Sparse Network Equivalent Based on Time-Domain Fitting: Single-port and Two-port Cases

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Abstract - **The paper presents an approach for the calculation of single-port and two-port network equivalents. These are meant to be used for the analysis of electromagnetic transients in power systems. The new approach is based on time-domain fitting and enforces some degree of sparsity for the equivalent. The calculated equivalent is appropriate for direct interface with the rest of the system in time-domain. The issues of order, stability, passivity, and initialization of the equivalent are discussed. Results demonstrating the accuracy and computational efficiency of the method are presented.**

Keywords: Network Equivalent, Electromagnetic Transients, Time-domain Fitting, Discrete Time.

I. INTRODUCTION

The design and operation of modern electric power systems rely increasingly on transient studies. In such studies there is usually a study zone, a restricted portion of the system, and an external system, which comprises the rest of the system. For the study zone, in contrast to the external system, we seek detailed information about its voltages and currents. If a complete representation is adopted for the external system, the computational effort required for the calculations may be excessive. This is particularly true when frequency dependent models are used for transmission lines. Even if only a part of the external system is to be used in the calculation, it is difficult to establish which part should be neglected. Network equivalents are used in this situation to reduce the computational burden for the transient calculation of the whole system by providing a suitable representation for the external system. They can be obtained using either frequency-domain or time-domain fitting techniques. Various methods relevant to network equivalent calculations with their merits and limitations are described in [1-8].

This paper presents a methodology for deriving a Sparse Network Equivalent (SNE) based on time-domain fitting procedures. The required data is obtained from the computed time-domain response of the network. The presented method enforces some degree of sparsity while preserving the accuracy of the equivalent. Rational models are used to represent the external system as either a single-port or a two-port equivalent. As it is derived in discrete-time domain, the SNE can be directly interfaced with the study zone for the simulation of electromagnetic transients. Several issues regarding its calculation, like the order, accuracy, sparsity, stability, and passivity of the equivalent, are discussed. The computational efficiency of the equivalent is ensured by its sparsity, especially when a small time step is needed.

The discrete-time nature of the equivalent easily handles initial conditions. When dealing with transient simulations from the steady state, the initial conditions for the external system, accounting for the presence of sources, can be readily implemented in the equivalent. This increases further the computational efficiency of the SNE. Accuracy and computational efficiency in the context of electromagnetic transients analysis have been discussed in [9-15].

The rest of this paper is organized as follows. Section II presents the details of the equivalent calculation. How the equivalent handles initial conditions is presented in Section III. In Section IV results are given for validating the methodology in terms of accuracy and efficiency (including or not simulations with initial conditions) as well as to show its limitations. Conclusions are stated in Section V.

II. NETWORK EQUIVALENT CALCULATION

The networks (external systems) to be equivalenced using this method are assumed to be linear networks. A linear system, seen as either a single-port or two-port network, can be fully characterized in the discrete-time domain by a linear constant-coefficients difference equation if time invariance and zero initial state are assumed [16,17]. Discrete-time domain is used here since it is closely related to the kind of data assumed for the equivalent. If the data is obtained from a transient calculation program, using a fixed time step, it is considered as accurate information for the system under study, disregarding the errors incurred by the discretization. Considering that current, $i(n)$, and voltage, $v(n)$, data sequences of length N taken at the ports of the external system, are available, the following difference equation, of order p (related to the output), characterizes the external system [16,17] and the equivalent

$$\sum_{k=0}^{p} A_k i(n-k) = \sum_{k=0}^{q} B_k v(n-k) \qquad (n = 0,\ldots,N-1), \quad (1)$$

where q is the number of past terms in the input, A_k and B_k, are constant coefficients that describe the equivalent and to

be determined. For the single-port case, A_k and B_k, in (1) are scalars and for the two-port case they are 2×2 matrices. Similarly, the currents and voltages in (1) are scalars for the single-port case and 2×1 vectors for the two-port. Rearranging the terms in (1), setting the value of A_0 to 1 (single-port) or to 2×2 identity matrix (two-port) and $q = p$ (see Section C below), we get the normalized equation

$$i(n) = B_0 v(n) + \sum_{k=1}^{p} (B_k v(n-k) - A_k i(n-k)). \quad (2)$$

The coefficient B_0, in (2), has the dimension of admittance and the term under the summation is calculated only from past values. Equation (2) is in the appropriate form to integrate the equivalent into a transient calculation program. The calculation of the equivalent boils down to identifying A_k and B_k. In the following, a procedure to determine A_k and B_k, while ensuring the sparsity of the equivalent, is presented. For multiphase systems, equivalents can be obtained using this procedure by treating each mode separately via modal decomposition.

