A New Approach for Determining the Periodic Steady-State of Circuits with Thyristor-Controlled Reactors

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Abstract - **A new technique is proposed for simulating the responses of circuits with thyristor-controlled reactors. The technique consists in connecting a voltage source across the thyristor module so that the circuit is separated into a linear part and a nonlinear part. Analysis of the two parts is simplified and the mismatch current flowing into the voltage source is determined. The objective of the proposed technique is to determine the voltage source for which the mismatch current is zero. The technique is general and it can be applied for simulating the transient and steady-state responses.**

Keywords: Flexible transmission, Controlled-devices, Nonlinear circuits, Dynamic systems, EMTP.

I. INTRODUCTION

The demand for electric energy in Brazil has been growing very rapidly and the increasing demand requires the installation of new generation and transmission capacity. However, resources for constructing generating stations and transmission lines are scarce and environmental restrictions against the expansion of the power system have also been growing. Therefore, it has become necessary to explore the possibility of increasing the capacity of the existing system.

Series and parallel compensators are being deployed in the system to increase the transmission capacity. Installation of a hvdc link in the south-east of Brazil has also alleviated the demand in that region. Recently, the application of Flexible AC Transmission System (FACTS) controllers has shown to be very effective for increasing transmission capacity [1]. These controllers apply the techniques of power electronics to control power flow and increase transmission capacity. A simple example of a FACTS controller is the thyristor-controlled series compensator (TCSC).

The transient response of the power electronic circuits is usually determined by reducing the linear part of the circuit to its Thevenin equivalent and then applying the Newton-Raphson procedure for determining the currents in the nonlinear elements. Subsequently, the compensation theorem is applied to obtain the nodal voltages in the linear part. Efficient methods have been described to automate the procedure and diminish the solution time [2] and these methods have been incorporated in programs such as the MicroTran [3], which is a version of the Electromagnetic Transients Program.

The steady state is determined, either by computing the transient response over a large number of periods at the fundamental frequency or by converting the response calculation into a two-point boundary value problem [4]. The former approach is a brute force technique whereas the latter approach converges rapidly to the steady state response.

Techniques for simulating power electronic circuits may be broadly classified into two categories. The first category of techniques treats the circuit as one of fixed topology and two-state resistors have been used to represent the switching devices [5]. The second category decomposes the circuit into a sequence of circuit topologies and the state equations for each configuration are determined. Digital simulation then consists of integrating the appropriate state equations over a small time-step and of subsequently establishing the topology for the next time-step [6]. In another approach, the time delay in accounting for topology changes is avoided by introducing linear output prediction and correction procedures [7]. Fixed topology methods are generally faster and easier to program than variable topology methods.

Several other algorithms for the transient analysis of power electronic circuits have been proposed [8-11]. However, few techniques for the steady state analysis of power electronic circuits have been presented.

The objective of this paper is to present a simple technique for determining the periodic steady state response of circuits containing thyristor-controlled reactors. In this technique, voltage sources are connected in parallel with the thyristor module and the mismatch currents through these sources are computed. The mismatch currents are the objective functions that are minimized iteratively by adjusting the parameters of the voltage sources. The proposed approach belongs to the fixed topology category and it reduces to the sequential analysis of the circuit, twice per iteration. This simple procedure is equivalent to the computation of Jacobian matrices and their inverses. It leads to a substantial improvement in accelerating the convergence to the steady state.

Detailed user-defined models for the semi-conductor devices may be easily incorporated in the proposed algorithm and control systems may be handled by treating them as nonlinear blocks. The technique does not require the determination of the conducting states of the switches. These instants are automatically obtained by the computational procedure. This is particularly the case for the turn-off instants of the thyristors.

Using this technique, the responses of TCSC circuits have been obtained as a function of the firing angle. Sudden changes in the conduction angle have been observed for certain firing angles. It has also been shown that there are firing angles for which a solution does not exist and this is due to a premature or delayed zero-crossing of the thyristor currents.

II. THYRISTOR-CONTROLLED REACTOR CIRCUITS

Consider the circuits shown in Fig. 1 and Fig. 2. The circuit of Fig. 1 is a single-phase static VAR system and the circuit shown in Fig. 2 is the Kayenta Advanced Series Compensator. Both circuits have been extensively studied in [12].

