Modeling of Soil Ionization for Calculation of the Transient Response of Grounding Electrodes

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Abstract –**The main difficulty encountered in lightning protection studies is the calculation of the transient response of grounding electrodes. Soil ionization might occur in several cases of lightning strikes, making necessary the appropriate modeling of the soil ionization phenomenon. In this paper, an attempt to approximate the response of a single ionised electrode is made. The electrode is modeled in two ways, using a lumped and a distributed parameter approach. Results are contrasted to experimental data and general type formulae are presented.**

Keywords: Grounding electrode, Soil Ionisation, Critical Electric Field Strength, EMTP.

I. INTRODUCTION

Modelling of soil ionisation, is very important for calculation of the transient response of a grounding electrode striken by lightning, although when it is neglected [1,2] the worst case results are provided. It occurs, when large current densities are injected in the electrode, and emanate from its surface to the soil. When the critical electric field strength E_c exceeds a particular value, breakdown of the soil occurs. As this happens, the electrode is surrounded by a cylindrical corona-type discharge pattern, which augments its effective radius and makes easier the dispersion of the current from its surface to the earth. Soil resistivity of the surrounding soil decays during ionization phase and recovers at the de-ionization phase. Attempts made in the past, in order to model soil ionisation, can be divided in two categories :

a) Methods considering the electrode having variable diameter. The electrode is considered having increasing dimensions during the ionisation phase, and being buried in non-ionized soil. According to this model, the electric field in the ionised region is null, as if it was short-circuited with the electrode. This model is far from corresponding to the actual physical phenomenon, but it can be easily realised both in EMTP simulations[14] and in simplified calculation formulae.

b) Those based in the variable resistivity approach. According to this approach the earth surrounding the electrode has an exponentially decreasing resistivity during the ionization phase, that recovers exponentially, during the de-ionisation phase. This model, although widely used [3-6] does not consider the variation of the electric field intensity inside the ionised region as it is mentioned in [7]. A beneficial variation of the above method is presented in [7] where the ionised electrode is assumed to be surrounded by equipotential shells. The formulae used to calculate the soil resistivity in the surrounding region are suitably modified.

In this paper two models are proposed for modeling the ionisation process, and their performance is investigated by comparison to actual field measurements. The first is a lumped parameter circuit model which is implemented in EMTP. Non linear resistance elements are used to represent the reduction of the shunt conductance. Two different kinds of such elements one for the ionisation phase and one for the de-ionisation phase have been incorporated in the model, and they are energised using suitable switching arrangements. Effect of segmentation of the electrode is investigated.

The second model is obtained from the first one by division into infinite number of lumped parameters circuits. Consequently it is a distributed paramenters model, which is used to write hand calculation formulae for determination of the impulse V_{max}/I_{max} ratio.

The two models are contrasted to experimental results [8]-[10] showing a good agreement. Techniques proposed in this paper are beneficial in terms of simplicity when constructing the model, and in terms of computational speed.

II. THEORETICAL BACKGROUND

A. Parameters of the Soil Model: One critical parameter that needs to be correctly determined is the critical field strength E_c of the soil. This is one of the most difficult parameters to predict with any degree of confidence. In this paper the values of critical field strength are given from experimental data, and they are contrasted to those obtained from a widely used formula proposed by Oettle[11] where the soil is considered having a breakdown strength E_c(kV/m) expressed as a function of soil conductivity σ as follows:

$$E_c = 241 \cdot \sigma^{-0.215} \tag{1}$$

B. Description of soil ionisation mechanism

As the increased impulse current is injected in the electrode, leakage current from the electrode to the ground, also increases. Soil ionization occurs when the current density which is branching off through the rod exceeds a critical value. The resistivity of the ionisation zone is much lower than the initial value of the soil resistivity. The zone dimensions increase as the injected current increases. The current density at a surface $S(r_d)$ surrounding an ionised grounding rod at a distance r_d will be :

$$J = \frac{I}{S(r_d)} \tag{2}$$

Followingly the voltage across a shell element is :

