NETWORK EQUIVALENT FOR THE ANALYSIS OF ELECTROMAGNETIC TRANSIENTS IN POWER SYSTEMS

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***Abstract* – It is characteristic for the frequency domain input admittance of a large network to have many, almost regularly distanced, peaks due primarily to the presence of the leading transmission lines. Because of this the traditional modeling of an equivalent using simple lumped parameter elements for each peak is often inefficient.**

The paper describes a novel approach for obtaining a low-order equivalent of a portion of a system (designated as external) in the calculation of electromagnetic transients, if it has transmission lines at the interface nodes. In our terminology, these lines form the surface-region of the external system. The rest constitutes the deep-region. The surface-region is represented by simplified (low-order) distributed-parameter lines. Its function is to match, as closely as possible, the spectrum of the external system in the desired frequency range. The function of the deep-region is to compensate for the difference between the spectrum of the external system and that of the surface region at low frequencies and to ensure the desired accuracy for the resultant equivalent. The equivalent can be either single-port or multi-port.

The paper describes the basic concepts of the proposed approach, highlights its merits, and provides application examples to demonstrate its computational efficiency and accuracy.

Keywords: Network Equivalents, Electromagnetic Transients, Digital Time-Domain Simulation.

I Introduction

The analysis of electromagnetic transients phenomena in power systems is traditionally based on digital time-domain simulation. The system is artificially divided into two sections: (i) study zone and (ii) external network. The study zone is the section with the source of the transient and which is most significantly affected by it. Therefore, the study zone, depending on the nature and characteristics of the phenomenon, must be represented by detailed component models, including frequency-dependency of the parameters and non-linearities. The external network has secondary impact on the phenomenon and can be approximated by linear component models, including however frequency-dependency of the parameters in the required band-width. The study zone usually contains a relatively small section of an interconnected system and the external network encompasses the remaining vast portion of it.

The linearity and size of the external network are the motivation for replacing it by an equivalent which reproduces with good accuracy the external network's frequency response in the required frequency band-width [1], [2]. An important characteristic of this frequency response is the presence of a number of peaks placed with certain regularity corresponding to resonances mainly on the leading transmission lines. Their modeling has required in existing programs the use of R,L,C,G elements for each peak which reduces the efficiency of the program especially if more than a low frequency range is of interest. Clearly, a remedy for this problem is to include transmission lines in the model of the equivalent.

This paper gives a solution to the above mentioned difficulty. It presents a new approach to represent an external network by using a hybrid equivalent. It is a "hybrid" equivalent since the mathematical model is composed of (i) approximated distributed-parameter line equations and (ii) rational transfer functions which represent lumped-parameter electrical components. The line equations approximate the external network over the desired frequency range and the rational functions compensate for the discrepancy between the approximated line models and the "exact" external network frequency-response characteristic. The main feature of the proposed hybrid equivalent, when compared with a conventional equivalent [1], [2], are (i) simplicity, i.e., low-order mathematical model, (ii) capability for wider frequency band representation, and (iii) computational efficiency.

II Concept of Hybrid Equivalent

An external electric network can be looked at as consisting of two regions: the first is the outer part or surface region, the second is the inner part or the deep region. The impact of the outer part and the inner part on the input admittance varies with frequency. At low frequencies, both inner and outer parts contribute to the admittance. However, at higher frequencies the role of the inner part diminishes drastically due to strong attenuation. This means that the inner part can be approximated by a smooth rational function with mainly real poles that affect the input admittance primarily at the lower frequency range.

The above considerations lead to the expectation that the system can be simply and accurately approximated by two regions that are topologically analogous to its outer and inner parts. The first is a set of simplified transmission line representations that cover the whole frequency spectrum. The second is a rational transfer function to compensate for the discrepancy between the exact transmission line models and their simplified models used in the lower frequency range.