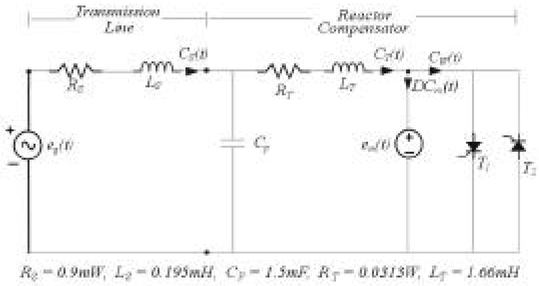


Fig. 1 – Single-phase Static VAR System

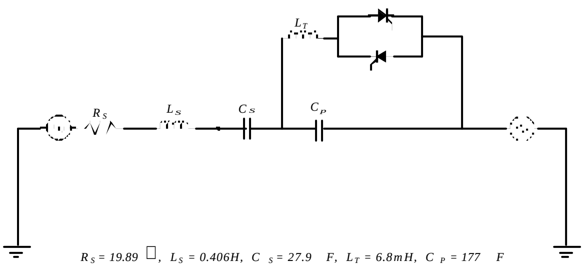


Fig. 2 – Kayenta Advanced Series Compensator

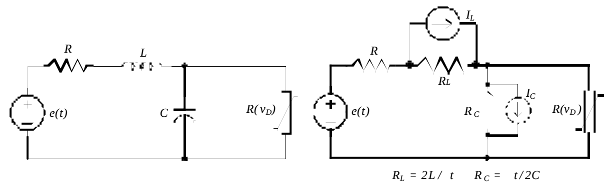
The technique used in [12] for simulating the periodic steady state response of thyristor-controlled reactor circuits consists in establishing the state equation for the system. The Poincaré map $f : x(t_0) \rightarrow x(t_0 + T)$ is defined, where $x(t)$ is the state vector at the instant 't' and T is the fundamental period of the excitation. The Poincaré map advances the system state by one period at the fundamental frequency and the fixed point of the Poincaré map is the steady-state solution.

The Jacobian of the Poincaré map is required for determining the fixed point and the stability of the steady-state solution. It has been computed in [12] after observing that the thyristor switch-off time may be regarded as a fixed parameter.

The Poincaré map has also been used in [13,14] to obtain the steady-state response and the stability of a TCSC system.

III. TECHNIQUE PROPOSED FOR DETERMINING THE TRANSIENT RESPONSE

Consider the nonlinear circuit shown in Fig. 3a. The nonlinear element is assumed to be a voltage-controlled resistor. In the existing EMTP solution methodology, the inductor and capacitor are replaced by their Norton equivalents consisting of resistances and fictitious current sources in parallel (Fig. 3b). The linear part of the circuit is then reduced to its Thévenin equivalent comprising a voltage source e_0 in series with the resistance R_{th}.



(a) (b)

Fig. 3 – Solution Methodology to nonlinear circuit
(a) nonlinear circuit (b) EMTP model

The voltage v_D across the nonlinear element is the solution of the equations:

$$F = v_D + R_{th}.i_D - e_0 = 0 \qquad (1)$$

$$i_D = f(v_D) \qquad (2)$$

where F is an objective function and $f(.)$ is the characteristic of the nonlinear element. The solution of (1), (2) is approached iteratively by following the Newton-Raphson procedure:

$$\Delta v_D^K = -F \Big/ \left(\frac{\partial F}{\partial v_D}\right)^K \qquad (3)$$

$$v_D^{K+1} = v_D^K + \Delta v_D^K \qquad (4)$$

where K is the iteration number, and Δv_D^K is the correction to be added to the solution estimate v_D^K.

An alternative solution procedure results if the correction Δv_D^K is written as:

$$\Delta v_D^K = -\frac{v_D^K + R_{th}.i_D^K - e_0}{1 + R_{th}\left(\frac{\partial i_D}{\partial v_D}\right)^K}$$

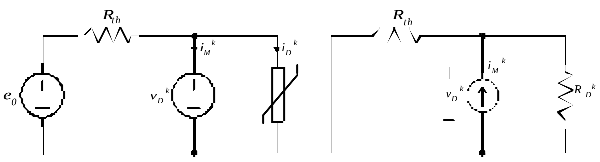
$$= \frac{R_{th}.R_D^K}{R_{th} + R_D^K}.\left(\frac{e_0 - v_D^K}{R_{th}} - i_D^K\right)$$

$$= \frac{R_{th}.R_D^K}{R_{th} + R_D^K}.i_M^K \qquad (5)$$

Where $R_D^K = \left(\frac{\partial v_D}{\partial i_D}\right)^K$ is the linearised (or small-signal) equivalent resistance of the nonlinear element. The Newton-Raphson procedure is therefore equivalent to the sequencial analysis of two circuits.

