

Real Time Simulation of Multiple Current Transformers connected to a Common Burden.

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Abstract— Differential protection schemes for busbars, generators or transformers connect multiple current transformers in parallel across a common burden. This paper describes the techniques used to simulate such an arrangement in real time.

Keywords. B-H loop, hysteresis, current transformers, simulation.

I. INTRODUCTION

Multiple current transformers feeding a common burden are found in differential protection schemes for busbars, machines and power transformers. In order to test the relays used in such schemes, it is necessary to reproduce the secondary level currents produced by the current transformers in the scheme. One method of producing these currents is by using a synthetic test plant [1] [2] to produce primary level currents into actual CT's but an alternative method which now presents itself is real time playback simulation or real time digital simulation [3] of the multiple CT currents. These simulations require only a few current amplifiers capable of reproducing the secondary level currents of the CT's in the scheme and are driven from the digital to analogue converter outputs of the simulator. In the limit, where only a spill current is required to drive the relay, a single amplifier will suffice. This is a much more compact and convenient way to test the relay but we must be sure that the simulators are capable of producing the same currents as appear on the synthetic test plant. By using a batch run feature considerable economies can be achieved in time taken to run comprehensive sets of tests.

This paper presents a mathematical formulation for multiple CTs feeding a common burden. The Jiles-Atherton theory [4] for ferromagnetic hysteresis is used as the basis for the current transformer model [5] as it can accurately represent the remanence flux in transformer cores. A case of three current transformers in a differential protection scheme is used to illustrate the formulation of the problem. The paper presents the results of an electro-magnetic transient simulation carried out using an RTDS running the proposed CT model, and compares the results with waveforms produced on a synthetic test plant. The mathematical model is presented in section III. and the results of simulations and

comparisons with test results are presented in subsequent sections for a 4 CT connection. A comparison is also made between the time taken to run the tests on the synthetic test plant and using the new method.

II. THE B-H LOOP

Detail of the parameters used in the simulation of the core are given in [5]. The resulting B-H loop used in the simulation is shown in figure 1

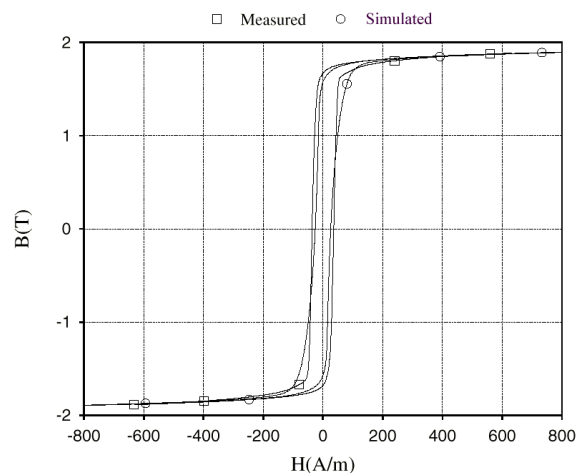


Figure 1 – The measured and simulated BH loop used in this study.

III. CURRENT TRANSFORMER MODEL

The problem of modelling the CT in an electromagnetic transient simulation program is to determine the changes in secondary currents when the changes in primary currents are given as inputs to the model. In order to develop the necessary equations, the 3 CT arrangement shown in figure 2 is considered in the following discussion. Each of the CT branches has an overcurrent element (shown as a small circle) of negligible impedance. The lowest branch will carry any spill current from the summation of the three CT secondary currents.

The following equations are obtained by applying Ampere's circuital law to the core of each CT ($i = 1, 2, 3$).

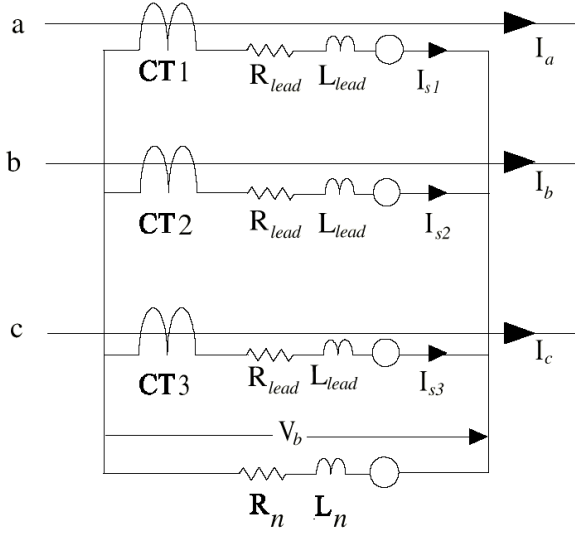


Figure 2 – Circuit diagram showing the 3CT connection.

