Non Uniform Line Tower Model For Lightning Transient Studies

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Abstract. **A technique to develop transmission tower models for lightning performance studies is presented in this paper. It consists in representing the tower body by a vertical transmission line and the tower arms by horizontal or inclined lines, as required. Whereas arm parameters are obtained from standard line formulas, tower body parameters are obtained from expressions derived here. The proposed technique is applied in the simulation of a field experiment reported elsewhere. The agreement between simulated and field results is satisfactory.**

Keywords: Vertical Transmission Lines, Tower Modeling, Transient Analysis, Lightning , Back-flashover.

I. INTRODUCTION

For a well designed line shielding, a lightning stroke hitting a phase conductor and causing an outage should be a very rare event. Nevertheless, lightning can still produce a large number of outages through the back-flashover mechanism. When a stroke hits a ground wire or a transmission tower, the injected current propagates towards the ground through the tower's body causing an instantaneous rise of the tower voltage. This may result in a back-flashover; that is, a flashover from the tower body to a phase conductor.

Field measurements and experiments provide the most effective means to assess the transient performance of a particular tower structure. Since this approach results very difficult an expensive, it is highly desirable to complement it with digital computer simulations. The specialized literature provides electromagnetic models only for certain classes of towers. These models usually consist in non-uniform lossless single line representations [2,3]. Often there is, however, a need for more general modeling techniques; as for instance, when the towers under study differ substantially from the geometries for which formulas are available. One may want, in addition, to analyze the effects of distributed losses, of variable wave speeds and even of tower arms. This paper presents a technique to construct transmission tower models for lightning performance studies which addresses the previously mentioned needs.

II. MODELING TOWERS WITH NON UNIFORM LINE SEGMENTS

The modeling methodology presented here focuses on truss-towers, like the one shown in Fig. 1a. These towers usually are composed by several truss modules; as for instance, the different stages of the main body, the arms and even perhaps few cross-sections. Each truss module consists typically of four long bars, hereafter called columns, which are joined by others usually shorter transversal and slanted bars. Propagation phenomena over the different elements of a transmission tower is a four dimensional problem involving spherical waves. A complete field solution would require using numerical methods such as Finite Difference Time Domain, Finite Element, Moments, etc. On assuming that the principal mode of propagation is TEM, it becomes possible to uniquely define a voltage difference between two points and, in consequence, it is now possible to deduce an analytical formula for the characteristic impedance. Although this TEM formulation is approximate, it provides a practical methodology that incorporates the travelling wave phenomenon into the tower model.

For the modeling technique being proposed here it is assumed that the lightning energy propagates only along the columns. It has been estimated in [4] that no more than 1 % of this energy propagates along the slanted bars and that practically zero energy propagates along the transversal ones.

Under the TEM assumption the columns of each module can be represented as multi-conductor lines which can be further reduced to single phase lines, either through Kron's reduction or through conductor bundling formulas[13,14]. Fig. 1b illustrates the representation of the tower in Fig. 1a by a number of interconnected non-uniform line segments. When the columns are horizontal, as in the arms and cross-sections, their line parameters can be obtained using well established methods [5,6,7]. If the columns of a module form a vertical multi-conductor system, as in the main body, their line parameters can be obtained through the methods provided in this work.

Once the line parameters have been calculated for all the modules in the tower under study, the non-uniform segments network representing the whole structure can be simulated either with the EMTP or with a finite difference method. The latter alternative is adopted in this paper. Specifically, the method of characteristics of reference [8] is applied in the simulation of a field experiment.