In the first circuit, a voltage source v_D^K is connected across the nonlinear element as shown in Fig. 4a and the current through this source is the mismatch current i_M^K. The connection of the voltage source in parallel with the nonlinear element separates the circuit into a linear part and a nonlinear part. Therefore, the determination of the mismatch current is very much simplified.

The second circuit (shown in Fig. 4b and referred to as the small-signal equivalent), is required for calculating the correction Δv_D^K. This circuit is obtained by removing the external source e_0, replacing the nonlinear element by its linearised equivalent resistance and finally, substituting the voltage source v_D^K by the mismatch current i_M^K with reversed polarity. The voltage across the current source i_M^K is the correction Δv_D^K to be added to the solution estimate v_D^K before proceeding to the next iteration. The procedure has converged when $\left|i_M^K\right|$ is less than the tolerance.



(a) (b)

Fig. 4 – Alternative Technique Solution.
(a) voltage source connection.
(b) correction Δv_D^K calculation

The extension of the proposed technique for determining the periodic steady-state response of nonlinear circuits is described in the next section.

IV. SOLUTION TECHNIQUE FOR DETERMINING THE STEADY-STATE RESPONSE

The proposed technique will be described with reference to Fig. 1. In this technique, a periodic voltage source $e_m(t)$ is connected across the thyristor module. Connection of this source separates the circuit into a nonlinear part comprising the thyristor module and a linear part comprising the source, the transmission line and the controlled-reactor without the thyristors.

The steady-state response of the linear part is obtained using the procedure described in the appendix and the steady-state currents $c_S(t)$, $c_T(t)$ are computed.

Analysis of the nonlinear part requires the firing instants for the thyristor valves. In this study, the firing pulses are synchronized either to the zero-crossings of the source voltage or to the zero-crossings of the line current. In the former case, the firing instants are obtained by adding the delay angle to the zero-crossings of the generator voltage. On the other hand, when the firing instants are synchronized to the line current, the zero-crossings of $c_S(t)$ are detected and the delay angle added to these instants. Irrespective of the firing strategy, the analysis of the linear part fixes the firing instants and enables the calculation of the thyristor-controlled reactor current $c_w(t)$ and the conductance $G_w(t)$ of the thyristor module.

The mismatch current flowing into the source $e_m(t)$ is given by

$$\Delta c_m(t) = c_T(t) - c_w(t) \tag{6}$$

The objective of the proposed technique is to determine the voltage source $e_m(t)$ for which $\left\|\Delta c_m(t)\right\|$ is negligibly small. This is achieved by adding at each iteration, a correction $\Delta e_m(t)$ to the periodic waveform $e_m(t)$.

The correction $\Delta e_m(t)$ is obtained from the small-signal equivalent circuit shown in Fig. 5 [15]. In this circuit, the generator voltage has been removed and the voltage source $e_m(t)$ has been replaced by the mismatch current with reversed polarity. The thyristor module has also been substituted by the time-varying conductance $G_w(t)$. The steady-state voltage developed across the mismatch current source is the correction $\Delta e_m(t)$ to be added to $e_m(t)$. The procedure is repeated until the mismatch current is below the specified tolerance.

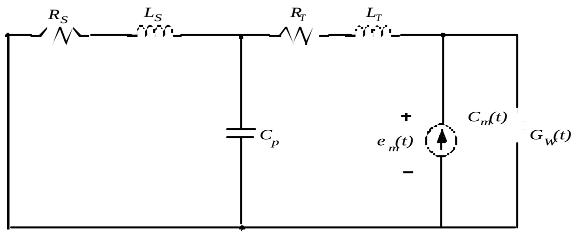


Fig. 5 – Small-signal equivalent circuit of single-phase Static VAR System

The technique described above may also be used for simulating the transient response. Closed-loop control may be included in the simulation by treating the control system as a nonlinear block. Simulation of power electronic circuits with closed-loop control using the proposed technique has been described in [16].