$$E=\frac{I\cdot\rho}{S(r_d)} \quad (3)$$

Where ρ is the resistivity of the zone around the electrode at distance r_d .Breakdown of the soil occurs when:

$$\frac{I\rho}{S(r_d)}\geq E_c \quad (4)$$

Consequently, the critical current density for soil ionisation to start is :

$$J_c\geq\frac{E_c}{\rho_0} \quad (5)$$

Where ρ_0 is the initial soil resistivity. Knowing that the area that surrounds the electrode has in the general case a shape as shown in fig.1a, and considering that condition (5) applies, the radius of the ionised channel is calculated, by solving (4):

$$r=\frac{\ell}{2}\left(-1+\sqrt{1+\frac{2\rho_0 I_{leakage}}{\pi\ell^2 E_c}}\right) \quad (6)$$

where $I_{leakage}$ is the total current that emanates from the outer surface of the zone to the ground, at the particular time instant.

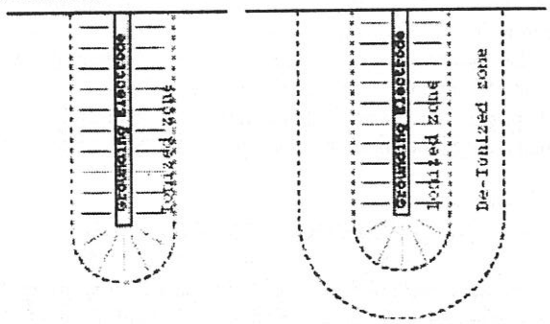

Fig.1a Fig.1b

Fig. 1.(a)Ionisation zone before peak current injection (b) Ionisation and De-Ionisation zones after peak current injection

Assuming that every point in the soil inside the ionised region has the same resistivity value at a particular time instant, the resistance of the electrode can be found by integration in the ionisation phase:

$$R=\frac{\rho_d}{2\pi l}\int_{r_0}^{r_d}\left(\frac{1}{x}-\frac{1}{x+l}\right)dx+\frac{\rho_0}{2\pi l}\int_{r_d}^{\infty}\left(\frac{1}{x}-\frac{1}{x+l}\right)dx$$

or (7)

$$R=\frac{\rho_0}{2\pi\ell}ln\left(1+\frac{\ell}{r_d}\right)+\frac{\rho_d}{2\pi\ell}ln\left\{\left(1+\frac{\ell}{r_0}\right)\Big/\left(1+\frac{\ell}{r_d}\right)\right\}$$

A similar expression applies also in case a part of the surrounding soil is recovering to the initial soil resistivity value. In (7), ρ_0 is the initial soil resistivity, ρ_d is the resistivity of the ionized region and r_d is the equivalent radius of the ionization zone. Formulae (7) is used to compare the maximum impulse resistance value to the one obtained from EMTP simulations as it will be shown next.

Formulae that can be used to calculate the soil resistivity as a function of time have been proposed by Liew and Darveniza[9], and by M.T.Correia de Barros[7]. These formulae are used in this paper.

Non-linear resistance of the electrode is also estimated by Hoki and Mita[12] who give a relation versus injected current

C. Calculation Methodology

Simulations are carried out using the measured current waveforms as energisation sources. Injection current has a double exponential waveform $I=I_0(exp(at)-exp(bt))$ where a and b are negative constants.

The soil in the area that surrounds the electrode, is divided in shells of elementary thickness. Each of the shells is modeled using a time varying resistor. This can be explained by the fact that the resistivities of the ionisation and de-ionisation zones are changing with time. The resistance of an elementary shell of thickness dr at a distance x from the axis of a grounding rod, is calculated according to the formula given in [9]:

$$dR=\frac{\rho_{ion}(t)dr}{2\pi(x^2+x\ell)}=\frac{\rho_{ion}(t)dr}{2\pi\ell}\left(\frac{1}{x}-\frac{1}{x+\ell}\right) \quad (8)$$

Accurate modeling of the elementary shell implies that a suitable capacitance is added in parallel to the resistor. It will be equal to $dC=\varepsilon\rho\cdot dG$ where $dG=dR^{-1}$.