Based on the previous discussion, the proposed approach is to substitute an exact representation of an external network, seen from its terminals, by (i) a set of simplified distributed-parameter transmission line models and (ii) a set of fictitious lumped-element network models representing the difference between the actual frequency response of the external network and the simplified line models. The deep-network is represented by a low-order rational transfer function with a relatively smooth frequency response in a wide frequency range. The simplified line models represent the external network in the whole frequency range of interest. The simplified line models can represent the system over a wider frequency range than lumped parameter equivalents adopted in conventional approaches. The lumped-element network compensates for the difference between the exact frequency response of the system and the one that the simplified line models provide. This compensation is primarily needed at the low-frequency range.

To demonstrate the principles of the proposed approach, let us consider the system of Fig. 1 (System-1). The external network consists of a single-phase transmission line terminated by an RL load. The line is 210 km long and the ground resistivity is 200 Ωm. The RL load is 500 + jω0.002 ohms. The complex plane method is used to account for the impedance of the ground return path [3]. The objective is to develop a hybrid equivalent of the external network observed from bus A. The transmission line terminal voltages and currents are related by

$$\begin{pmatrix} Y_A & Y_{AB} \\ Y_{BA} & Y_B \end{pmatrix}\begin{pmatrix} V_A \\ V_B \end{pmatrix}=\begin{pmatrix} I_A \\ -I_B \end{pmatrix} \tag{1}$$

where V_A, V_B, I_A, I_B are voltages and currents as shown in Fig. 1, and Y_A, Y_B, Y_{AB}, Y_{BA} are the elements of the transmission line nodal admittance matrix.

We define Y_{input} and Y_{load} ($=Z^{-1}$; see Fig. 1) by

$$Y_{load}V_B = I_B, \quad Y_{input}V_A = I_A \tag{2}$$

Substitution for I_A and I_B from (2) in (3) leads to

$$\begin{pmatrix} Y_A - Y_{input} & Y_{AB} \\ Y_{BA} & Y_B + Y_{load} \end{pmatrix}\begin{pmatrix} V_A \\ V_B \end{pmatrix}=\begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{3}$$

Thus solving for Y_{load} yields

$$Y_{load} = Y_{BA}\left(Y_A - Y_{input}\right)^{-1} Y_{AB} - Y_B \tag{4}$$

Equation (4) shows that if instead of the elements of the nodal admittance matrix of the line their simplified approximations are used, then Y_{load} must deviate from the original function Z^{-1} to maintain the original frequency response characteristic of the system with respect to bus A. Fig. 2(a) shows Y_{input} of System-1 corresponding to two representations of the line. One characteristic corresponds to the exact line representation. The other corresponds to the case where (i) the line characteristic impedance is approximated as a constant value over the whole frequency range and (ii) the line propagation function is represented by a second-order rational function. The RL load is kept the same for both plots of Fig. 2(a). Fig. 2(b) shows the difference (absolute values) between the corresponding values of the input admittances plotted in Fig. 2(a). Fig. 2(b) illustrates that the difference between the two input admittances is more noticeable at low frequencies than at high frequencies. Therefore, the deep-region approximation must primarily compensate for the low frequency discrepancy. Higher impact of the deep-region approximation at lower frequency range can be predicted based on the nature of wave propagation in transmission lines. Due to the high attenuation of a line at high frequencies, the range of propagation of high frequency waves is limited. This leads to the idea that the deep-region approximation should be approximated by a simple rational transfer function corresponding to the low-frequency range. Thus, the approach for the proposed hybrid equivalent can be summarized as follows.

- The external network is partitioned into two regions, a surface-region and a deep-region.
- The surface region is approximated by a set of low-order line models corresponding to the transmission lines that exist in reality in the surface region.
- The next step is to find a lumped-element approximation for the deep-region. One way to go, is to use the approximation of the surface-region in conjunction with the input admittance to find the deep-region input admittance; then use Vector Fitting [4] to find the rational-function approximation desired for the deep region. We investigate this method and describe the difficulties associated with it. Another approach is to have a set of measurements for the deep region alone and directly find a rational function approximation for the deep-region.

The development of the above steps is described in depth in the following sections.