V. ILLUSTRATIVE EXAMPLES

The simulation results for the single-phase static SVC and the Kayenta TCSC are presentad in this section. A 333 MHz Pentium II microcomputer has been used in all the computations. The number of time-steps per fundamental period has been chosen to be 16384, which allows very accurate determination of the transient responses. The individual thyristor have benn modelled as two-state resistors, with Roff=50MΩ and Ron=1/Roff. More detailed models for the thyristors may be easily included in the simulations.

A. Single-phase static VAR system

The system has been previously studied using the Poincaré map [12]. Generator voltage synchronization has been used for triggering the thyristors. Fig. 6. shows the variation of the thyristor conduction angle for the steadystae solutions, as a function of the firing angle. Two separate sets of periodic solutions have been obtained corresponding to the two branches in Fig. 6.

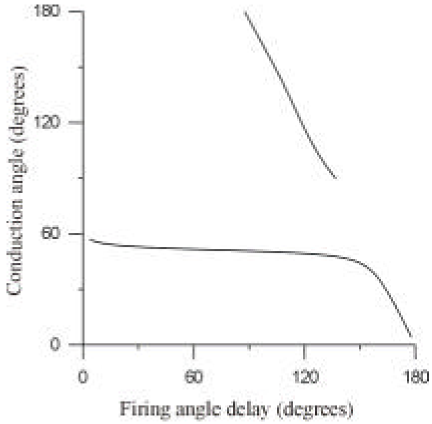


Fig. 6 – Conduction angle of single-phase Static VAR Compensator

The first branch starts with zero conduction-angle and it corresponds to the thyristors being totally blocked, i.e., a firing angle delay of 180°. The firing angle delay has been progressively reduced resulting in steady-state solutions with increasing conduction angles. The minimum firing angle delay is 3.86° and this corresponds to a conduction angle of 57.2°. When the firing angle delay is further reduced, the extinction of the thyristor is missed and a steady-state solution is not obtained. Fig. 7. shows the reactor current transient when the SVC is in the steady-state corresponding to the delay angle of 3.86° and the delay is suddenly reduced by 1°.

The second branch of steady-state solutions starts from full conduction, i. e., conduction angle of 180°, and the subsequent steady-state solutions are obtained by progressively increasing the firing delay. Fig. 8 shows the reactor current when the delay angle is 137° and the conduction angle is 89.8°. Any further increase in the delay angle leads to a new zero-crossing of the reactor current and a steady-state solution is no longer obtained.

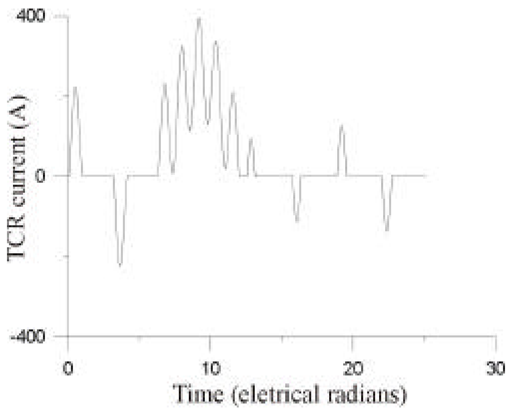


Fig. 7 – Reactor current transient

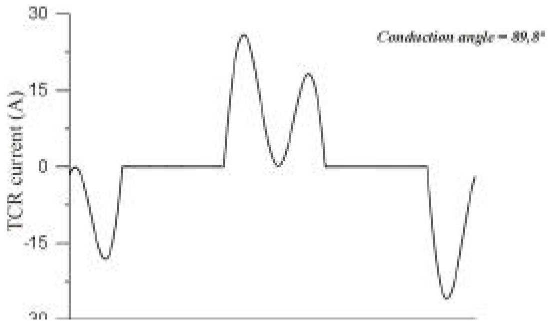


Fig. 8 – Steady-state reactor current

Fig. 9. show the eigenvalues of the Jacobian for each of the steady-state solutions and they are all seen to lie within the unit circle.

