and thenceforth, decaying until the normal steady-state magnetising conditions in the transformer are reached. This transient sometimes may last tens of seconds and even several minutes.

It is interesting to note that  $\Delta \varphi$ , which is the difference between the residual flux and the steady-state flux at the instant the voltage is applied, plays a crucial role in the transformer saturation phenomenon as it is responsible for offsetting the steady-state flux within a transformer.

### III. VOLTAGE SAG CHARACTERISTICS

Voltage sags are reductions in the rms voltage values during a short period of time. The interest in this phenomenon is mainly due to the interruption they cause on several industrial process as some piece of equipment may trip out when the rms voltage drops below certain value for longer than one or two cycles.

Momentarily reduction in rms voltage may be caused by starting of large motors, transformer energising, etc., but most of the current interest in this phenomenon is directed to voltage sags due to short-circuits faults. A very good insight on this matter is given in [8].

The voltage sag characteristics are basically related to the rms value of the fundamental frequency component of the voltage. However, to analyse more accurately the effect of voltage sags on the behaviour of some electric equipment it is necessary to determine the exact point-on-voltage-wave of the "starting" and "ending" of the event, the so-called "point-on-wave of sag initiation" and the "point-on-wave of sag recovery" [9, 10].

The point-on-wave of sag initiation is the phase angle of the fundamental voltage wave which the voltage sag starts. This angle corresponds to the angle at which the short-circuit faults occurs. The point-on-wave of voltage recovery is the phase angle of the fundamental voltage at which the main recovery of the voltage takes place. This angle corresponds to the angle at which the short-circuit faults is cleared.

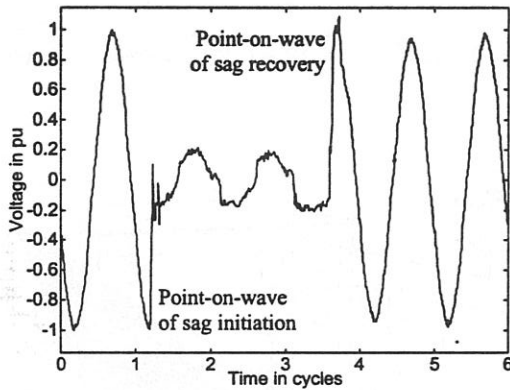


Figure 1 - Measured voltage sag [11]

An important characteristic of voltage sags is the phase-angle jump of the voltage during a short-circuit fault. This phase-angle jump occurs due to the change in the system caused by the short-circuit, which shifts the zero cross of the instantaneous voltage. Large phase-angle jumps occur in distribution system with underground cables. Small positive phase-angle jumps may occur in transmission systems [7].

The missing voltage is another characteristic of voltage sags which has been proposed recently by the IEEE [10]. The concept is important when dimensioning series-connected voltage-sources devices to compensate for voltage sag. The missing voltage is the difference between the pre-event voltage and the sagged voltage.

### IV. MAGNETIC FLUX WITHIN A TRANSFORMER

To investigate the magnetic flux within a transformer before, during and after the event of a voltage sag it is assumed, for the sake of simplicity, some assumptions as following:

- All voltage waveforms are considered as having only the fundamental component (no voltage distortion)
- The sagged voltage is in phase with the pre-sagged voltage (no phase-angle jump)
- The recovered voltage (pos-sagged voltage) has the same magnitude, phase-angle and wave shape of the pre-sagged voltage

Considering that the voltage sag occurs in the interval of time between the instants  $t_s$  and  $t_r$  and the pre-sagged voltage is given by:

$$v_{pre}(t) = V_m \cos \omega t \quad t < t_s \quad (7)$$

then

$$v_{sag}(t) = k_s V_m \cos \omega t \quad t_s < t < t_r \quad (8)$$

and

$$v_{pos}(t) = V_m \cos \omega t \quad t_r < t \quad (9)$$

where  $v_{sag}$ ,  $v_{pre}$  and  $v_{pos}$  are respectively the sagged, pre-sagged and pos-sagged voltages,  $t_s$  and  $t_r$  are the "starting" and "ending" instants of the sag, and  $k_s$  is the ratio between the magnitude of sagged voltage and the pre-sagged voltage.