A) Surface Layer and Deep Layer Convention

As stated earlier, the first step is to divide the system into two parts or layers; see Fig. 3. The first one is approximated by a set of modified transmission lines, the second as a lumped parameter network. The surface layer can extend to more than one layer of buses within the network. It may even contain some lumped parameter elements, like capacitors or inductors, if they existed in the original network.

The decision of how to divide the system is largely left to experience. However, some basic guidelines should be obeyed in order to achieve the simplest possible equivalent. Those guidelines can be stated as:

- The topology of the surface layer must be respected.
- We depend on attenuation in the transmission lines to use a simple approximation. Therefore it is the damping produced by the surface layer elements that determines how far should the surface layer proceed into the external network. The damping depends on the length of the transmission lines and the damping provided by loads in the system. The longer the transmission lines, the more damping they provide, and the less is the impact of the deep-network on the input admittance. Thus, if the surface-region contains long transmission lines, it is likely that we need only a one-bus layer as surface layer. If the lines are short and the surface layer does not contain loads or other elements that would produce extra damping, then it is most likely that there is a need to reach for a two- or even three-bus layer to be considered as surface layer.
- If a difficulty arises in the rational-function approximation of the deep-region, it can be treated by including more buses into the surface region and thus making it wider.

B) Surface Layer Approximation

The second step in the approach is to find an approximation of the surface layer. In order to keep the equivalent simple, it is preferred to keep the lumped parameters, if any, as they are and try to find a simple approximation of the transmission lines within the surface region. It is worthwhile to shed some light on transmission line approximation by the following.

C) Transmission Line Approximation

A transmission line is characterized by a propagation function H_p and a characteristic impedance Z_c. Both H_p and Z_c are frequency dependent functions. The propagation function H_p defines the relationship between the incident wave (V_{incid}) and the reflected wave (V_{refl}) by

$$V_{incid} = H_p V_{refl} \tag{5}$$

For a lossless line H_p becomes $H_o=e^{-j\omega\tau}$, which corresponds to a time delay τ along the line. In general, H_p can be interpreted as the product of two factors, a delay component H_o and a shaping component H_{sh} [5]:

$$H_p = H_0 H_{sh} \tag{6}$$

where $H_o=e^{-j\omega\tau}$, τ is the time delay along the line, and H_{sh} is the shaping component. H_{sh} can be approximated as a

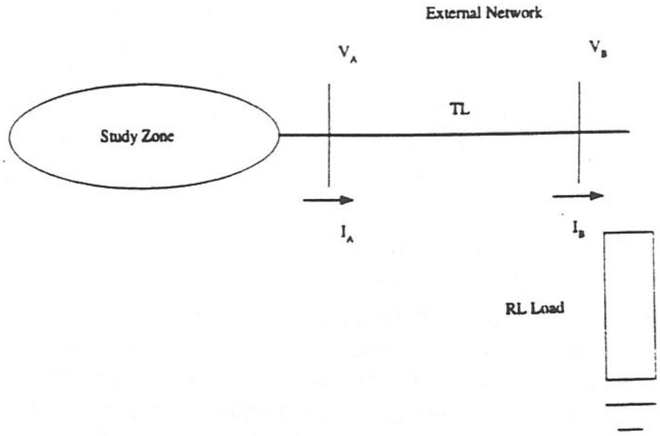

Fig. 1. Test System.

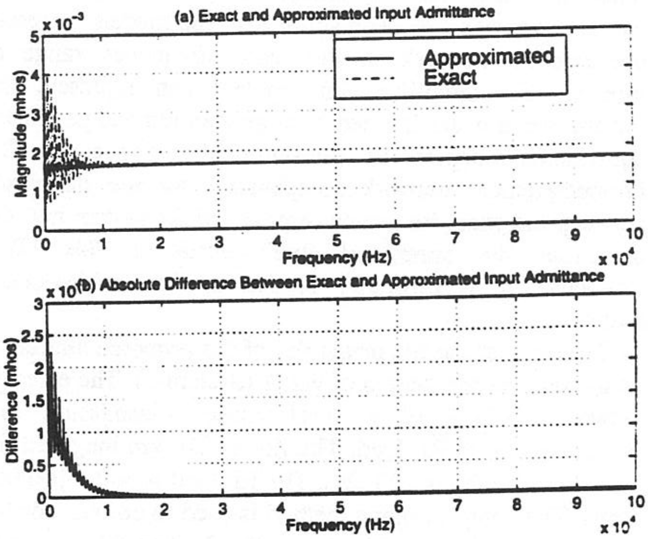

Fig. 2. Exact and equivalent input admittance of external network of test system.

strictly proper rational function. It can be as simple as two exponential functions (in the time domain), i.e., only two partial fraction terms.