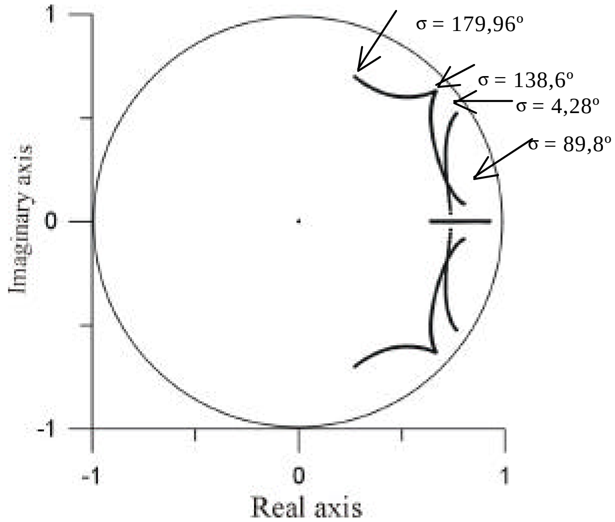


Fig. 9 – Eigenvalues of the Jacobian

B. The Kayenta Advanced Series Compensator (ASC) [12]

In the final design of the Kayenta ASC, the firing pulses have been synchronized to the zero-crossings of the line current.

Fig. 10 shows the conduction angle as a function of the firing-angle delay. Fig. 11 shows the ASC voltage and the reactor current for a conduction angle of 140^0.

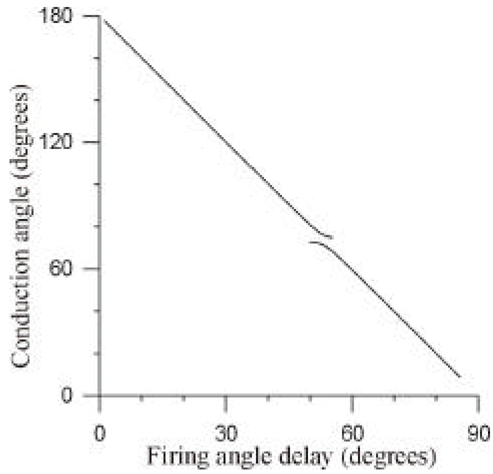


Fig. 10 – Conduction angle of Kayenta compensator

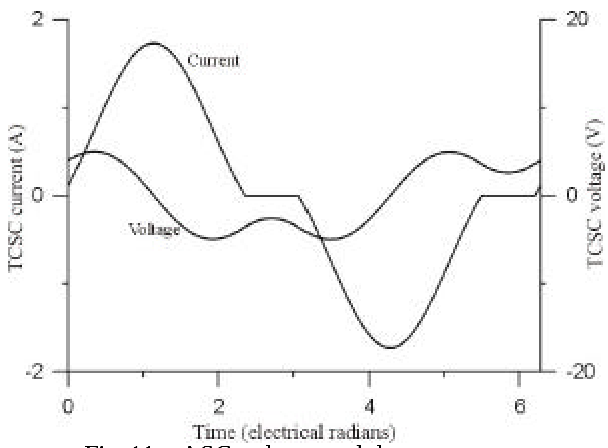


Fig. 11 – ASC voltage and the reactor current (σ=140º)

VI. CONCLUSIONS

A new technique has been proposed for simulating the responses of circuits with thyristor-switched reactors. The transient and periodic steady-state responses may be computed with this technique. The small-signal equivalent for the thyristor-controlled reactor circuit has been established and the stability of the steady-state responses is determined from this equivalent.

VI. APPENDIX

A1. Steady-state response of a linear circuit with periodic excitation

The time-domain technique for determining the periodic steady-state response of a linear circuit will be briefly described, since it forms the basis for the proposed technique for simulating the thyristor-switched reactor circuit.

Consider a linear circuit whose state is defined by a n-vector. Let the initial state vectors Qj, j=1,2,...,n, be defined by

$$Qj = [0,0,...,0,1,0,...,0]$$

where the non-zero element of Qj is in the j^{th} position. The zero-excitation response $Z_j(t)$ is computed for each of the initial state vectors Qj. Let Pj be the corresponding state of the circuit after one period at the fundamental frequency of the excitation. It should be noted that the transformation Qj->Pj is the Poincarê map. Finally, the zero-state response Zo(t) is computed and the state B after one period of excitation is determined.

Let J be the nxn matrix whose columns are Pj, j=1,2,...,n, and let S be the vector given by

$$(J\text{-}I).S = -B$$

where I is the identity matrix. It may be shown that the transient response of the circuit with the excitation applied and starting from the initial state S, is the steady-state response of the circuit. It is a linear combination of Zj(t) added to Zo(t).

The matrix J is the Jacobian of the Poincaré map and the eigenvalues of this matrix will define the stability of the steady-state response. This response is exponentially stable if all the eigenvalues lie within the unit circle in the complex plane.

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