The thickness of each shell is influencing the accuracy of the methodology. In order to determine the number of shunt elements that will be used to construct the circuit model, a first estimation of the maximum radius of the ionisation zone is done using (6) where the maximum total current dispersion to the ground is used instead of $I_{leakage}$.

Inside each elemental shell, it is assumed that soil is homogeneous at a particular time instant, and soil resistivity is equal everywhere. It is also assumed that in all points inside this shell, ionisation starts at the same time instant i.e. when the dispersed current density exceeds the critical value in the outer shell surface.

1) Parameters of the electrode model: The grounding electrode is modeled using a series resistance R_e and a series inductance L_e . The shunt elements of the model will be initially a conductance G_e and a capacitanc C_e, but will change as soon as ionisation or de-ionisation occurs. The values of R_e L_e C_e G_e are given from known formulae[13]. The propagation effect of ionisation along the electrode, is not considered. For this reason, series resistance R_e and series inductance L_e, are considered not to be affected.

2) Circuit model of the electrode : The circuit model used has some common points with the model used to simulate corona phenomena in transmission lines [16].

The effect of segmentation of the electrode is important as it results in different accuracy in most of the calculation cases. When short electrodes under an impulse strike are simulated, they can be considered concentrated, and the voltage when they are not-ionized can be taken constant along them, even under the injection of very fast impulse waveforms. Consequently they can be calculated using only one or two elementary segments.

Soil ionisation and deionisation processes are incorporated in the circuit model of the grounding electrode, using three branches which are alternatively energised as the different phases of the phenomenon are diagnosed by a TACS sub-network which controls the state of the switches. The lumped circuit model of an elementary segment of the electrode, suitably modified for ionization studies is shown in fig.2.b.

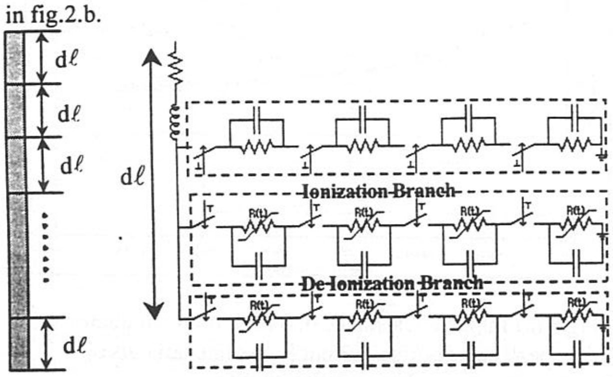

Fig. 2.(a)Division of a grounding electrode into segments of length d ℓ, (b) Circuit model of an elementary segment for ionisation studies.

3) Distributed model of the electrode : A distributed type model of the electrode is needed in order to approximate more accurately the physical phenomenon. Moreover, a simplified solution is used to provide a simple calculation formula for the estimation of the impulse impedance of an ionised grounding electrode. Calculation of the impulse impedance of an ionised electrode, using its distributed model is done in the following stages :

3.a.) Non-linear time dependent shunt resistors are replaced by lumped resistances R_{ion} and inductances L_{ion}. These resistances and inductances are suitably chosen in order to match the resistance characteristics for the particular waveshape of the injected current. The de-ionisation phase is simulated using a shunt RC branch with parameters evaluated in a similar way to the one used for determination of the parameters during the ionisation phase.

3.b.) Differential equations are written for the total network of lumped elements that simulates the grounding electrode, assuming that it is divided in n elementary segments (Appendix I).

3.c.) The limit of the solution of differential equations when n tends to infinity is calculated. Formulation of the expressions of current and voltage distributions is shown in Appendix I, and they are as follows:

$$I(x,t)=\frac{cosh(\gamma_\alpha(x-2\ell))-cosh(\gamma_\alpha x)}{cosh(2\gamma_\alpha \ell)-1}e^{at}-\frac{cosh(\gamma_\beta(x-2\ell))-cosh(\gamma_\beta x)}{cosh(2\gamma_\beta \ell)-1}e^{\beta t} \quad (9)$$