A. Identification of A_k, B_k

A unit step voltage is used to produce the voltage and current sequences of (1) as it provides information on both high and low frequency characteristics of the external system. Equation (1) is rewritten in matrix form leading to a set of linear equations. For the single-port case we seek the scalars a_k (a_1,..., a_p) and b_k (b_0,..., b_p). Using the following convolution matrices from the corresponding sequences $i(n)$ and $v(n)$

$$I = \begin{bmatrix} 0 & 0 & . & 0 \\ i_0 & 0 & . & 0 \\ . & . & . & . \\ i_{p-2} & i_{p-3} & . & 0 \\ \hline i_{p-1} & i_{p-2} & . & i_0 \\ . & . & . & . \\ i_{N-2} & i_{N-3} & . & i_{N-p-1} \end{bmatrix}, \quad V = \begin{bmatrix} v_0 & 0 & . & 0 \\ v_1 & v_0 & . & 0 \\ . & . & . & . \\ v_{p-1} & v_{p-2} & . & 0 \\ \hline v_p & v_{p-1} & . & v_0 \\ . & . & . & . \\ v_{N-1} & v_{N-2} & . & v_{N-p-1} \end{bmatrix}, \quad (3)$$

where the subscripts in the variables i and v indicate the time index in the sequences, we build (4), which is used for the determination of a_k and b_k:

$$\begin{bmatrix} I & -V \end{bmatrix} \begin{bmatrix} a_k \\ b_k \end{bmatrix} = -i. \quad (4)$$

This method of solution is related to Prony's work [18], which deals with the impulse response of linear systems and would lead to a different set of equations.

For the two-port case, we choose to work with only one set of coefficients related to the output (currents). This means that the two ports will share the same set of poles and the matrices A_k will be diagonal with equal elements. The symmetry and stability (see Section D below) of the equivalent is thus facilitated. For the determination of A_k and B_k, we use 5 sequences obtained in the following way. With port II short-circuited, we take the voltage at port I, $v_{\rm I}(n)$, the current at port I, $i_{\rm I}(n)$, and the current at port II, $i_{\rm I,II}(n)$. Similarly, with port I short-circuited, we take the voltage at port II, $v_{\rm II}(n)$, and the current at port II, $i_{\rm II}(n)$. Due to the symmetry of the external system, it is not necessary to use $i_{\rm II,I}(n)$, since it is equal to $i_{\rm I,II}(n)$. These sequences are used to obtain each element of the matrices A_k and B_k as

$$A_k = \begin{bmatrix} a_k & 0 \\ 0 & a_k \end{bmatrix}, \quad B_k = \begin{bmatrix} b_{\rm Ik} & b_{\rm I,IIk} \\ b_{\rm I,IIk} & b_{\rm IIk} \end{bmatrix}, \quad (5)$$

using (6), which was built using the convolutions matrices of the voltages and currents listed above:

$$\begin{bmatrix} I_{\rm I} & -V_{\rm I} & 0 & 0 \\ I_{\rm II} & 0 & -V_{\rm II} & 0 \\ I_{\rm I,II} & 0 & 0 & -V_{\rm I} \end{bmatrix} \begin{bmatrix} a_k \\ b_{\rm Ik} \\ b_{\rm IIk} \\ b_{\rm I,IIk} \end{bmatrix} = \begin{bmatrix} -i_{\rm I} \\ -i_{\rm II} \\ -i_{\rm I\text{-}II} \end{bmatrix}. \quad (6)$$

The number of unknowns we seek is $2p$+1 or $4p$+3, solving (4) or (6), for the single-port and two-port cases, respectively. We use N greater than $2p$+1, thus, (4) and (6) are overdetermined and a solution in the least squares sense can be obtained for the unknowns. Our studies indicate that a redundancy around 2 results in a good overall accuracy. Obtaining a solution for A_k and B_k, one should evaluate if (*i*) the desired accuracy is achieved, and (*ii*) the deduced equivalent is both stable and passive. Equations (4) and (6) produce non-sparse equivalents, the sparsity is treated next.