$$\Delta H_i = \frac{N_{pi}}{l_i} \Delta I_{pi} - \frac{N_{si}}{l_i} \Delta I_{si} \quad (1)$$

N_p is the number of turns in the primary; N_s is the number of turns in the secondary; l is the length of the flux path, ΔH is the change in applied magnetic field intensity, ΔI_p is the change in primary current and ΔI_s is the change in secondary current. The subscripts 1, 2 and 3 are used to identify the three CTs. We are making the assumption of a mean path length and uniform flux density in the core.

Equation 2 is obtained by equating the rate of change of flux linkage to the sum of voltage drops in the secondary circuit (for $i = 1, 2, 3$) and using the Trapezoidal rule of integration.

$$\begin{aligned} \frac{N_{si} Area_i}{\Delta t} \Delta B_i &= R_{si} (I_{si}^{old} + \frac{\Delta I_{si}}{2}) \\ &+ \frac{L_{si}}{\Delta t} \Delta I_{si} + V_b^{old} + \frac{\Delta V_b}{2} \end{aligned} \quad (2)$$

$Area$ is the cross section area of the core of the CT, Δt is the integration time step, ΔB is the change in flux density in the core, R_s is the sum of secondary winding and lead resistances, L_s is the sum of secondary winding leakage and lead inductance, I_s^{old} is the secondary current during the previous time step, ΔV_b is the change in burden voltage (voltage that is common to all three CTs, see figure 2) and V_b^{old} is the burden voltage during the previous time step.

Note the common term $V_b^{old} + \frac{\Delta V_b}{2}$ in all three equations which therefore couples the flux densities in the three CTs.

Let s_1, s_2 and s_3 be the slopes of the $M-H$ characteristics for the three CT's [5]. Applying the relationship $\Delta B = \mu_0 (\Delta H + \Delta M)$ to the three current transformers ($i = 1, 2, 3$);

$$\Delta B_i = \mu_0 \Delta H_i + \mu_0 s_i \Delta H_i \quad (3)$$

The voltage across the common burden can be expressed as:

$$\begin{aligned} V_b^{old} + \frac{\Delta V_b}{2} &= R_n (I_{s1}^{old} + I_{s2}^{old} + I_{s3}^{old}) \\ &+ \frac{1}{2} (R_n + \frac{2L_n}{\Delta t}) (\Delta I_{s1} + \Delta I_{s2} + \Delta I_{s3}) \end{aligned} \quad (4)$$

The above equations can be manipulated to derive the matrix equation given in equation 5.

$$[G][\Delta I_s] = [K][\Delta I_p] + [J] \quad (5)$$

[G] is defined as:

$$\begin{aligned} G_{ii} &= \left(\frac{Z_{si} + Z_n}{2} \right) + \frac{N_{si}^2 Area_i \mu_0}{l_i \Delta t} (1 + s_i) \quad i = 1, 2, 3 \\ G_{ij} &= \frac{Z_n}{2} \quad i \neq j; i, j = 1, 2, 3 \end{aligned} \quad (6)$$

[K] is a diagonal matrix with the diagonal elements given by:

$$K(i, i) = \frac{N_{si} N_{pi} Area_i \mu_0 (1 + s_i)}{l_i \Delta t} \quad i = 1, 2, 3 \quad (7)$$

[J] is a vector that contains past values of I_s

$$\begin{aligned} J(i) &= -R_{si} I_{si}^{old} - R_n (I_{s1}^{old} + I_{s2}^{old} + I_{s3}^{old}) \\ i &= 1, 2, 3 \end{aligned} \quad (8)$$

$$Z_s = R_s + \frac{2L_s}{\Delta t} \quad (9)$$

$$Z_b = R_n + \frac{2L_n}{\Delta t} \quad (10)$$

Note that the diagonal elements of [G] and [K] are functions of s_1, s_2 and s_3 . Therefore the slopes of the $M-H$ curve (i.e. s_1, s_2 and s_3) must be known to solve equation 5 for ΔI_s for a given ΔI_p . The slope is a function M, H and δ , where δ is defined as: $\delta = +1$ when H is increasing and $\delta = -1$ otherwise. The slope at the beginning of a time step can be used to solve the matrix equation 5. If the integration time step is sufficiently small the solution obtained using the *slope* at the beginning of the time step would give sufficiently accurate solutions. In order to compute the slope, the value of δ at the beginning of the time step is used. At the end of the time step δ is updated depending on the value of ΔH . This is the method employed when running the algorithm in real time on a real time digital simulator with a time step of the order of $50\mu s$ [7].