$$V(x,t)=\frac{sinh(\gamma_\alpha x)-sinh(\gamma_\alpha(x-2\ell))}{cosh(2\gamma_\alpha \ell)-1}\cdot\frac{\gamma_\alpha}{Z_{p_\alpha}}\cdot e^{at}-\frac{sinh(\gamma_\beta x)-sinh(\gamma_\beta(x-2\ell))}{cosh(2\gamma_\beta \ell)-1}\cdot\frac{\gamma_\beta}{Z_{p_\beta}}\cdot e^{\beta t}+const\cdot A$$

where $\gamma_\alpha=\sqrt{Zs_\alpha Zp_\alpha}$, $\gamma_\beta=\sqrt{Zs_\beta Zp_\beta}$

$$Zs_\alpha=R_e+aL_e,\; Zp_\alpha=\frac{1}{R_{ion}+aL_{ion}}+aC_e \text{ and}$$

$$const=\frac{sinh(\gamma_\alpha\cdot 2\ell)}{2cosh(2\gamma_\alpha\ell)-2}\cdot\frac{\gamma_\alpha}{Z_{p_\alpha}}\cdot e^{a}-\frac{sinh(\gamma_\beta\cdot 2\ell)}{2cosh(2\gamma_\beta\ell)-2}\cdot\frac{\gamma_\beta}{Z_{p_\beta}}e^{\beta}$$

$$A=e^{t\cdot\frac{-C_eR_{ion}+\sqrt{C_e^2R_{ion}^2-4C_eL_{ion}}}{2C_eL_{ion}}}+e^{t\cdot\frac{-C_eR_{ion}-\sqrt{C_e^2R_{ion}^2-4C_eL_{ion}}}{2C_eL_{ion}}}$$

Formulae giving R_{ion}, L_{ion} in order to match the experimental results of an horizontal electrode are:

$$R_{ion}=\left[\left[a+b\ln\left(I_{peak}\right)\right]/\left(R_{ss}\rho_o\right)\right]\cdot d\ell/\ell,$$

$$a=.500855\,\rho_0-4.545102\,\pi R_{ss}*2$$

$$b=.1214931\,\rho_0+.62281\,\pi R_{ss}*2 \quad (10)$$

$$L_{ion}=R_{ion}\cdot 2E-7$$

where R_{ss} is the steady state resistance

The capacitance and resistance $C_{de\text{-}ion}$ and $R_{de\text{-}ion}$ of the shunt branch that is used to simulate the electrode during the de-ionisation phase, depend on the final simulation time. Using the expressions (9), maximum current and maximum voltage are analytically determined and lead to analytical expression for the impulse impedance of the ionised electrode $Z=V_{max}/I_{max}$.

III. APPLICATION RESULTS

A. Results from the proposed circuit model:

The proposed methodology has been applied to the cases illustrated in [9].

A 0.61m long grounding rod of 0.15m diameter in 50Ohm-m soil is examined. *Ec* is equal to 110kV/m, much lower than the value calculated using (1) which is 558kV/m. Parameters of the lumped circuit model are evaluated : R_e=5.775E-7Ohm, L_e=3.028E-4mH, C_e=2.734E-4 μF, G_e=0.030811mho.

Plots of the response of the electrode are shown in the following figures:

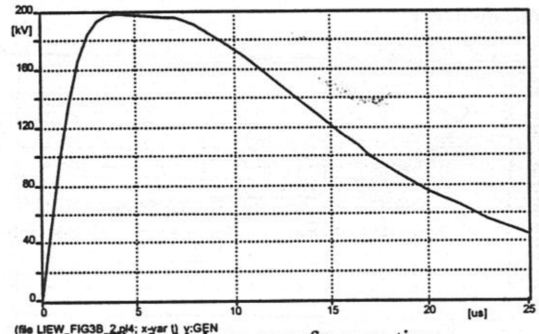

Fig.3: Voltage waveform vs. time

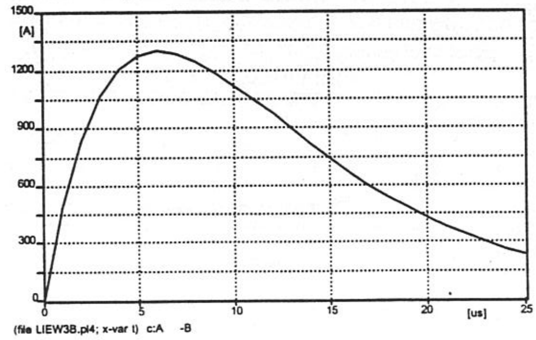

Fig.4: Injected Current waveform vs. time

A good aggreement is observed between EMTP results and those in[9]. Resistance calculated using (7) is 14.5 Ω and 15.4 Ω from EMTP.