B. Sparsity in the identification of A_k, B_k

When used in the simulation of transients, the non-sparse equivalents obtained from (4) or (6) usually require less computational effort than the calculations with the complete representation would require. However, if the ultimate goal is speed rather than maximal accuracy, a sparse network equivalent can be obtained. This requires a few modifications of the above procedure. There is a trade-off between the degree of sparsity and accuracy: the higher the sparsity the lower the accuracy. The sparse network equivalent usually has the same (or even higher) dynamic order p as the non-sparse equivalent but at a reduced computational burden. To calculate a sparse equivalent we eliminate some of the unknowns by setting them to zero. The procedure is exemplified for the single-port case as follows (the two-port case is similar):

⇒ Step 1: Obtain a basic solution for a_k and b_k using (4);

⇒ Step 2: For the desired sparsity N_S (number of non-zero coefficients for each set a_k and b_k), the calculated a_k and b_k from Step 1 are scanned to identify the positions of the N_S largest coefficients. These values are of course not the best ones and they have to be recalculated as follows;

⇒ Step 3: All the coefficients other than those N_S identified in Step 2 will be zeroed and do not contribute to (4). To build the sparse version of (4), modify matrices I, V keeping only the columns corresponding to the N_S largest coefficients for a_k and b_k;

⇒ Step 4: Solve (4), built with the sparse versions of I and V, and address the solution to the corresponding positions of the N_S largest coefficients, resulting in sparse sets of coefficients a_k and b_k;

⇒ Step 5: Check for stability and passivity of the equivalent, using the criteria presented in Section D. Check for the desired overall fitting error. If the stability, passivity and fitting requirements are not satisfied, N_S is increased and the procedure is repeated from step 2 to step 5 iteratively until the requirements are satisfied. Another

search strategy is to start with non-sparse sets of coefficients and iterate to increase the sparsity as much as the criteria for fitting, stability and passivity permit.

For the two-port case we deal with sets a_k, b_{Ik}, b_{IIk} and $b_{I,II}k$. Usually the number of non-zero coefficients for a_k is larger than the other sets, since it carries the burden of fitting 3 current sequences. It is also possible to reduce N_S for the set $b_{I,IIk}$ as it has many leading zeros accounting for the time delay between the two ports.

Equation (4) or (6) does not usually result in a full rank problem, especially when the ground mode is considered (see Section C below). Then the basic solution from step 1 is best obtained using methods based on QR decomposition. This is due to the fact that these methods can automatically set to zero a number of variables corresponding to the rank deficiency [19], thus providing ab initio some sparsity.

C. Determination of the order of the equivalent

The following analysis is based on the single-port case and the implications for the two-port case are presented in the sequel.

As shown before, we use the same number of past terms, $q = p$, for both sets of coefficients, a_k and b_k. This is not a restrictive assumption; it is based on the analysis of the current wave reflections in an open-ended single-phase transmission line excited by an ideal voltage source. Thus, to account for time delays and multiple reflections due to the transmission line, one must use the same order for a_k and b_k. This analysis also provides information about the value of p, the order of the equivalent, which must be twice the travel time of the transmission line divided by the time step. It can be observed from this simple example that the order of the equivalent is determined by the occurrence of time related events such as arrival of the reflected current wave at the sending bus. As the velocity of propagation of the line mode is larger than that of the ground mode, one expects larger order for the latter, since the reflected waves take longer to arrive at the sending bus.

The value of p determines the number of variables to be calculated, $2p + 1$, and the rank condition of (4). A larger order implies more variables to be determined and the rank of (4) should increase accordingly. However, the analysis of the condition of the matrices in (4) for the line and ground modes indicates a lower rank for the ground mode. This is because the corresponding high loss attenuates the traveling waves and degrades the information associated with farther nodes. In the following, this problem is further examined and two approaches for determination of p are discussed.

One approach to determine a suitable range for the order of a_k and b_k utilizes the "length", in time steps, of the network, and the number of lumped energy storage components. The lower limit for the "length" of a network is defined as the double of the time required for a signal to arrive from the boundary bus to the farthest node in the network through the shortest path possible, divided by the time step. This approach is adequate for radial networks only. If there are loops in the network, then the order could be close to the total length of the network. This is equal to the double of the sum of all transmission line travel times divided by the time step. This is an upper limit. Usually a suitable value for the order is between these two limits.