IV. COMPARISON OF SIMULATION AND TEST RESULTS

The primary currents for injection into the model are derived from a simple system simulation which consists of a voltage source supplying a resistive load through an impedance (impedance of the power system). The impedance of the system can be adjusted to achieve a desired X/R ratio under fault conditions. This contrasts with the special techniques required in the synthetic test plant to achieve the desired an X/R ratio [1]. Faults can be applied to the system at predetermined points-on-wave (POW). Usually, the chosen POW corresponds approximately to the maximum dc off-set in the fault current. “Faults” are represented by shorting out the load in the phase or phases concerned leaving the voltage source driving only the system impedance in these phases. In the synthetic test plant special techniques are required to get a long enough time constant for the exponential component of the fault current

When a fault is applied, the waveform of the secondary current in a given CT depends on the flux in the core. If the initial (remanent) flux is high and the dc off-set is in such a direction as to further increase the flux then the CT will saturate much more quickly than had the initial flux been small.

In order to compare the simulation results with the experimental results the initial flux density in the core must be identical in the two cases. In practice it is not possible to measure the initial remanence. A simple way to perform the comparison is to apply a sequence of faults with maximum offset. This will drive the flux in the same direction during the faults and set a known remanent flux level at the end of the sequence.

Figure 3 shows the flux density and the secondary current of phase a CT when three consecutive faults (phase a to ground) were applied with an initial remanence of -1.0 T. It can be seen that even with initial remanence of the opposite polarity the first fault drives the CT into saturation in the positive flux density direction and thus for the second fault the initial flux density will be the same irrespective of the initial remanence for the first fault. Whether the first fault drives the core into saturation also depends on the burden of the CT and in the case presented above, the burden is enough to saturate the core during the first fault.

It is also important to control the point of interrupting the fault current, so that the second and subsequent faults are applied when the core is at a known value of flux density. The fault can be interrupted at a zero crossing of current either when the current is increasing or decreasing. These two instances correspond to the negative and positive peaks of the flux swing when the burden is principally resistive. For large burdens the difference in flux density corresponding to *rising* and *falling* zero-crossings is significant as the flux swing is large. Therefore for the purpose of comparison of simulation and test results it is important to record whether the fault was interrupted at a *falling* zero-crossing or a *rising* zero-crossing. The same conditions must then

be applied in the simulation to produce the necessary waveforms for comparison. For example, in figure 3, the first fault has been cleared at a *rising* current-zero whereas the subsequent faults have been cleared at *falling* current-zeros. The first peak of the current for the second fault is therefore larger than that for the third fault.

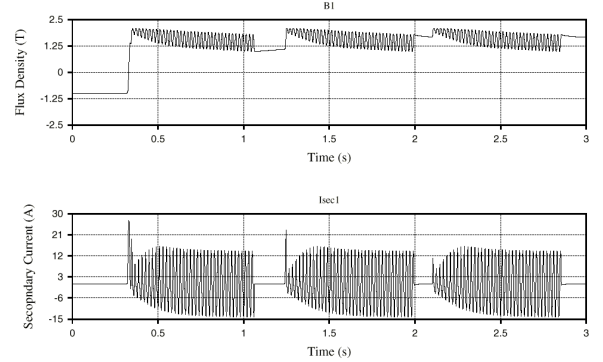


Figure 3 – Flux density and secondary current for 3 consecutive faults with initial remanent flux of -1 T..

Tests were performed on the synthetic test plant for different values of lead impedances and for line-to-ground faults and line-to-line faults. In the synthetic test plant the nominal settings and the actual values of the resistances were not identical. Therefore these values were measured and the correct values were used in the simulation. The inductances of the leads were also measured and the measured values were used in the simulation. These measured values are given in the Appendix. Only the currents in the phase or phases involved in the fault are shown in the figures. The currents in the other phases are extremely small and below the measurement range of the recording instruments.