B. Results from the distributed electrode model:

Validation of the results obtained from the distributed electrode model differential equations are contrasted to experimental results in terms of comparing impulse resistance. Here the impulse resistance is defined as $Z=V_{max}/I_{max}$

An 1.5m long ground rod has been examined. The impulse impedance of the rod is calculated and contrasted to experimental results [10]. Injected current peak values vary from 5kA to 15 kA. The rod is buried in 450Ohm-m soil and the radius is 0.007m. Voltage waveforms produced from simulations are contrasted to measured data in fig.5.

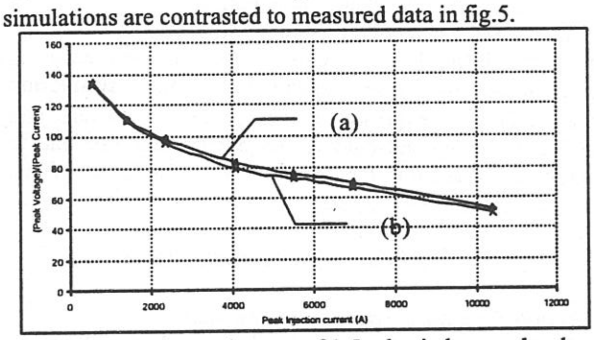

Fig.5 : (a)Impulse resistance of 1.5m buried ground rod vs. peak energisation current (experimental curve), (b)simulation results

A buried counterpoise is calculated next. In the three cases examined, the soil resistivity is 436 Ωm 505 Ωm and 522 Ωm and the electrode length is 8.15m, 17m and 34m correspondingly. Results from measurements are contrasted to experimental data [8]as shown in figs 6,7 and 8. E_c varies from 600kV to 1000kV and it is 890-925 kV/m according to (1).

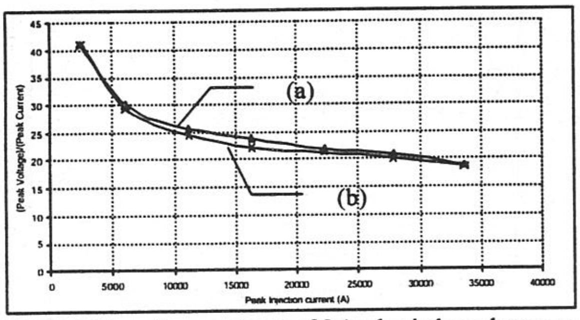

Fig.6 : (a)Impulse resistance of 8.1m buried conductor vs. peak energisation current (experimental curve), (b)simulation results

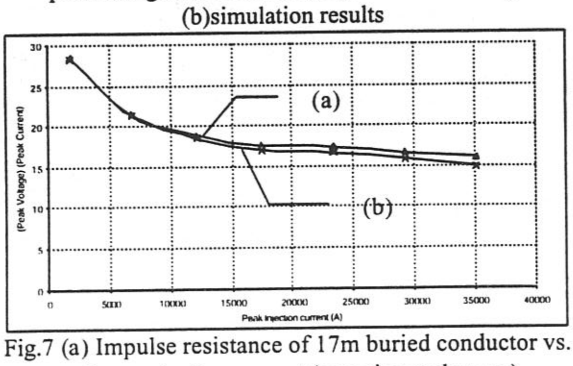

Fig.7 (a) Impulse resistance of 17m buried conductor vs. peak energisation current (experimental curve), (b)simulation results