III. TRANSMISSION LINE REPRESENTATION

Each truss module from a transmission tower is represented here as a single phase non-uniform line segment. As such, its electromagnetic response to lightning surges is modeled by the following line equations [8]:

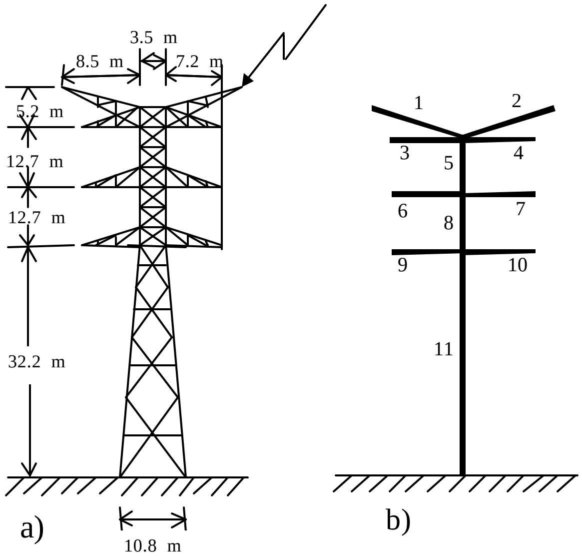

Fig. 1.- a).-Truss transmission tower, b).-tower representation by non-uniform line segments.

$$-\frac{\partial V}{\partial x} = RI + L\frac{\partial I}{\partial t} \quad \text{and} \quad -\frac{\partial I}{\partial x} = C\frac{\partial V}{\partial t}, \qquad \text{(1a,b)}$$

where L, R and C are the respective inductance, resistance and capacitance line parameters. The parameter L is formed by three terms as follows [5]:

$$L = L_G + L_E + L_C, \qquad (2)$$

where L_G is the geometric inductance due to the magnetic flux through the air, L_E is the inductance due to the finite conductivity of the earth plane and L_C is the conductor's internal inductance. The parameter R is formed by the following two terms [5]:

$$R = R_E + R_C \qquad (3)$$

where R_E and R_C are the resistance due to the finite conductivity of the earth and of the metallic tower bars, respectively. As for the parameter C, this can be considered as consisting only in the geometric or ideal line capacitance C_G [5].

Equations (1a) and (1b) are based on the assumption that line parameter variations with respect to frequency are negligible. The parameters L_E, L_C, R_E and R_C must be calculated at a frequency representative of the transient phenomenon under study. The following rule of thumb is applied. For a line segment of length "l" (in meters) the representative frequency is [8]:

$$f = 3\times10^8/(4l) \qquad (4)$$

IV. LINE PARAMETERS OF HORIZONTAL BARS

The columns of a horizontal truss module can be regarded as a horizontal multi-conductor system. Its line parameters can thus be obtained by well established methods, such as the ones provided in references [5,6,7]. The multi-conductor line representing the truss module is further reduced into a single conductor equivalent line by applying Kron's reduction as described in section VIII [10].

V. ELECTROMAGNETIC REPRESENTATION OF VERTICAL CONDUCTORS

Assume that the columns of a vertical truss module are equivalent to cylinders. Fig. 2a depicts a vertical column of radius "r_C" with a "dh" element at a height "h" above a perfect ground plane. This differential element can be considered also an element of the inverted conical line shown in Fig. 2b. This element is located at a distance "r" from de cone apex [11]. The fields of the transversal electromagnetic wave supported by the conical line are given as follows [11]:

$$H_\varphi = \frac{A e^{-j\beta r}}{r \sin\theta} \qquad (5)$$

$$E_\theta = \sqrt{\frac{\mu}{\varepsilon}}\, H_\varphi \qquad (6)$$

and

$$E_r = E_\varphi = H_\theta = H_r = 0 \qquad (7)$$

where A is the magnitude of the magnetic vector potential, $\beta=\omega\sqrt{(\mu\varepsilon)}$ and θ is the angle defined in Fig. 2b.