The second approach is based on the rank analysis of the matrices of (4) via singular value decomposition (SVD) [20]. The data from $i(n)$ or $v(n)$, used to build I or V, is corrupted by round-off noise due to the limited number of significant digits (usually 6) available in the output format from transient calculation programs. The effect of noise is easily noticed using SVD analysis, and a procedure to determine p based on this information is formulated in the following. Some remarks are important to be made with respect to the use of SVD:

- The upper partitions (denoted by the dashed line) of I and V, I_U and V_U, contribute to the rank of (4) with p linearly independent rows, regardless of how large p is, due to the upper triangles filled with zeros. These zero values account for the initial conditions. If (4) is well-conditioned, the remaining N–p lines should have other p+1 linearly independent rows, completing the rank. Therefore, the information about the rank must be searched in the lower partitions (I_L and V_L) of matrices I and V. The upper partitions should not be used for obtaining p because of their triangular shape. If p is overestimated, they lead to erroneous information regarding the intrinsical order of the system.
- The SVD analysis can be performed on the lower partitions I_L and V_L, either separately or together. It reliably reveals the rank of the partition being used. If the source used to obtain $i(n)$ and $v(n)$ is a voltage source, one uses the data from $i(n)$ – matrix I_L. In the case of a current source, one uses the data from $v(n)$ – matrix V_L. If a complex source is used, the SVD analysis must be performed on the whole lower partition (matrices I_L and V_L).
- Losses inherent to ground mode highly attenuate the travelling waves associated with farther nodes. Since losses for the line mode are not very high, the SVD indicates a reliable value for p. However, the value of p for the ground mode indicated by the SVD is not reliable. Despite the increased travel times, due to the high losses, the SVD leads to a p smaller than that of the line mode. Therefore, a value of p for the ground mode is set to be greater than that obtained for the line mode, by a factor based on the differences in the travel times. Determination of a_k and b_k for the ground mode is usually a rank deficient problem.

To determine p for the line mode we calculate the SVD of I_L built for a p much larger than the order assumed for the external subsystem, for instance equal to half the number of available data points which is presumably much higher than the correct order for the system. A suitable range of values for p is indicated by the indexes of the singular values which are about $10^2 \sim 10^8$ times larger than the smallest singular values (corresponding to the round-off noise). Usually, the larger the order, up to a certain limit, the higher the accuracy. A plot of singular values of partitions I_L for the determination of p for both line and ground modes is presented in Section IV, Fig. 1.

We conclude by emphasizing that the order p we choose for the realization (2) of an external system must not be smaller than its intrinsical order. In the computational problem (4), because both a_k and b_k appear as unknowns, we have matrices with $2p$+1 columns. Their

numerical rank is increased accordingly due to their upper part (as discussed before) and only their lower part properly reflects the intrinsical rank or order of the network.

The two-port case can be treated as three single-port cases attached by a common set of a_k coefficients, as the row blocks of (6) denotes. Thus, we perform the above described SVD analysis in the partitions I_{II}, I_{III} and $I_{I,III}$ and take the highest of the three values indicated for p as the order for the two-port equivalent.

D. Stability and passivity requirements

Stability and passivity are inherent characteristics of a passive network and an equivalent for such a network must exhibit these characteristics. The stability of a linear system described by a constant-coefficients difference equation can be checked by the analysis of its natural modes corresponding to its zero-input response. The zero-input response of a system described by (1) is a sequence $i_h(n)$ that satisfies

$$\sum_{k=0}^{p} A_k i_h(n-k) = 0 . \qquad (7)$$

As in the case of constant-coefficients differential equations, (7) is referred to as the homogeneous equation and $i_h(n)$ is the homogeneous solution [16]. We seek a solution for (7) of the form

$$i_h(n) = \text{const.} z^n . \qquad (8)$$

Substituting (8) in (7) results in the requirement that the complex numbers z be the roots of the polynomial equation

$$\sum_{k=0}^{p} A_k z^{-k} = 0 . \qquad (9)$$

For stable sequences, the absolute value for each z must be less than 1. As we use only one set of coefficients related to the output in the two-port case, instead of a matrix polynomial in (9) we have a single polynomial. Thus, the check for stability is the same for both single-port and two-port cases.