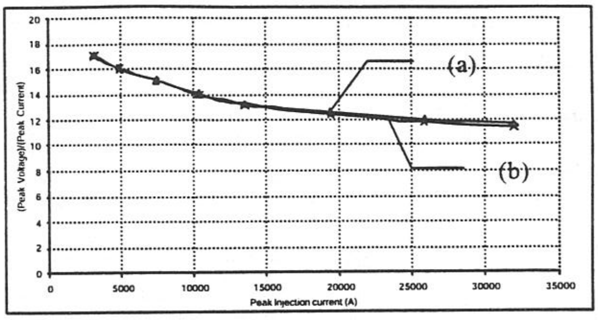

Fig.8 (a) Impulse resistance of 34m buried conductor vs. peak energisation current (experimental curve), (b)simulation results

A good agreement can be observed between measured and experimental results in all cases examined.

In the above figures only the impedance values $Z=V_{max}/I_{max}$ are contrasted to measured results. A good agreement is also observed, between the waveshapes of measured data and simulation results. In all the above examined cases current has a 3/7 μs waveform and voltage has 2.5/7 μs one.

IV. DISCUSSION OF THE RESULTS

Results from the two models presented in this paper show a good agreement with measured data. However, the circuit model assumes a segmentation of the electrode which introduces an error within acceptable limits. Additional error is produced due to the trapezoidal rule of integration that is used to handle the differential equations which describe the total circuit of the electrode. When the distributed model is used, mathematical accuracy and simplicity is achieved. The error introduced in the previous case due to segmentation of the electrode does not further exist. The only cause of error is the fact that the nonlinear resistance that dispersion current faces when leaks into the soil, is replaced by a shunt branch with lumped R-L linear elements. The nonlinearity of ionisation phenomenon is introduced here by the response of inductive element.

An advantage of the circuit mo del is that it is suitable for incorporation to larger network studies using EMTP.

The disadvantage of using a formulae that gives impulse resistance R(i) versus injection current[12] is that the two phases ionisation and de-ionisation are not clearly distinguished. According to this formulae resistance R(i) is the same when the same amount of current disperses into the soil, either during ionisation or de-ionisation phase. Consequently, the formula does not describe accurately the real phenomenon. Improvement on this formula has been introduced by Sekioka [18], considering the dependency on the injected amount of energy.

It is suggested that methodologies and formulae that allow the distinction of ionisation and de-ionisation phases are used. Those are formulae of Liew and Darveniza[9], De Barros[7] and Sekioka [18].

IV. CONCLUSIONS

Modeling of soil ionisation process of major importance when lightning response of grounding systems is examined. Various authors have proposed methodologies for analysis of ionisation phenomenon. In the most accurate of these methodologies two phases are distinguished, the ionisation and the de-ionisation phase.

In this paper, a circuit and a distributed type model are applied in order to calculate the transient response of an electrode in ionised soil. Both models show good aggreement with experimental results. Although the lumped parameters circuit model of the electrode is preferred when the electrode is part of a simulated electrical network, the distributed type model is simpler and mathematically accurate.

V. REFERENCES

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VI. Appendix 1.

The grounding electrode is considered having distributed parameters along its length. A corresponding lumped parameters network of the electrode during the ionisation phase is shown in fig.A.1. This network is constructed connecting in series each pi-circuit that represents one elementary segment. Series resistance R_e inductance L_e and shunt capacitance C_e of the electrode, are calculated as if ionisation is neglected. The lumped parameters R_{ion} and L_{ion} represent the non-linear resistance that leakage current finds when it disperses into the soil during ionisation.

A similar type network is modeling the electrode during the de-ionisation phase and it is shown in fig.A.2.