$$k_s = \frac{v_{sag}(t)}{v_{pre}(t)} \quad (10)$$

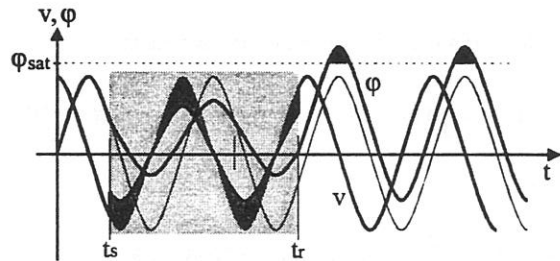


Figure 2 - Behaviour of the magnetic flux within a transformer before, during and after a voltage sag

Before the occurrence of the voltage sag, the transformer is operating normally and, therefore, the magnetic flux within the transformer is a sinusoidal steady-state flux given by:

$$\varphi_{pre}(t) = \Phi_m \sin \omega t \quad t < t_s \quad (11)$$

where  $\Phi_m$  is given by (5).

When the voltage sags, the magnetic flux within the transformer becomes thereafter:

$$\varphi_{sag}(t) = \varphi_{res}^{sag} + \frac{1}{N} \int_{t_s}^t v_{sag}(t) dt \quad (12)$$

The “residual” flux within the transformer at the instant  $t_s$  is the magnetic flux  $\varphi_{pre}$  at that instant, then:

$$\varphi_{sag}(t) = \varphi_{pre}(t_s) + \frac{1}{N} \int_{t_s}^t v_{sag}(t) dt \quad (13)$$

Solving (13):

$$\varphi_{sag}(t) = \Phi_m[(1 - k_s)\sin\omega t_s + k_s\sin\omega t] \quad (14)$$

Similarly, after the voltage recovery the flux within the transformer is given by:

$$\varphi_{pos}(t) = \varphi_{res}^{pos} + \frac{1}{N} \int_{t_r}^t v_{pos}(t) dt \quad (15)$$

or

$$\varphi_{pos}(t) = \varphi_{sag}(t_r) + \frac{1}{N} \int_{t_r}^t v_{pos}(t) dt \quad (16)$$

Solving (16)

$$\varphi_{pos}(t) = \Phi_m[(1 - k_s)(\sin\omega t_s - \sin\omega t_r) + \sin\omega t] \quad (17)$$

or

$$\varphi_{pos}(t) = (1 - k_s) \int_{t_r}^{t_s} v_{pre}(t) dt + \Phi_m \sin\omega t \quad (18)$$

Re-writing (18) in a general form results in

$$\varphi_{pos}(t) = - \int_{t_s}^{t_r} [v_{pre}(t) - v_{sag}(t)] dt + \Phi_m \sin\omega t \quad (19)$$

or

$$\varphi_{pos}(t) = - \int_{t_s}^{t_r} \Delta v_{miss}(t) dt + \Phi_m \sin\omega t \quad (20)$$

where

$$\Delta v_{miss}(t) = v_{pre}(t) - v_{sag}(t) \quad (21)$$

is the missed voltage owing to the voltage sag.  
Also, (18) can be written as

$$\varphi_{pos}(t) = -\Delta\varphi_{miss} + \Phi_m \sin\omega t \quad (22)$$

where

$$\Delta\varphi_{miss} = \int_{t_s}^{t_r} \Delta v_{miss}(t) dt \quad (23)$$

It should be noted that (22) is similar to (4). This suggests that the magnetic flux within a transformer after a voltage sag behaves as that within a transformer being energised, with the instant of switching being the instant of recovery voltage ( $t_r$ ). The difference between the “residual flux” and the steady-state flux at the instant the voltage is applied is therefore  $-\Delta\varphi_{miss}$ .