To develop the passivity criterion we specify the admittance of the equivalent, in terms of the sets of coefficients, as function of the frequency. We analyse the single-port case and then extend it to the two-port case. Consider the equivalent excited by a single frequency voltage source

$$v(n) = v(t_n) = v(n\Delta t) = v_\omega e^{j\omega n\Delta t} = v_\omega (e^{j\omega\Delta t})^n . \qquad (10)$$

Using the auxiliary variable $z = e^{j\omega\Delta t}$, we have

$$v(n) = v_\omega z^n . \qquad (11)$$

Similarly, the corresponding current can be expressed as

$$i(n) = i_\omega z^n . \qquad (12)$$

Setting $v_\omega = 1$, implies $i_\omega = y_\omega$, the admittance of the equivalent at frequency ω. Substituting (11) and (12) in (1) and rearranging the terms, we obtain

$$y_\omega = g_\omega + jd_\omega = \frac{\sum_{k=0}^{p} b_k z^{-k}}{1 + \sum_{k=1}^{p} a_k z^{-k}} . \qquad (13)$$

g_ω is the real part of the polynomial fraction in (13). The criterion for passivity of the equivalent, single-port case, is $g_\omega > 0$. This must be checked for a reasonable number of frequencies (typically 1000), in the range from 0 to $\pi/\Delta t$.

In the two-port case, the admittance of the equivalent is a 2×2 matrix, rather than a scalar:

$$Y_\omega = \begin{bmatrix} y_{\omega I} & y_{\omega I,II} \\ y_{\omega I,II} & y_{\omega II} \end{bmatrix} = G_\omega + jD_\omega . \qquad (14)$$

The elements of Y_ω are obtained using (13), substituting b_k by b_{kI}, $b_{kI,II}$ or b_{kII}, and the two-port a_k set. G_ω is the real part of Y_ω. To ensure passivity, both eigenvalues of G_ω must be greater than zero or G_ω must be positive definite [14,17,21]. As for the single-port case, this must be checked for several frequencies.

The procedure presented in this paper does not address the issues of stability and passivity as constraints imbedded in the calculation of the equivalent. However, a stable and passive equivalent is usually obtained if losses are represented in the network and a redundancy of around 2 is observed for (4) or (6).

III. SIMULATIONS WITH NON-ZERO INITIAL CONDITIONS

The discrete nature of the equivalent derived in this work permits its initialization very easily. From (2), we see that including initial conditions is just a matter of calculating an appropriate set of past values of voltages and currents. This process is especially facilitated if the initialization refers to the steady state for a given frequency ω_0 (corresponding, for instance, to 50 Hz or 60 Hz). In this situation, the voltage and current sequences are related by the value of the admittance of the equivalent at ω_0, Y_{ω_0}, which is calculated using (13) or (14), for single-port and two-port cases, respectively. We exemplify this using the two-port case.

It is desired to perform a simulation from the steady state at a given ω_0, for which the voltages at port I and II are specified as phasors. The corresponding current phasors are calculated as:

$$\begin{bmatrix} i_{0,I} e^{j\phi_{0,I}} \\ i_{0,II} e^{j\phi_{0,II}} \end{bmatrix} = Y_{\omega_0} \begin{bmatrix} v_{0,I} e^{j\theta_{0,I}} \\ v_{0,II} e^{j\theta_{0,II}} \end{bmatrix} . \qquad (15)$$

The initial conditions for voltages and currents are thus given by the sequences in (16) and (17), where $n = -1, -2, -3, \ldots, -p$, respectively for the ports I and II.

$$v_I(n) = v_{0,I}\sin(\omega_0 n\Delta t + \theta_{0,I}),\ i_I(n) = i_{0,I}\sin(\omega_0 n\Delta t + \phi_{0,I}) \qquad (16)$$

$$v_{II}(n) = v_{0,II}\sin(\omega_0 n\Delta t + \theta_{0,II}),\ i_{II}(n) = i_{0,II}\sin(\omega_0 n\Delta t + \phi_{0,II}) \qquad (17)$$

Naturally, when the equivalents are obtained for each mode, the voltages and currents used in the initialization process are modal quantities. Thus, for power frequency steady state, only the line-mode has non-zero initial conditions.

IV. ILLUSTRATIVE EXAMPLES

The three-phase network presented in Fig. 1, considered as the external system, is used to demonstrate the accuracy,

computational efficiency and limitations of the equivalencing technique for both single-port and two-port cases. Non-sparse and sparse equivalents are calculated for the line and ground modes of the network shown in Fig. 1. All the transmission lines are assumed to be balanced.

A detailed time-domain electromagnetic transients simulation model of the test system is developed to serve as reference for the comparisons of accuracy and computational efficiency of the equivalent. This model is also used to generate the voltage and current sequences for the calculation of the equivalent. Frequency dependent models [9,10] are used for all lines, and the propagation functions and characteristic impedances are represented by ten partial fractions for both line and ground modes. The partial fractions are obtained using vector fitting [22]. The routines for (*i*) obtaining the network equivalent and (*ii*) performing the electromagnetic transient computations using the equivalent or the detailed network representation were developed in Matlab [19].