The voltage of the "dh" element on the cone, with respect to the ground, is given by:

$$V = \int_{\theta_1}^{\pi/2} E_\theta \, r \, d\theta \qquad (8)$$

On substituting (6) in (8) and solving:

$$V = \sqrt{\frac{\mu}{\varepsilon}} A\, e^{-j\beta r} \ln\left[\cot\left(\frac{\theta_1}{2}\right)\right] \qquad (9)$$

The current flowing through this "dh" element of the cone is found applying Ampere's law around the transversal cone circle with radius r_C:

$$I = \int_0^{2\pi} H_\varphi \, r_c \, d\varphi \qquad (10)$$

On replacing (5) in (10) and solving:

$$I = 2\pi A\, e^{-j\beta r} \qquad (11)$$

Using equations (9) and (11) the characteristic impedance of the inverted conical line is obtained as follows:

$$Z_C = \frac{V}{I} = \frac{1}{2\pi}\sqrt{\mu/\varepsilon}\ \ln\left[\cot\left(\frac{\theta_1}{2}\right)\right] \qquad \text{(12a)}$$

From Fig.2 it can be shown after some algebraic manipulation that:

$$\cot\left(\frac{\theta_1}{2}\right) = \frac{1+\cos\theta_1}{\text{sen}\ \theta_1} = \frac{\sqrt{h^2+r_C^2}+h}{r_C} \qquad \text{(12b)}$$

Expression 12a also corresponds to the characteristic impedance of the vertical cylinder at the height h, as shown in Fig. 2a. Then while for a conical line Z_C remains constant, for a vertical cylinder it will change along its height.

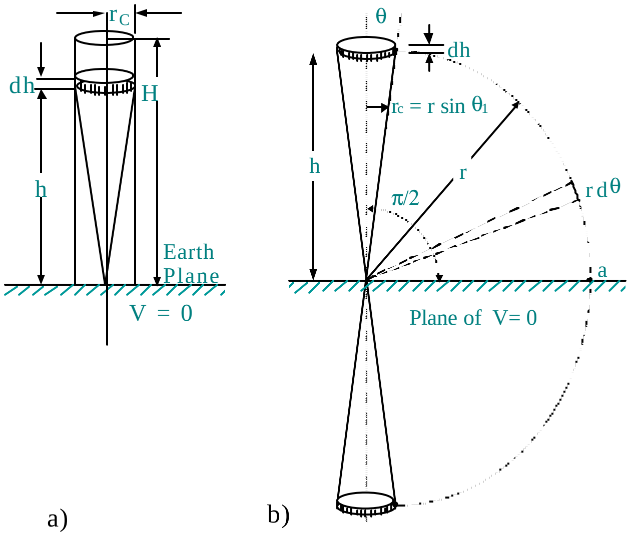

Fig. 2.- a) Vertical Cylinder above an Ideal Ground Plane, b) conical line representation.

VI. MUTUAL IMPEDANCE OF TWO VERTICAL CONDUCTORS

Consider now the two parallel columns shown in Fig. 3 along with the two differential elements A and B at a height "h" above the earth plane. A and B also are elements of the inverted cones depicted in this figure. The current I_i flowing through element A produces an electrical field E_θ given by (6). The voltage of element B is obtained integrating E_θ from C to B along the circular trajectory of radius r' indicated in Fig. 3:

$$V_j = A\sqrt{m/e}\; e^{-jb\, r'} \ln\left[\cot\left(q_2/2\right)\right] \qquad (13)$$

The current I_i flowing through element A is already given by expression (11). The mutual characteristic impedance between columns "i" and "j" is thus:

$$Z_C^{ij} = \frac{V_j}{I_i} = \sqrt{\frac{\mu}{\varepsilon}}\,\frac{e^{-j\beta(r'-r)}}{2\pi}\ln\left[\cot\left(\frac{\theta_2}{2}\right)\right] \qquad (14)$$

