

nous currents viewed from the armature terminals is negative. When this negative resistance exceeds the sum of armature and network resistances, the electrical system is self-excited. Such self excitation would be expected to result in excessive voltages and currents.

However, the above classification is somewhat ambiguous: For example, [3] explains IGE as a purely electrical phenomenon while it can bring about large oscillatory shaft torques like TI. [4] emphasizes that “induction generator effect and torsional interaction are not mutually exclusive and will co-exist”. Is there a way to ideally separate the two phenomena? The answer to this question depends very much on the definitions agreed upon, but as will be discussed below, it is possible to differentiate between certain phenomena in the process, by some assumptions. In order to understand the implications associated with each set of assumptions, we need to leave out all unnecessary details. Much of the complexity comes from the “coupling” between the mechanical and electrical subsystems which occurs in the air gap of the machine. So we begin to simplify the machine model up to the point where SSR can still be captured. The machine model provided for the IEEE Second Benchmark Model [5] is used for this purpose. This is shown in sections II, III and IV. Then, in section V the simplified system is used to study subsynchronous resonance. Section VI concludes the paper.

II. STUDY SYSTEM

Fig. 1 shows the system we want to simplify. The synchronous machine is represented by seven windings as is normally done in turbo-generator transient studies. The mechanical system consists of two rotating masses (not shown in the figure). \mathbf{q} is defined as the angle between the axis of phase ‘a’ of the stator and the d axis of the rotor. There is no transformer, implying that the nominal voltage of the machine is the same as that of the network. The following assumptions are made for the machine:

- The windings are sinusoidally distributed, so each current-carrying winding produces a sinusoidal magnetic field in the air gap.
- Permeability of the core is infinite, so the magnetic circuit is linear (no saturation) and all the magnetic field is concentrated in the air gap.
- Saliency in the machine is ignored so the magnetic characteristics along the d and q axes are the same.
- Magnetic flux leakage is ignored. This means that all the flux produced by a winding passes through the core.

With these assumptions it is possible to completely describe the self and mutual inductances of the machine in terms of its dimensions[6]:

$$L_{mW_i} = (N_{W_i}/2)^2 \mu_0 r l g^{-1} \quad (1)$$

$$L_{W_i W_j} = (N_{W_i}/2)(N_{W_j}/2) \mu_0 r l g^{-1} \cos(\angle W_i W_j) \quad (2)$$

where L_{mW_i} is the self inductance of winding i and $L_{W_i W_j}$ is the mutual inductance between windings i and j . Also:

N_{W_i} = Number of turns of winding i

$\mu_0 = 4 \pi 10^{-7} \text{ Hm}^{-1}$

r = Radius of the rotor in m

l = Axial length of the generator winding in m

g = Air gap length in m

$\angle W_i W_j$ = Angle between the axes of windings i and j

In this machine, only the mutual inductances between the stator and rotor are time-varying. In the next section, numerical values for the simplified machine and other parameters of the system are derived from the IEEE Second SSR Benchmark Model.

III. NUMERICAL VALUES OF SYSTEM PARAMETERS

MicroTran[®] (UBC version of EMTP) accepts conventional synchronous machine data and produces in the output file the list of machine parameters in SI units using the most accurate data conversion whenever possible. If this conversion fails then the program automatically uses approximate conversion recommended in [5]. For the Second Benchmark Model the following values are calculated[7] by the program:

X_d	$= 1.331000$	Ω
X_{sfd}	$= 3.666667 \cdot 10^1$	Ω
X_{sD}	$= 3.666667 \cdot 10^1$	Ω
X_{fdD}	$= 1.103753 \cdot 10^3$	Ω
X_{mfd}	$= 1.190476 \cdot 10^3$	Ω
X_{mD}	$= 1.200240 \cdot 10^3$	Ω
R_{fd}	$= 7.017414 \cdot 10^{-1}$	Ω
R_D	$= 1.173057 \cdot 10^1$	Ω
X_q	$= 1.282600$	Ω
X_{sg}	$= 3.520971 \cdot 10^1$	Ω
X_{sQ}	$= 3.520971 \cdot 10^1$	Ω
X_{gQ}	$= 1.059895 \cdot 10^3$	Ω
X_{mg}	$= 1.360042 \cdot 10^3$	Ω
X_{mQ}	$= 1.113874 \cdot 10^3$	Ω
R_g	$= 6.559317$	Ω
R_Q	$= 8.484922$	Ω
f	$= 60$	Hz