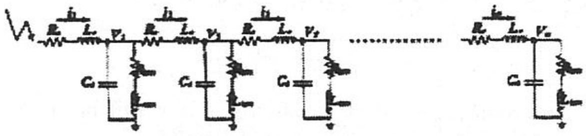

Fig. A.1. Lumped parameters electrode model during the ionisation phase

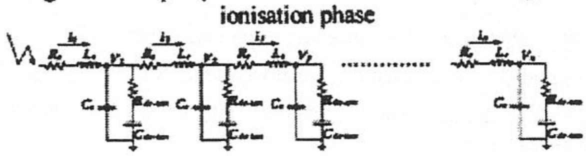

Fig. A.2. Lumped parameters electrode model during the de-ionisation phase

Differential equations are firstly formed for each of the pi-circuits that corresponds to an elementary segment of the electrode. Giving an example, at the start of the circuit model shown in fig.A.1. it is :

$$i_1 = i_2 + \left(\frac{1}{R_{ion} + L_{ion}D} + C_e D \right) V_1 \qquad \text{(A.1)}$$

$$V_1 = V_2 + (R_e + L_e D) i_2$$

Where D=d/dt is the differential operator. Writing similar differential equations at any point of the lumped parameters network model, a set of differential equations is obtained. For known current source of double exponential waveform, this set accepts the partial solution

$$i_k = C_{\alpha k} e^{at} - C_{\beta k} e^{\beta t} \qquad \text{(A.2)}$$

$$V_k = \frac{C_{a,k} - C_{a,k+1}}{\left(\frac{1}{R_{ion} + L_{ion}\alpha} + C_e \alpha\right)} e^{at} - \frac{C_{\beta,k} - C_{\beta,k+1}}{\left(\frac{1}{R_{ion} + L_{ion}\beta} + C_e \beta\right)} e^{\beta t}$$

at any middle point k.

where $C_{a,k}$ are coefficients determined as :

$$C_{\alpha,k} = \frac{\alpha_1^{k-2}\left(\beta_1^{2n-2} - 1\right) + \beta_1^{k-2}\left(\alpha_1^{2n-2} - 1\right)}{\beta_1^2\left(\beta_1^{2n-2} - 1\right) + \alpha_1^2\left(\alpha_1^{2n-2} - 1\right)} \qquad \text{(A.3)}$$

where $\alpha_1 = \dfrac{p + \sqrt{p^2 - 4}}{2}, \alpha_1 + \beta_1 = p,$

$$p = 2 + (R_e + \alpha L_e)\left(\frac{1}{R_{ion} + L_{ion}\alpha} + \alpha C_e\right)$$

and $C_{b,k}$ are coefficients determined by substitution of a with â to (A.3). The above partial solution needs to be added to the solution of the homogeneous differential equations in order to be complete. However, in the examined cases where electrode lengths are shorter than 40-50m the latter can be neglected

Expressions of current and voltage distributions along the electrode vs. time, are obtained when the limit of the number of the elementary segments tends to infinity at the expressions (9) of the solution.

Using the expressions (9), maximum current and maximum voltage are analytically determined and an analytical expression for the impulse impedance of the ionised electrode $Z=V_{max}/I_{max}$ is obtained.

VI. Appendix II.

Formulae giving the resistivity of the soil in the area that surrounds the ionised electrode are [9]:

$\rho = \rho_0 e^{-\frac{t}{\tau_1}}$ in the ionisation phase, where τ_1 is the ionization time constant, in most cases equal to $2.0\mu s$. In the de-ionization phase the resistivity of the soil recovers following the law:

$$\rho = \rho_i + (\rho_0 - \rho_i)\left(1 - e^{\frac{-t}{\tau_2}}\right)\left(1 - \frac{J}{J_c}\right)^2$$

where τ_2 is the de-ionization time constant and ρ_i is the resistivity of the shell when the current density is equal to J_{critic} during the decay period.

Proposed formulae by De Barros [7] are a modification to those proposed by Liew and Darveniza. Resistivity of an ionized shell inside the ionization region is :

$\rho_k = \dfrac{E_c}{I} A_k$ where A_k is the outer surface of the shell, and I the current injected from the surface of the electrode to the ground. Resistivity of a shell in the de-ionization region will be equal to :

$$\rho_k = \rho_{k_i} + (\rho_0 - \rho_{k_i})\left(1 - e^{\frac{t}{\tau_1}}\right)\left(1 - \frac{E_k}{E_c}\right)^2$$

where τ_1 is here the de-ionization time constant E_k is the electric field related to shell k, and ρ_{ki} is the resistivity of the shell k at electric field intensity E_{critic} during the decay period .