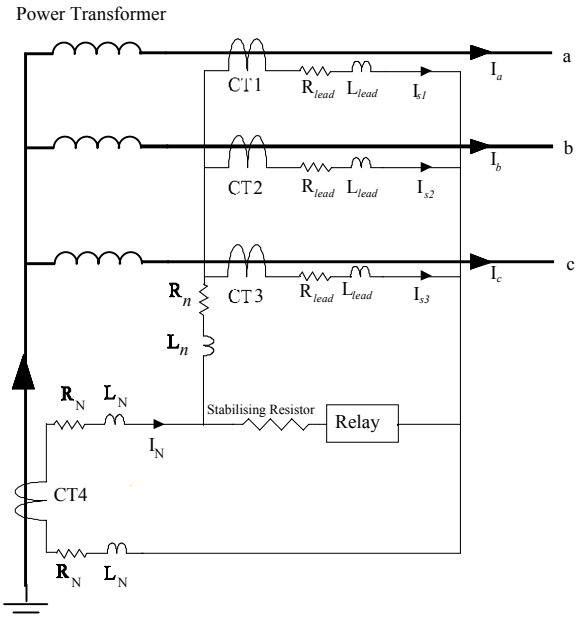


Figure 4 - A 4CT connection used in a restricted earth fault protection for a power transformer.

A. Tests on a 4CT configuration

Equations similar to those in equations 2 and 4 were constructed for the 4CT configuration shown in figure 4. Two through fault cases were applied as follows

Case1: Phase to ground on phase *a* and

Case2: phase to phase

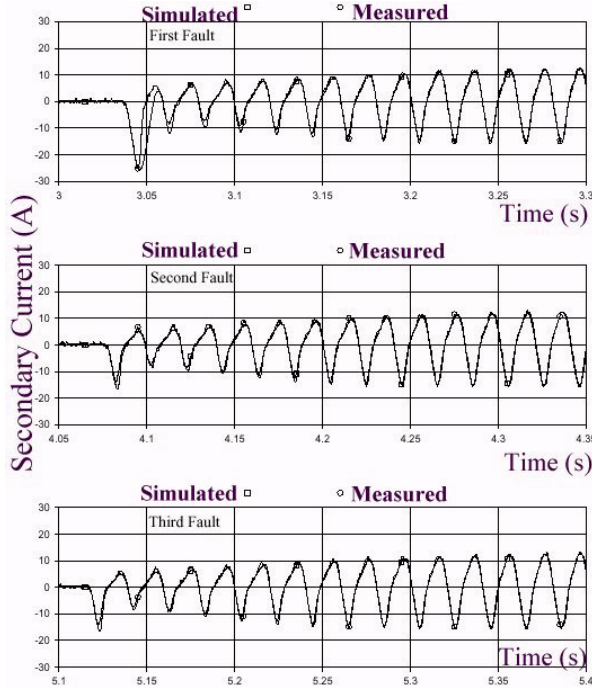


Figure 5 - Comparison of the measured and simulated phase *a* secondary currents for a through line to ground fault.

In both cases the line CT leads on the test plant were on the 0Ω nominal setting and 4Ω nominal setting in the neutral CT leads. The accurate values are given in the appendix. The relay impedance is negligible.

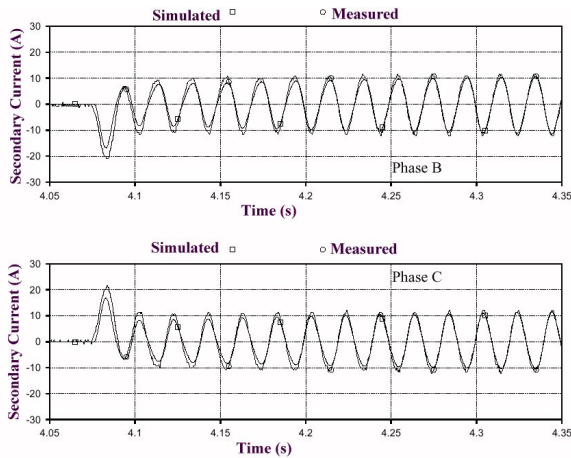


Figure 6 - Comparison of measured and simulated secondary current waveforms for a *bc* fault.

Comparisons of the currents in the faulted phases are shown in figures 5 and 6. In both cases the CT's are

preconditioned by 3 successive *abc* fault shots with full offset in phase *a*. While this will drive all line CT's into a known condition it leaves the neutral CT in an unknown condition. This explains the slight disagreement in the first fault shot waveforms of **Case1** in figure 5. The second and third shots are better because the neutral CT has been preconditioned by the previous fault shot. For **Case2** figure 6 shows the phase *b* and *c* currents for the second fault shot. In all cases there is good agreement between the recorded and simulated waveforms when the plant CT's and the simulated CT's start from the same initial condition.