where X_d and X_q are the d and q axis equivalent impedances of the stator windings, respectively. For our machine model, we have ignored saliency and leakage altogether. An immediate conclusion is $X_d = X_q = (3/2) X_{ms}$, where X_{ms}

is the self inductance of a stator winding. Also, it is apparent from the above data that the corresponding reactances of the assumed rotor windings on the two axes are not much different from each other. Hence further simplification can be achieved by assuming the same number of turns for all the rotor windings to make the self inductances and the mutual inductances equal. The following set of data is suggested for the reduced model:

$$\begin{aligned} L_{ms} &= 2.3537 \cdot 10^{-3} \quad H \\ L_{mfd} &= L_{mD} = L_{mg} = L_{mQ} = 3.226 \quad H \\ R_{fd} &= 0.702 \quad \Omega \\ R_D &= 11.731 \quad \Omega \\ R_g &= 6.560 \quad \Omega \\ R_Q &= 8.485 \quad \Omega \end{aligned}$$

The mutual inductance values are now uniquely defined by (2):

$$\begin{aligned} L_{sfd} &= L_{sD} = L_{sg} = L_{sQ} = 87.138 \cdot 10^{-3} \quad H \\ L_{fdD} &= L_{gQ} = 3.226 \quad H \end{aligned}$$

It is understood that for the mutual inductance between one stator and one rotor winding, cosine of the angle between the winding axes has to be considered. For instance the mutual inductance between phase 'b' winding of the stator and the 'Q' winding would be

$$L_{sQ} \cos\left(\mathbf{q} + \frac{\mathbf{p}}{2} - \frac{2\mathbf{p}}{3}\right).$$

Fig. 2 shows the d and q axis equivalent circuits with per unit values (indicated by lowercase letters) in the L_{ad} - base reciprocal per unit system[8].

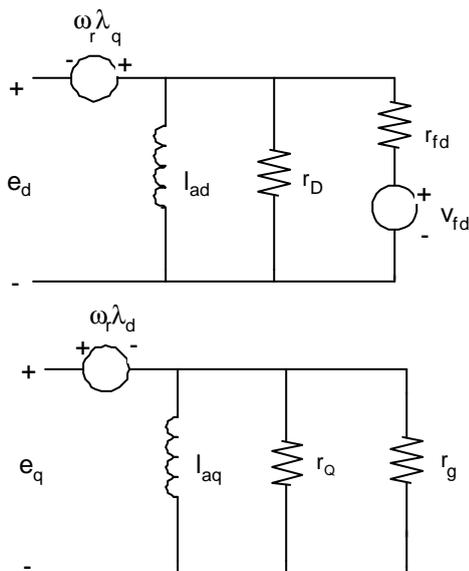


Fig. 2: d and q axis equivalent circuits of the simplified machine

Based on the machine rated parameters (600 MVA, 22 kV) these values are:

$$\begin{aligned} l_{ad} &= l_d = 1.65 \quad p.u. \\ l_{aq} &= l_q = 1.65 \quad p.u. \\ r_{fd} &= 9.5 \cdot 10^{-4} \quad p.u. \\ r_D &= 1.6 \cdot 10^{-2} \quad p.u. \\ r_g &= 1.15 \cdot 10^{-2} \quad p.u. \\ r_Q &= 8.9 \cdot 10^{-3} \quad p.u. \end{aligned}$$

For the shaft model, data of the LP turbine and generator rotor is used without modification:

$$\begin{aligned} K &= 132.8295 \cdot 10^6 \quad Nm/rad \\ J_{GEN} &= 7425.254 \quad kgm^2 \\ J_{2ndM} &= 13094.151 \quad kgm^2 \end{aligned}$$

The mechanical natural frequency is thus:

$$f_{mech} = \frac{1}{2p} \sqrt{\frac{K(J_{GEN} + J_{2ndM})}{J_{GEN}J_{2ndM}}} = 26.65 \quad Hz \quad (3)$$

To study SSR, the numerical values of the L and C in the external circuitry are chosen to give a natural electrical frequency to complement f_{mech} with respect to 60 Hz:

$$\begin{aligned} L &= 0.865 \quad H \\ C &= 26.22 \quad \mu F \\ R_s &= 0.0 \quad \Omega \end{aligned}$$

The simplified model was next validated with the transients simulation program NETOMAC[®] by Siemens. Fig. 3 shows the torque developed on the shaft between the two masses for 0.5 seconds. The development of a growing oscillatory shaft torque is evident. Phase 'a' capacitor voltage is also shown in Fig. 3 for comparison reasons.