As a measure of accuracy, the overall fitting error, F_{err}, is calculated (using voltages as an example) as

$$F_{err} = \|V_{eq} - V_{fn}\| / \| V_{fn} \|, \qquad (18)$$

where the subscripts "*eq*" and "*fn*" refer to the sequences calculated using the equivalent and the full network representation, respectively. The sparsity of the equivalent is measured by its "occupancy", which is defined as the percentage of non-zero coefficients in the sparse equivalent compared to the non-sparse equivalent. For the two-port case, this is done taking into account all the sets of coefficients at once. The criterion for computational efficiency is the number of floating point operations (flops) needed to perform a transient calculation. Instead of time comparisons, flops are chosen because they do not depend on the platform or the compiler used.

A. Single-port case

For the single-port case, the external system is the network (Fig. 1) seen from bus I. The voltage and current sequences used for the calculation of the equivalent were obtained from a transient simulation due to a step voltage applied to bus I using a 20 μs time step. The determination of p uses the SVD approach. Fig. 2 shows the singular values obtained from the SVD of I_L (built for p = 300) for the line mode. Several equivalents for the line mode could be calculated using values of p from 105 to 160 providing different fitting errors, stability and passivity conditions. The non-sparse equivalent for the line mode is calculated

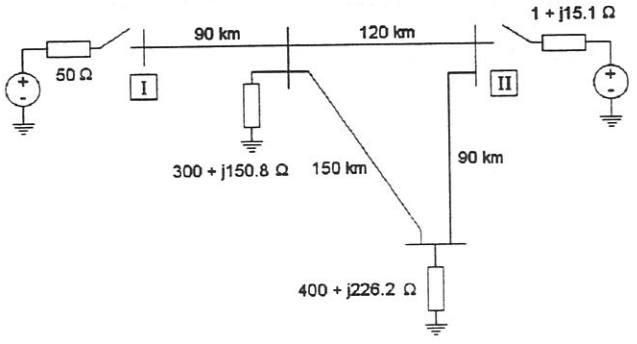

Fig. 1 – Single line diagram of the three-phase network taken as the external system.

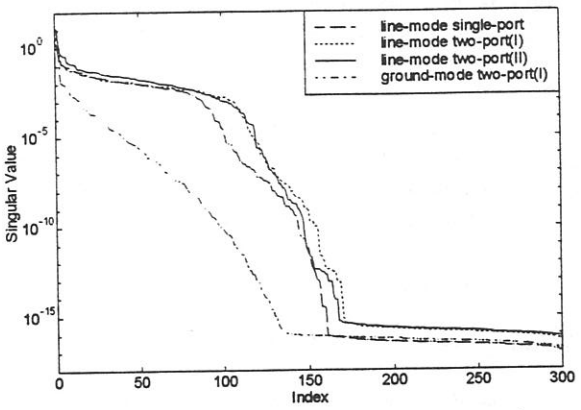

Fig. 2 – SVD for the determination of p, single-port and two-port cases (line and ground modes).

using p = 150. It is stable, passive and presents an F_{err} (compared to the current sequence used for its calculation) of 3.48×10^{-7}. The sparse equivalent (SNE) is searched decreasing the spartsity, or, increasing the value of N_S (the number of non-zero coefficients in the sets a_k and b_k), initially set to 10. A stable and passive SNE, with an F_{err} of 0.0177, is obtained for N_S = 17, which provides an occupancy of only 11%. For the ground-mode, a stable and passive non-sparse equivalent, with $F_{err} = 9.30\times10^{-8}$, is calculated using p = 215. A stable and passive SNE for the ground mode is also obtained decreasing the sparsity and resulting in N_S = 23 and F_{err} = 0.0024 (occupancy of 10.7%).

To further probe the accuracy of the equivalent and its computational efficiency, an energization of the network at bus I is examined. The energization is calculated using full representation and both non-sparse and sparse equivalents. It is a monopolar switching operation (using a 60 Hz voltage source and 50 Ω resistance) performed at bus I, phase A, while phase B is short-circuited and phase C is left open. The results regarding accuracy and computational efficiency are presented in Table 1. For the accuracy, voltages on phases A and C, and current on phase A and B, all at bus I, are compared. Fig. 3 shows the voltages on phase C calculated using full representation and the sparse equivalent. The accuracy of the non-sparse equivalent is not compared because it presents negligible fitting errors (less than 10^{-5}).