V. THE NEW TEST ENVIRONMENT.

The new real time digital simulator and conditioning amplifiers are shown in Figure 7. This is a much more compact arrangement than the old analogue test transmission line and the synthetic test plant. One of the major features of the new arrangement is the ability to automate a series of tests in which parameters such as the lead resistance, the point on wave of fault incidence, the primary system X/R ratio and many others parameters are automatically changed from one case to the next. The various cases can be altered, compiled, loaded and executed using a script file written in a "C" like language. Once the script file has been written the process does not require the engineer to be available during the runs. This contrasts with the old test system which required the test engineers to change the settings, parameters etc., run the test and process the results. As with any batch type testing care must be taken not to exceed the thermal rating of the relay under test. At low current levels ($<5A$) 200 tests can be executed in 33 minutes. At high current levels ($>20A$) cooling time must be programmed into the test script thereby lengthening the time for 200 tests to 3.25 hours. A test script which covers a wide spectrum of secondary lead burdens and other parameter variations for one fault type performs typically 1400 tests in around 12.5 hours. If we run 3 fault type script files in succession (A-N, B-C, A-B-C) then we can perform 4,200 tests in 37.5 hours. Parallel testing with 5 test stations gives a maximum capability of 21,000 tests in 37.5 hours. This is a significant increase in testing efficiency compared to the synthetic test plant procedure.

VI. CONCLUSIONS

A multi-CT application simulation algorithm based on the Jiles/Atherton theory of ferromagnetic magnetization has been shown to give accurate results under fault conditions involving residual flux in the CT core or cores. The particular cases highlighted in this paper involve 3 or 4 CT's connected in parallel to a common burden but the technique is suitable for extension to schemes with larger numbers of CT's.

VII. APPENDIX

Magnetic material data: (all CTs are identical)

$$\begin{aligned} M_s &= 1.72e6 & \alpha &= 1.32e-5 & k &= 5.0e-5 \\ a_1 &= 2730 & a_2 &= 3209 & a_3 &= 20294 \\ c &= 0.1 \end{aligned}$$

Other data:

$$\begin{aligned} N_p &= 200 & N_s &= 20 & Area &= 2.26e-3 \\ l &= 0.638 & R_{sec} &= 0.5\Omega & L_{sec} &= 0.8mH \end{aligned}$$

Lead resistances and inductances of the test plant for 0 Ω setting for **Case1** and **Case2**.

$$\begin{aligned} R_a &= 0.967\Omega & R_b &= 0.861\Omega & R_c &= 1.163\Omega \\ L_a &= 846.5\mu H & L_b &= 845.7\mu H & L_c &= 846.2\mu H \\ R_n &= 0.305\Omega & L_n &= 23.73\mu H & R_{stab} &= 178.8\Omega \\ R_N &= 2.341\Omega & L_N &= 88.08\mu H & L_{stab} &= 300\mu H \end{aligned}$$

- [4] D.C. Jiles, D.L. Atherton, "Theory of Ferromagnetic Hysteresis.", *Journal of Magnetism and Magnetic Materials*, vol. 61, p48, 1986.
- [5] U.D. Annakkage, P.G. McLaren, E. Dirks, R.P. Jayasinghe, A.D. Parker, "A current transformer model based on the Jiles/Atherton theory of ferromagnetic hysteresis." *IEEE Transactions on Power Delivery*, January 2000, Vol. 15, No. 1, p57
- [6] D.C. Jiles, J.B. Thoeke and M. Devine, "Numerical determination of hysteresis parameters using the theory of ferromagnetic hysteresis.", *IEEE Transactions on Magnetics*, Vol. 28, 1992, pp. 27-35



Figure 7 - The new simulator set up with the RTDS's and amplifiers on the right with the controlling workstations on the left.

REFERENCES

- [1] GEC Protective Relays Application Guide.
- [2] P.A. Crossley, A.D. Parker, "Design and Evaluation of a Circulating Current Differential Relay Test System.", *IEEE Transactions on Power Delivery*, Vol. 13, No. 2, April 1998, p427.
- [3] P.G. McLaren, R.P. Jayasinghe, "Simulation of CT's operating in parallel into a differential current protection scheme." *IEEE Wescanex 97*, Winnipeg, June 1997

- [7] R. Kuffel, J. Giesbrecht, T. Maguire, R.P. Wierckx, P. McLaren, "RTDS-A fully digital power system simulator operating in real time.", *Proceedings of the CEA Operating Division Conference*, Montreal, March 1996.

VIII. ACKNOWLEDGMENTS

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