In the same manner as with (12a) and (12b), one can obtain the following relationship:

$$\cot\left(q_2/2\right) = \left(\sqrt{h^2+d^2}+h\right)/d \qquad (15)$$

VII. LINE PARAMETERS FOR A SYSTEM OF VERTICAL CONDUCTORS

The characteristic impedance matrix for a vertical multi-conductor system is readily obtained with (12) and (14). From basic Electromagnetic theory [11], the geometric inductance matrix for this system $\mathbf{L}_G$ would be related to the surge impedance matrix $\mathbf{Z}_C$ as follows:

$$\mathbf{L}_G = \sqrt{m\,e}\;\mathbf{Z}_C \qquad (16)$$

From this expression and from (12) and (15), the self and mutual terms of $\mathbf{L}_G$ are respectively:

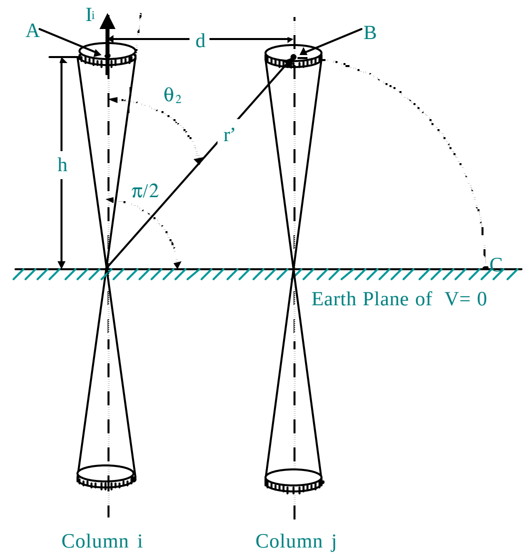

Fig. 3.- Two vertical lines.

$$L_G^{ii} = (m/2p)\,\ln\left(\left(\sqrt{h^2+r_C^2}+h\right)/r_C\right) \qquad (17)$$

and

$$L_G^{ij} = \frac{m}{2p}\exp\left[-jb\left(r'-r\right)\right]\times\ln\left(\frac{\sqrt{h^2+d^2}+h}{d}\right) \qquad (18)$$

To calculate the geometric capacitance matrix for the vertical multi-conductor system, one can use the following well known relationship [11]:

$$\mathbf{C}_G = \mu\,\varepsilon\,(\mathbf{L}_G)^{-1} \qquad (19)$$

Losses due to finite ground conductivity can be accounted for in (17) and (18) by means of the complex depth of images method [6,7]. Let "*p*" be the complex skin depth in the ground [6,7]:

$$p = \frac{1}{\sqrt{jwms}} \qquad (20)$$

where μ and σ are, respectively, the soil magnetic permeability and conductivity. ω is the frequency deemed representative of the surge phenomenon under study. Let now "*h*" be replaced by "*h+p*" in (17) to obtain a modified inductance L^{ii}_M:

$$L_M^{ii} = \frac{\mu}{2\pi}\ln\left[\frac{\sqrt{(h+p)^2+r_C^2}+(h+p)}{r_C}\right] \qquad (21)$$

From basic mathematical transformations it follows that this expression is composed by the geometric inductance and a complex inductance L^{ii}_{CPX}:

$$L_M^{ii} = L_G^{ii} + L_{CPX}^{ii} \qquad (22)$$

with L_G^{ii} as in (17) and L^{ii}_{CPX} as follows:

$$L_{CPX}^{ii} = \frac{\mu}{2\pi} \ln \left[\frac{\sqrt{(h+p)^2 + r_C^2} + (h+p)}{\sqrt{h^2 + r_C^2} + h} \right] \quad (23)$$

The real part of L^{ii}_{CPX} is related to L_E^{ii} as follows:

$$L_E^{ii} = \mathbf{Re} \left\{ L_{CPX}^{ii} \right\} \quad (24)$$

The imaginary part of L^{ii}_{CPX} is related to R_E^{ii} as follows:

$$R_E^{ii} = -w \; \mathbf{Im} \left\{ L_{CPX}^{ii} \right\} \quad (25)$$

When the frequency becomes high, the generalized inductance given by (23) becomes small and in the limit when $\omega \to \infty$ it is zero, as it should be since p is zero and no field penetration in the earth occurs. If the earth is assumed to be perfectly conducting p is zero again and the earth return inductance is zero likewise. In both cases only the contribution of the geometric inductance given by (17) remains. When $\omega \to 0$, p becomes infinite and a magnetoestatic approach should be applied. Since the earth is non-magnetic at low enough frequencies it should have no influence on the magnetostatic field and it should be the same field as that of the wire current in free space.