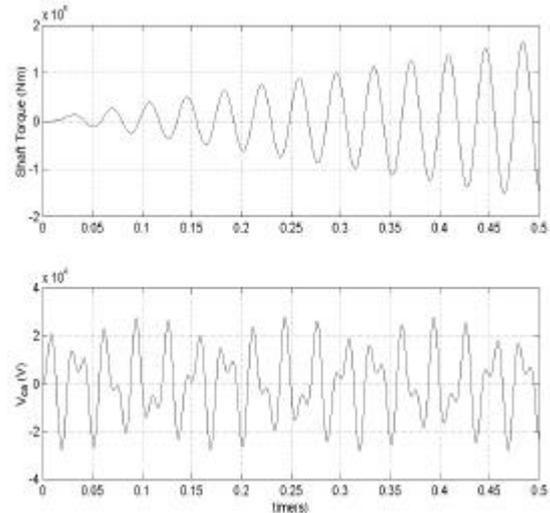


Fig. 3: NETOMAC model, 4 windings on the rotor

IV. FURTHER SIMPLIFICATION

The differential equations of the above system can be easily described either in phase quantities or in Park's dq0 frame. They must be solved numerically however. It is already known that transient simulation is a stiff problem[8] meaning that the probability of failure of explicit integration methods increases with modeling detail. This was specifically observed in an attempt to use MATLAB® ordinary differential equation solvers which use Runge Kutta methods, to solve system equations with 4 rotor windings. While this problem can best be skipped by resorting to implicit integration, here in line with our simplification, we can proceed by assuming only one winding on the d axis or two windings on the d and the q axes. Simulation results in Fig. 4 show that the SSR phenomena can still be captured and there is good match between the results of NETOMAC with 4 windings and those obtained by MATLAB ODE solvers using only 1 winding on the rotor.

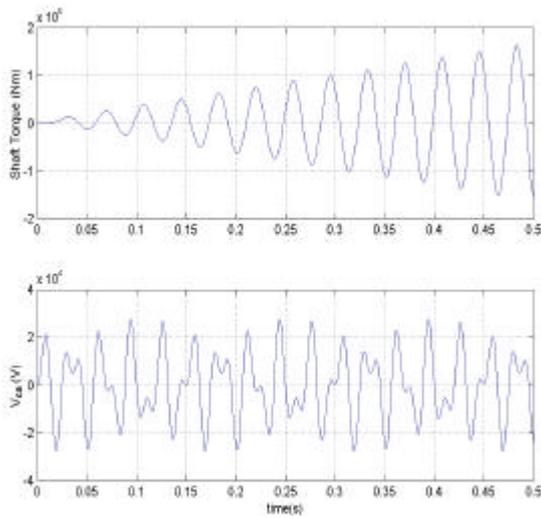


Fig. 4: MATLAB model with only one winding on the rotor

The last simplification step omits three elements from the d and the q axis equivalent circuits, and yields the model in Fig. 5.

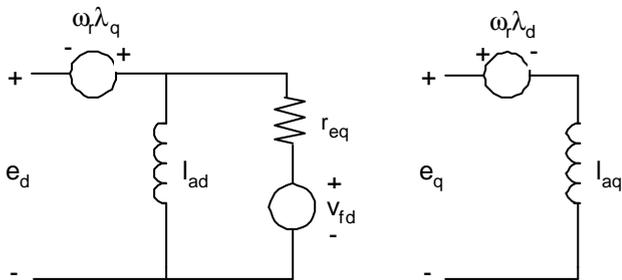


Fig. 5: Equivalent circuits of the final model

In the following the value of r_{eq} is varied according to whether TI or IGE is studied.