The rational approximation of the characteristic impedance is proper and can be written as the sum of partial fractions [4]

$$k_0 + \sum_{i}^{n} \frac{r_i}{s + p_i} \tag{7}$$

where k_0 is a constant term, p_i is the ith pole, r_i is the residue for the ith fraction, and s stands for $j\omega$. All poles and residues can be real or complex.

D) Deep Layer Approximation

The first question is how to obtain the frequency response of the deep-region. Two options are available. The first is by "extracting" it from the input admittance using (4), provided that the nodal admittance matrix of the surface-region approximation is known. The other is by directly measuring the frequency response of the deep-region. In this paper we concentrate on the first method, while the second approach is treated in depth in a paper submitted to the IEEE [6].

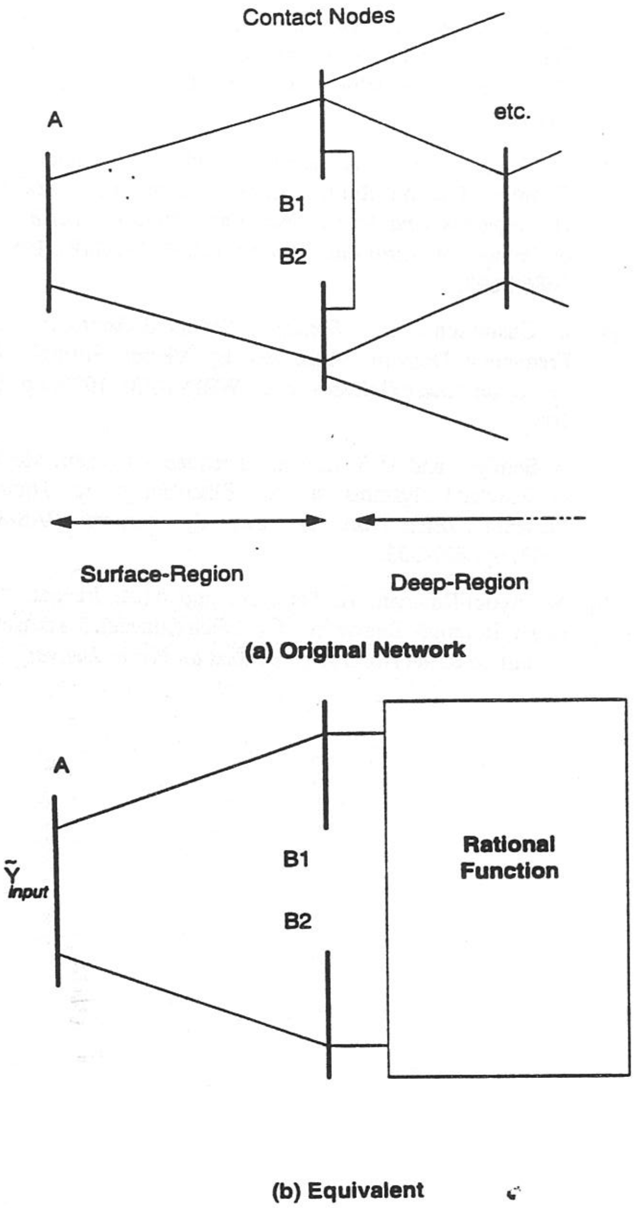

Fig. 3. External system partitioned into surface-region and deep-region.