A close analysis of Figure 2 shows that  $\Delta\varphi_{miss}$  could be also calculated taking the difference between the value of the steady-state magnetic flux that would be produced by the pre-sagged voltage (as if there was no voltage sag) and the flux produced by the sagged voltage, at the instant  $t_r$ . From (11) and (14), it can be written that:

$$\varphi_{pre}(t_r) - \varphi_{sag}(t_r) = \Phi_m \sin\omega t_r - \Phi_m[(1 - k_s)\sin\omega t_s + k_s\sin\omega t_r]$$

$$\varphi_{pre}(t_r) - \varphi_{sag}(t_r) = (1 - k_s)\Phi_m(\sin\omega t_r - \sin\omega t_s)$$

$$\varphi_{pre}(t_r) - \varphi_{sag}(t_r) = (1 - k_s) \int_{t_s}^{t_r} v_{pre}(t) dt$$

$$\varphi_{pre}(t_r) - \varphi_{sag}(t_r) = \int_{t_s}^{t_r} [v_{pre}(t) - v_{sag}(t)] dt$$

$$\varphi_{pre}(t_r) - \varphi_{sag}(t_r) = \int_{t_s}^{t_r} \Delta v_{miss}(t) dt = \Delta\varphi_{miss} \quad (24)$$

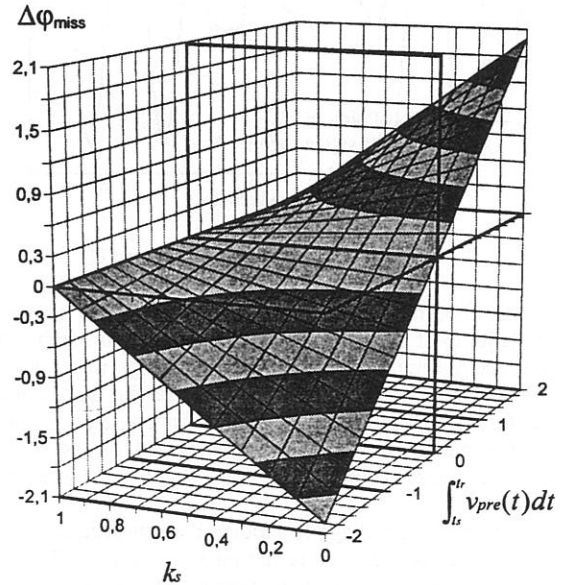


Figure 3 -  $\Delta\varphi_{miss}$  as a function of  $k_s$  and the magnetic flux that would be produced by  $v_{pre}$  between  $t_s$  and  $t_r$ .

Figure 3 shows the values of  $\Delta\phi_{miss}$  as a function of the magnitude of the voltage sag ( $K_s$ ) and the magnetic flux that would be produced by the pre-sagged voltage in the interval between the instants  $t_s$  and  $t_r$ . Considering that transformers are normally designed to operate with 1.0 pu of flux density and that they saturate at approximately 1.2 - 1.3 pu, it means that after a voltage sag a transformer should saturate when  $\Delta\phi_{miss}$  is greater than  $\pm 0.3$  pu.

It is important to note that, actually, the "starting" point of the voltage sag is more likely to occur near maximum voltage. This is because most faults are associated with a flashover and they are more likely to occur near voltage maximum than near voltage zero. Also, as fault clearing takes place at current zero cross and the power system is essentially inductive, the "ending" point of voltage sag occurs near to voltage maximum as well. In addition to that, the magnitude ( $K_s$ ) of most voltage sags are above 0.7 pu. Fortunately, these conditions make the magnetic flux  $\Delta\phi_{miss}$  small, explaining the reason why transformer saturation following a voltage sag recovery does not occur so frequently.

## V. SIMULATION RESULTS

In order to illustrate the transformer saturation phenomenon after a voltage sag recovery, several cases were simulated. A single-phase electric system (Figure 4), composed of a 50 Hz voltage-source supplying a transformer through a series impedance was used. The transformer model was developed to be used in the SABER simulator [12]. The voltage sag was generated by switching of a small resistor ( $R_{sc}$ ) through a controlled switch.

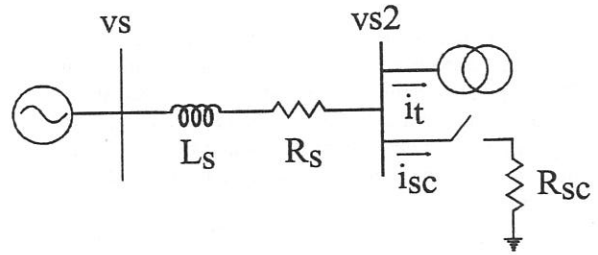


Figure 4 - Electric circuit used in the simulation

Figure 5 shows the results of one case where the voltage sag starts at the instant the voltage crosses zero being positive ( $t=0.04s$ ) and finishes at the short-circuit current ( $i_{sc}$ ) crosses zero ( $t=0.7s$ ). At the top of Figure 5, it can be seen the voltage on the bus-bar to which the transformer is connected ( $vs2$ ) and the voltage ( $vs$ ) generated by the source. Note that the sagged voltage presents a phase-angle jump.