V. SSR STUDY WITH THE DEVELOPED MODEL

Back to the question posed in the introduction section, to separate torsional interaction from induction generator effect we do as follows: If we assume a constant field current then the rotor is basically a permanent magnet. The voltage induced in the rotor winding does not influence the field current so induction in the rotor does not play a role in the torque developed in the machine. This becomes clearer by examining the electromagnetic torque equation which is a function of the rotor and stator currents and the rotor angle but not of any voltages explicitly:

$$T_{el}(t) = -L_{sfd} i_{fd} \times [i_{as} \sin \mathbf{q} + i_{bs} \sin(\mathbf{q} - 2\mathbf{p}/3) + i_{cs} \sin(\mathbf{q} + 2\mathbf{p}/3)] \quad (4)$$

The induction effect on the stator currents is still present however and affects the torque. But this induction effect is not *induction generator effect*. For induction generation we first need to have an induction effect on the rotor circuits to be reflected later on the stator side. With the assumption of constant field current the induction effect on the rotor is not reflected. So we can claim to have suppressed the induction generator effect totally.

Fig. 6 shows the shaft torque for the case of constant current source on the rotor ($I_{fd} = 546.8$ A, this value of field current produces nominal voltage at the terminals of the machine at no load conditions). Since $I_{fd} = \text{const.}$, r_{eq} is of no concern. There is no independent mechanical source in the system (turbine power is zero) and both masses are initially rotating at synchronous speed. The electrical subsystem has zero initial conditions. Three voltage sources are mounted on the stator side. An FFT analysis of the electromagnetic torque reveals the existence of a torque component with the natural frequency of the mechanical system.

To obtain only the induction generator effect and suppress torsional interaction, the simulation should start with zero rotor currents without any independent source on the rotor. To force an interaction between the stator and rotor circuits we need nonzero initial conditions and/or sources in the stator side. This way, if the currents flowing in the stator windings produce a magnetic field in the air gap which moves slower than the rotor, then the effect of the currents induced in the rotor circuits is to turn the machine into an induction generator. Now it becomes more a matter of definition whether to assume constant rotor speed, in which case the effect will be completely electrical, or to allow for rotor speed deviations which will then incorporate mechanical effects as well. Here we assume the latter and consider the effect on the shaft torque.

Fig. 7 shows the shaft torque for the case of a rotor winding with 11.7Ω resistance ($r_{eq} = r_D$ to imitate damper winding), together with the FFT of the electromagnetic torque developed in the machine. The masses are rotating at synchronous speed. Like the TI case there is no mechanical source in the system. The initial electrical conditions are all zero including the rotor currents. Three voltage sources are mounted on the stator side. The torque

still grows, but it is small compared to the TI case. Note that the scales of Fig. 6 and 7 are different.

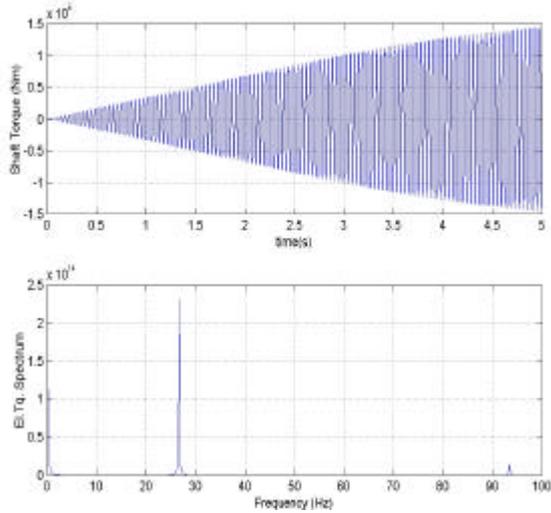


Fig. 6: Growing of shaft torque with TI

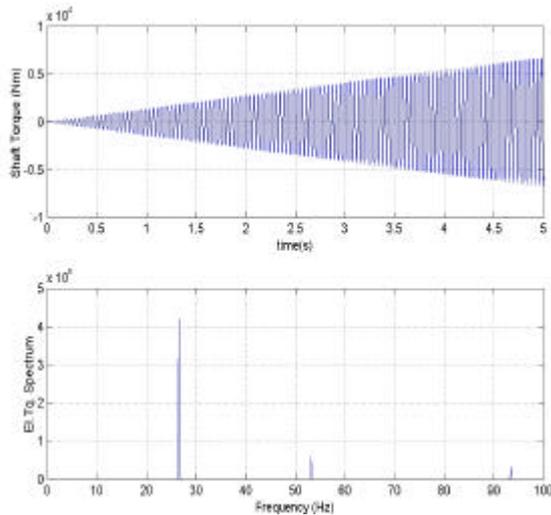


Fig. 7: Growing of shaft torque with IGE

It is helpful to look at the two SSR types within the classification which results from different sets of assumptions in our simplified model:

a) *Constant current field winding*: Only torsional interaction is present as a result of which currents and voltages at subsynchronous frequency in the stator and the shaft torque grow.

b) *Damper winding on the rotor with zero initial current*: Only induction generator effect is present as a result of which currents and voltages at subsynchronous frequency in the stator, current in the rotor and the shaft torque grow.

c) *Constant current field winding, constant synchronous speed*: Interaction at subsynchronous frequency between the stator and rotor circuits stops. If there is no resistance in the stator, the subsynchronous currents and voltages resulting from initial conditions continue to exist without growing. Also the electromagnetic torque component on

the generator rotor is present, but since with our constant speed assumption we have actually placed an independent torque source on the generator rotor which completely compensates this torque, the mechanical oscillations do not grow.

d) *Damper winding with zero initial current, constant synchronous speed*: Interaction at synchronous frequency between the stator and rotor circuits stops. The electrical induction effect on the stator side results in currents and voltages at subsynchronous frequency to grow.

VI. CONCLUSION

In this paper we have shown that SSR can be studied with very much simplified models without loss of generality.

The main concern in SSR studies is the development of an oscillatory torque on the generator rotor which can cause excessively large torsional stress between the rotating masses. The results presented in the previous section demonstrate that either TI or IGE can produce growing torsional oscillations, although the severity of torsional interaction for a typical synchronous generator is higher. The growing torque development in both cases can be explained by noting that the induction effect is similar on the stator side (helping to sustain the electrical oscillator at the subsynchronous frequency).

A word is in order about the accuracy of the simplified model. Fig. 8 shows the shaft torque developed in three different models: the IEEE second benchmark model, the model of Fig. 2 and the model of Fig. 5 with $r_{eq} = r_{fd}$. For a meaningful comparison, the mechanical data of the LP turbine and the generator are used. Therefore, only the electrical data of the generators are different in the three cases. As shown, our simplifications have reduced the magnitude of the shaft torque, however for the chosen level of series compensation (31% of the line reactance), SSR will occur nevertheless.

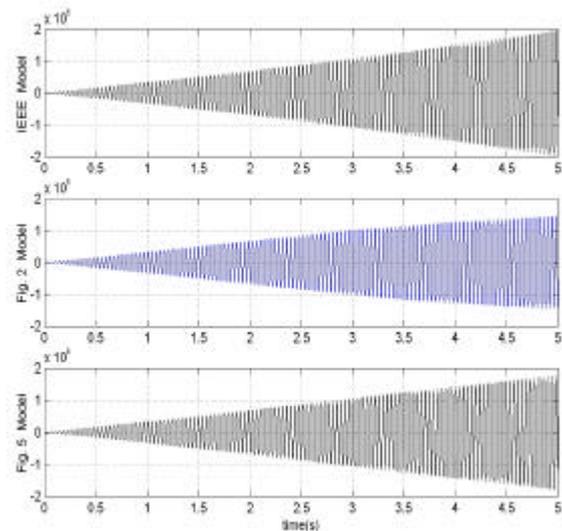


Fig. 8: Comparison of the shaft torques

While for a practical design study, detailed models are necessary, the simplicity of our model allows us to isolate

and demonstrate the effects of TI and IGE. With this model, the authors hope to make a contribution to a better understanding of the physical phenomena inherent to SSR.

SSR can be viewed as a “coupled oscillator” problem, where an electrical and a mechanical oscillator interact to bring about large shaft torques. The nonlinearity of the coupling, is the reason why energy of oscillations can grow once they have started. The discussion of energy flow in the simplified system is an interesting issue which will appear in another publication.

VII. REFERENCES

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