The deep-region is represented by a lumped parameter equivalent with emphasis on the low frequency behavior. This lumped parameter equivalent must be stable. It generally takes the form of an $n \times n$ matrix, where n is the number of the contact nodes between the surface-region and the deep-region: see Fig. 3

III Equivalent of a Single-Line System

The objective of this section is to demonstrate the feasibility and accuracy of the proposed method by developing a hybrid equivalent of a single-line external network within the frequency band-width of 0 to 100 kHz. The system under consideration is System-1 (Fig. 1).

The line model consists of a 4th-order propagation function, and a 4th-order characteristic impedance. The deep-network is extracted up to 10 kHz and extrapolated up to 100 kHz. It is approximated by a 2nd-order rational function. It is worth mentioning that the choice of the order of the transmission line approximation for both the propagation function and characteristic impedance, and for the deep-region is achieved via exhaustive search. We are looking for the least order that will provide sufficient accuracy.

Fig. 4 compares the exact input admittance of the external network and the one obtained from the hybrid equivalent of the external network. Close agreement between the corresponding results of Fig. 4 indicates that the hybrid equivalent represents the external network of Fig. 1, in the desired frequency range, with an acceptable accuracy.

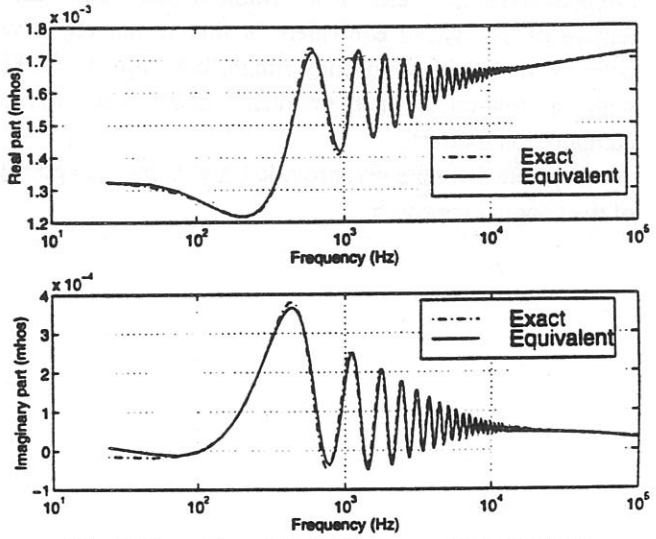

Fig. 4. Exact Input Admittance and Equivalent.

IV Multi-Port Equivalent

The previous section dealt only with single-port series networks. However, we note that (4) is not necessarily valid for the general case of a multi-port network. For a multi-port case, the elements of (3) are not scalars, but vectors and matrices. (3) is a set of homogeneous equations with the vector $V=[V_A\ V_B]^T$ as a non-trivial solution. This means that the admittance matrix in (3) must be singular:

$$\begin{vmatrix} Y_A - Y_{input} & Y_{AB} \\ Y_{BA} & Y_B + Y_{load} \end{vmatrix} = 0 \qquad (8)$$

It is obvious that for the case Y_{load} is a scalar we have a unique solution. otherwise, we have one equation in more than one unknown. Thus, an infinite number of solutions for Y_{load} will satisfy (8). The problem can be formulated as the optimization problem

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$$\text{minimize:} \qquad \sum \| Y_{load} \|_2^2 \qquad (9)$$

subject to (8). This approach has the merit of using only one set of measurements.

However, the solution must be stable, minimum-phase, and positive-real. In addition, the simplest solution is pursued. The details of this topic are discussed in our paper submitted to IEEE [6].

V Conclusions

This paper introduces the concept of a "hybrid equivalent" for large interconnected electric networks for the analysis of electromagnetic transients. The approach is based on representing a network by a combination of (i) simplified distributed-parameter transmission line equations and (ii) a rational transfer function corresponding to lumped-parameter electrical components. The salient feature of the hybrid equivalent is that it can represent a network in a wide frequency range, e.g., up to 100 kHz, with a low-order mathematical model and minimal computational effort.

A simple example was provided that shows the potential of the adopted approach.

VI References

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