The waveforms at the bottom of Figure 5 are the magnetic flux within the transformer and the magnetising inrush current. As can be seen, the flux behaves as explained previously, with the transformer saturating after the voltage recovery. It is interesting to observe that the magnetic flux becomes flat during saturation and the magnetising current increases. The distortion on the recovered voltage is due to the voltage drop across the system impedance caused by the magnetising current.

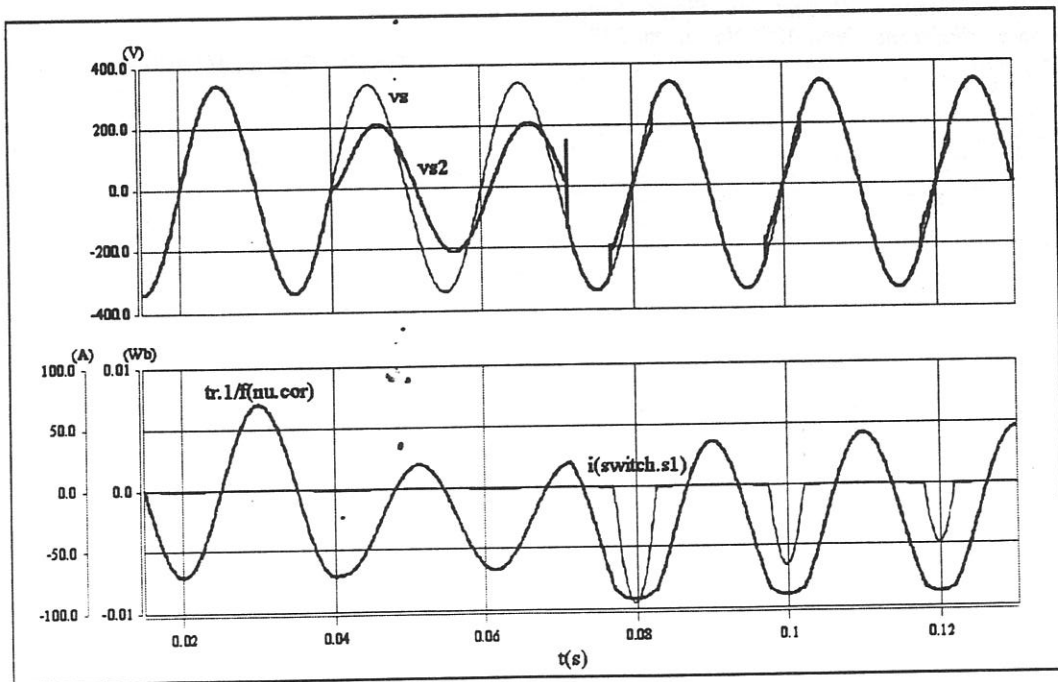


Figure 5 - Simulation results of transformer saturation due to voltage sag recovery

## VI. CONCLUSIONS

The phenomenon of transformer saturation after a voltage sag recovery has been qualitatively and quantitatively investigated in this paper. It has been shown that saturation occurs due to the voltage recovery following a voltage sag, with the magnetic flux within the transformer behaving itself in the same way as that in a transformer being energised.

When this phenomenon occurs, the inrush magnetising currents in the transformers connected to the system will add together causing voltage drops across the system impedance. As a result, the voltage recovers relatively slowly as the decay of these inrush magnetising currents are normally slow.

This problem, which is generally more severe in weak system, may lead to tripping under-voltage and over-current relays. Also, if the harmonics of the magnetising current coincide with resonance points in the system, harmonic voltages and currents of high magnitude will build up, causing the operation surge arrests and capacitor bank over-current relays.

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Recife - Brazil, 21/Jan/2001