Table 1 shows that the SNE has a major impact in the computational efficiency while the non-sparse equivalent has not. This is due to the small time step used, which increases the order of the equivalents. The absence of a study zone in these simulations restricts the computational burden to the calculation of the external system only, thus

TABLE 1:Computational efficiency and accuracy comparisons.

Single-port equivalent: computational efficiency:			
	Full network representation	Non-sparse equivalent	SPARSE equivalent
Mflops(%):	100%	50.1%	**7.8%**

Single-port equivalent: accuracy (fitting error, F_{err}):				
	Voltage (phase A)	Voltage (phase C)	Current (phase A)	Current (phase B)
SPARSE equiv.:	0.0021	0.0256	0.0109	0.0487

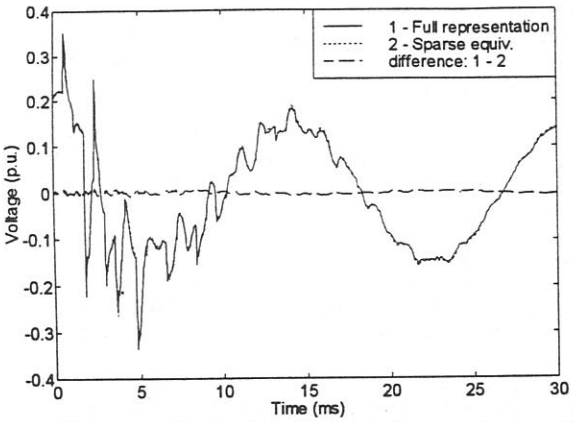

Fig. 3 – Voltages at bus I, phase C, due to the energization of the network presented in Fig. 1 (SNE: single-port).

permitting a direct comparison. The accuracy comparisons are not affected since the equivalents are properly integrated into a transient calculation routine to interact with the voltage sources. The fitting errors presented by the SNE, shown in Table 1 and Fig. 3, are acceptable in view of the computational efficiency provided.

B. Two-port case

The sequences for the calculation of the equivalents were obtained applying step voltages at buses I and II (Fig. 1), one at a time, as described in Section II.A. The values for p were determined using the SVD approach (Fig. 2). Table 2 shows the characteristics of the equivalents for line and ground modes regarding order, fitting errors and sparsity. The non-sparse equivalents for both modes are stable, passive and present negligible fitting errors (6.9×10^{-6} for the worst case). While both SNEs are stable and present acceptable fitting errors, only the equivalent for the ground mode is passive. The passivity of G_ω (14) for the line mode SNE was checked for 1000 frequencies resulting in 63 negative eigenvalues. This violation of the passivity criterion makes the SNE unusable, as it may lead to instability. To check the two-port SNE behavior, the same monopolar switching operation used in the single-port case was performed at port I. The voltages (phase C) resulting from this energization, calculated using full representation and the SNE, are shown in Fig. 4. The instability starts to

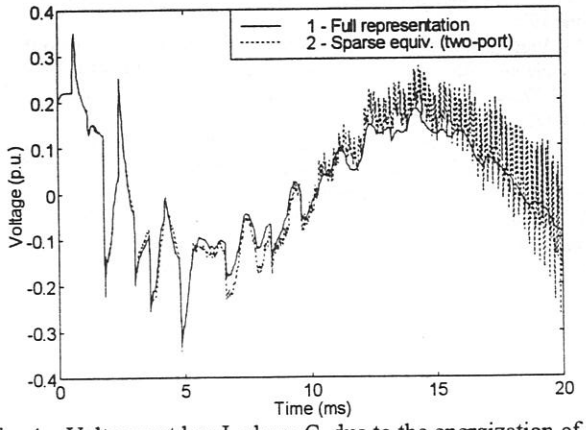

Fig. 4 – Voltages at bus I, phase C, due to the energization of the network presented in Fig. 1 (SNE: two-port).

TABLE 2: Two-port equivalent characteristics.