In the same form as before, "h" is replaced by "$h+p$" in (18) to yield a modified mutual inductance L^{ij}_M:

$$L_M^{ij} = \frac{\mu}{2\pi} \left(\frac{e^{-j\beta r'}}{e^{-j\beta r}} \right) \ln \left[\frac{\sqrt{(h+p)^2 + d^2} + (h+p)}{d} \right] \quad (26)$$

A geometrical mutual inductance L^{ij}_G, as given by (18), can be extracted from (26) to obtain the following complex mutual term:

$$L_{CPX}^{ij} = \frac{\mu}{2\pi} \left(\frac{e^{-j\beta r'}}{e^{-j\beta r}} \right) \ln \left[\frac{\sqrt{(h+p)^2 + d^2} + (h+p)}{\sqrt{h^2 + d^2} + h} \right] \quad (27)$$

The real part of L^{ij}_{CPX} yields L^{ij}_E:

$$L_E^{ij} = \mathbf{Re} \left\{ L_{CPX}^{ij} \right\} \quad (28)$$

The imaginary part of L^{ij}_{CPX} is related to R^{ij}_E as follows:

$$R_E^{ij} = -\omega \; \mathbf{Im} \left\{ L_{CPX}^{ij} \right\} \quad (29)$$

Diagonal matrices L_C and R_C account for the penetration of electromagnetic fields inside the columns. The following high frequency expression is used to evaluate the diagonal elements of these matrices [5,7]:

$$jw \, L_C^{ii} + R_C^{ii} = \frac{\sqrt{jwm r}}{2p \; r_C} \quad (30)$$

where r_c is the equivalent radius of the i-th vertical column and ρ is the resistivity of the column material.

There are several formulas, which are summarized in [13], for calculating the characteristic impedance of a vertical conductor. All of them work well when $h>>r$. However, near the bottom of the line they fail, but the CIGRE formula. The formulas proposed here allow calculating the characteristic impedance in the case of small values of h where $h < r$.

VIII. COLUMNS GROUPING

Considering the columns of a truss module as a multi-conductor system, the following frequency domain line equations can be applied:

$$-d\mathbf{V}/dx = \mathbf{Z}\,\mathbf{I} \quad \text{and} \quad -dI/dx = \mathbf{Y}\,\mathbf{V} \qquad (31a,b)$$

where $\mathbf{Z=jwL+R}$ and $\mathbf{Y=jwC_G}$. Let V_1, V_2, , V_n be the elements of $\mathbf{V}$ and I_1, I_2, , I_n the elements of $\mathbf{I}$, with "n" the number of columns in the truss module. By assuming that:

$$dV_1/dx = \ldots = dV_n/dx = dV/dx \quad (32)$$

$$I = I_1 + I_2 + \ldots + I_n \quad (33)$$

one can apply Kron's reduction [10] in (31a), thus obtaining:

$$-dV/dx = Z_{eq}\, I \quad (34)$$

From Z_{eq} one can extract R and L as follows and apply these values to time domain equation (1a):

$$Z_{eq} = R + jw\, L \quad (35)$$

In a similar manner from equation (31b), on assuming (32) and (33), one can obtain:

$$-dI/dx = Y_{eq}\, V \quad (36)$$

with:

$$Y_{eq} = j\omega\, C_{eq} = j\omega \sum_{i,\,j=1}^{n} C_G^{ij} \quad (37)$$

The obtained C_{eq} corresponds to C of equation (1b).