Order p (NON-sparse equivalent):				
LINE mode:	150	GROUND mode:	215	
Fitting errors (F_{err}) – SPARSE equivalent:				
Current seq.:	$i(n)_{\mathrm{I}}$	$i(n)_{\mathrm{II}}$	$i(n)_{\mathrm{I,II}}$	
Line mode:	0.0013	0.0033	0.0039	
Ground mode:	0.0040	0.0042	0.0020	
Sparsity (N_S):				
Coeff. set:	a_k	$b_{\mathrm{I}k}$	$b_{\mathrm{II}k}$	$b_{\mathrm{I,II}k}$
N_S (line):	35	29	29	24
N_S (ground):	48	40	40	33
Occupancy (%)				
LINE mode:	19.6%	GROUND mode:	18.8%	

be pronounced after around 5 ms. This is a limitation of the presented method and the authors intend to overcome it by using a constrained solution of (6) to enforce passivity.

The computational efficiency of two-port equivalents is about 3 times lower than that of single-port equivalents if similar order and sparsity are used. This comes from the fact that the single-port equivalent uses only two sets of coefficients while the two-port uses six sets. It may turn the use of two-port SNEs of little advantage and the non-sparse equivalents of none, unless we deal with simulations from the steady state, as shown in the following.

The steady state initialization of transient calculation routines using full representation may not be as easy as it is for the discrete-time equivalents used in this work. The initialization of both sparse and non-sparse equivalent has practically no computational burden if the method presented in Section III is used. To compare the computational efficiency provided by the use of equivalents in simulations with initial conditions, we conduct a short-circuit transient calculation for the network of Fig. 1. The network is excited at bus II by a three-phase voltage source with impedance of $1+j15.1\ \Omega$. The short-circuit is performed at bus I, phase B, when the simulation time is 0.2 s and it is cleared after 10 ms. The 0.2 s is needed for the simulation using full representation to reach the steady state. The transient is calculated using full representation, non-sparse equivalent and the SNE. Table 3 presents the results regarding the computational efficiency comparisons considering or not the initialization of the equivalent. These results show that for simulations from the steady state the use of the equivalents has a major effect on the computational burden. The efficiency of the equivalents is increased by a factor of about 8 if the initialization is used.

Regarding the accuracy comparison, it is done only for the non-sparse equivalent, as the SNE exhibits instability problems. The voltages at bus I, phase B, due to the short-circuit transient calculation, using full representation and the non-sparse equivalent, are shown in Fig. 5. The difference between the two curves is magnified by a factor

TABLE 3:Computational efficiency comparisons.

Two-port equivalent: computational efficiency:			
WITHOUT initialization	Full network representation	Non-sparse equivalent	SPARSE equivalent
Mflops(%):	100%	141.8%	**39.1%**
WITH initialization	Full network representation	Non-sparse equivalent	SPARSE equivalent
Mflops(%):	100%	**18.5%**	**5.1%**

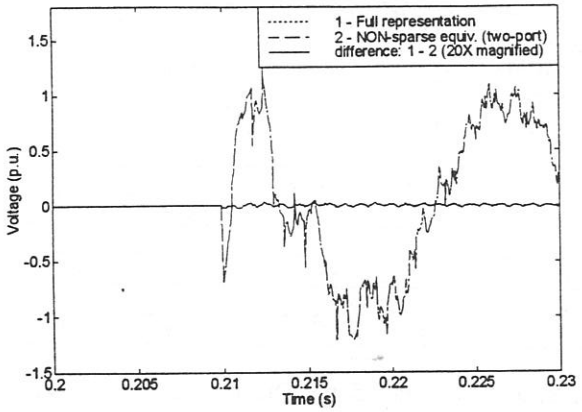

Fig. 5 – Voltages at bus I, phase B, due to the short-circuit transient calculation (NON-sparse quivalent - two-port).

of 20. This small difference is not due to inaccuracy of the equivalent, but to the fact that the steady state initialization of the equivalent is perfect and 0.2 s is not sufficient for the complete initialization of the full representation. Nevertheless, the errors are too small, $F_{err} = 1.7 \times 10^{-3}$ for the worst case.

V. CONCLUSIONS

The paper presents a methodology for calculating sparse network equivalents based on time-domain fitting. Single-port and two-port equivalents are studied. Criteria for checking stability and passivity of both equivalents are formulated. A method for the initialization of the equivalents is also presented.

The results show the superior computational efficiency of the sparse equivalent for transient calculations when compared to either the non-sparse equivalent or the full network representation. The study results indicate that the overall fitting error of the sparse equivalent is satisfactory. The initialization feature of the equivalents further increases their efficiency.

The presented methodology could not be successfully applied to obtain passive two-port equivalents. The solution to this problem will be presented in a sequel to this paper.

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