There are cases for which the reduction to a single line can be attained applying conductor bundling techniques [10]. These cases correspond to columns being evenly distributed on a circumscribing circle whose radius is much smaller than the height of the horizontal truss module.

IX. SIMULATION OF A FIELD EXPERIMENT

The previously described modeling technique is applied as follows in the simulation of a field experiment reported in [1] . It consisted in the injection of a step of current into one of the ground wire arms of the tower depicted in Fig. 1a.. The voltages at the insulator strings were then recorded. Fig. 4 shows the diagram for the experiment's set up with tower 7 being the one in which the current step was injected. Note in this figure that the ground wires are solidly connected to towers 5,7 and 8 and are insulated from tower 6. Phase conductors are solidly connected to tower structures 5 and 8 and are insulated from towers 6 and 7 through their insulation strings.

Fig. 5 depicts the model employed here in the simulation of the current injection experiment. The excitation is represented by an ideal current source that delivers the 3 A stepwave shown in Fig. 6. This source is in parallel with a 0.004 S admittance. Tower Nr. 7 is modeled as a network of eleven non uniform lines, as it is shown in Fig. 1b. The data on Table I are assumed here to generate the line parameters of these segments.

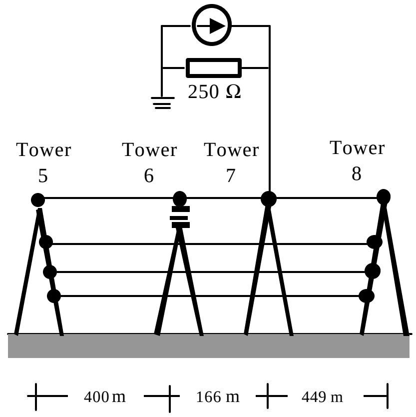

Fig. 4.- Experimentset up diagram.

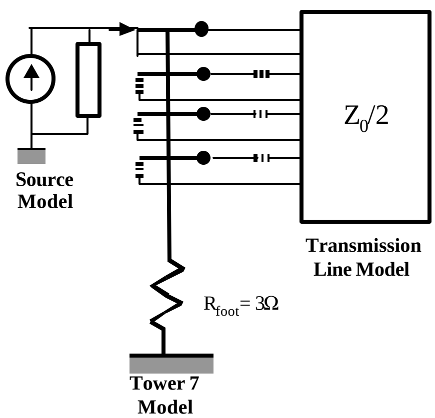

Fig. 5.- Model for experiment’s set up.

TABLE I.- Data assumed in the modeling of tower 7

Tower material:	Iron
Tower resistivity	$9.09 \times 10^{-7} \Omega$m
Tower magnetic permeability:	$4\pi \times 10^{-4}$ Hy/m
Ground resistivity:	30 Ωm
Main body.	
Number of columns	4
Column equivalent Radius.	0.1 m
Maximum distance between columns:	10.7 m
Minimum distance between columns:	3.5 m.
Tower arms.	
Number of columns	4
Column equivalent Radius.	0.05 m
Maximum distance between columns:	3.5 m
Minimum distance between columns:	0.0 m
Phase arm length:	7.5 m
Ground wire arm length:	8.5 m

As for the transmission line (*i. e.*, the phase and ground conductors), this is represented by a matrix of impedances that corresponds to one half the line’s surge impedance Z_0 [12]. Data of the ground wires are: diameter of 1.57 cm, medium height of 72.2 m and horizontal distance to tower center of 10.4 m. For the phase conductors the data are: four conductors of 3.84 cm of diameter per bundle, bundle radius of 70.7 cm, horizontal distances to tower center of 10 m and medium heights of 59.4, 46.7 and 34.0 meters for the upper, middle and lower phases, respectively. Notice that this line representation neglects the effects of towers 5, 6 and 8. The reflections from tower 6 are negligible and the reflections from the solid connections to towers 5 and 8, will arrive at tower 7 after 3 μs. One additional detail concerning the impedance matrix representing the line is that its off-diagonal elements are provided with delay operators. The effect of these operators is to provide delay times that account for the time required by a disturbance in one conductor to exert an influence on another parallel conductor. The delay times correspond to the distance between conductors divided by the light speed.

Fig. 7 shows both, the simulated and the measured waveforms at the insulator string on the upper arm which is at the same side as the ground wire arm with the current injection. Note that the agreement between the two waveforms is very satisfactory. Figs. 8 and 9 show respectively the middle and lower insulators voltage waveforms.

X. CONCLUSIONS

A technique to construct transmission tower models for lightning performance analysis has been presented in this paper. This technique has been based mostly on standard line theory and is applicable to a large class of truss towers. Single-phase line representation of tower arms has been obtained using well known procedures [5,7,10]. As for the single-phase line representation for the tower body, formulas for vertical line parameters have been developed and provided here.

The proposed technique is applied in the simulation of a field experiment reported in [1]. The agreement between field measurements and simulations has been found satisfactory. Since the proposed methodology has been based on conventional line concepts, these authors consider it very attractive for practical analysis.

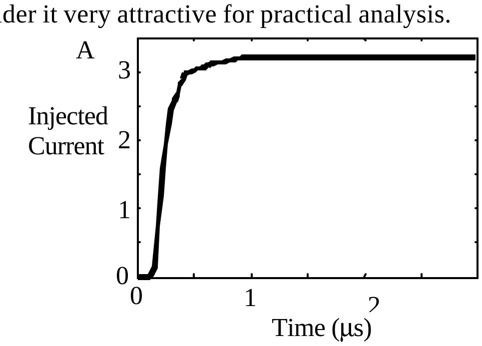

Fig. 6.- Injected current waveform.

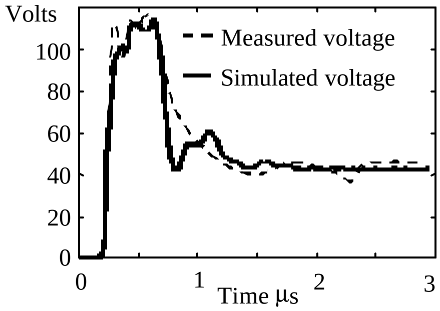

Fig. 7.- Comparison between measured and simulated voltage waves at the upper insulator string.

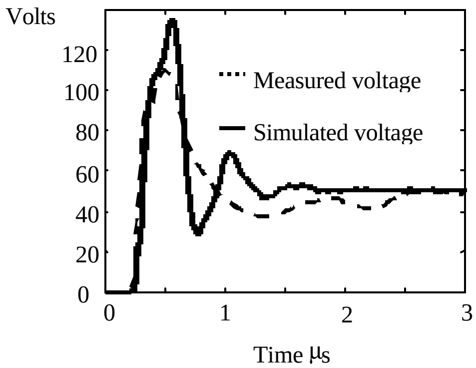

Fig. 8.- Comparison between measured and simulated voltage waves at the medium insulator string.

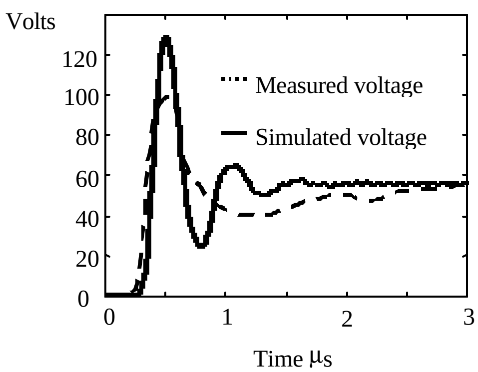

Fig. 9.- Comparison between measured and simulated voltage waves at the lower insulator string.

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