A New Finite Difference Time Domain Scheme for the Evaluation of Lightning Induced Overvoltages on Multiconductor Overhead Lines

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Abstract – **LEMP to transmission line coupling equations can be dealt with either in the frequency domain or in the time domain. A time domain approach allows handling in a more straightforward way non-linearities which appear when considering corona effect, and/or when protective devices such as surge arresters are present. This is the approach proposed by Agrawal et al. to solve their transmission line coupling equations. In particular, *Agrawal* et al. proposed a 1st order point centered Finite-Difference Time Domain (FDTD) integration scheme. In this paper, we propose a 2nd order FDTD scheme for solving the Agrawal coupling equations. The algorithm applies to multiconductor lines above a frequency-dependent lossy ground, with multiple grounding of shielding wires. 1st and 2nd order FDTD techniques are compared. It is shown that the proposed 2nd order technique leads to more stable numerical results when considering frequency-dependence and/or non linearities. The developed 2nd order FDTD algorithm for the analysis of overhead multiconductor lines illuminated by an external electromagnetic field is also interfaced with EMTP96.**

Keywords: FDTD second order, Lightning-induced voltages, Numerical Methods, EMTP interface.

I. INTRODUCTION

Most studies on lightning-induced voltages on overhead power lines, use a direct time domain analysis because of its straightforwardness in dealing with insulation coordination problems and its ability to handle non-linearities, which arise in presence of protective devices such as surge arresters, or corona phenomenon.

One of the most popular approaches to solve the transmission line coupling equations in time domain is the finite difference time domain (FDTD) technique [1]. Such a technique was used indeed by *Agrawal* et al. in [2] when presenting their field-to-transmission line coupling equations. This algorithm has been later extended by the authors in [3] to the case of an overhead line above a frequency-dependent lossy ground and then to a line with periodical groundings of shielding wires [4]. In the above-mentioned publications, partial time and space derivatives were approximated using the first-order FDTD technique.

In this paper, we propose an integration scheme of the *Agrawal* et al. transmission line coupling equations based on the 2nd order FDTD technique and give the relevant equations. The proposed scheme is translated into a computer code which allows for the treatment of multiconductor lines above a lossy ground characterized by a frequency-dependent impedance. Presence of periodical groundings along the line as well as corona effect is also dealt with in the new algorithm. The relevant equations and a comparison between the results obtained from the new 2nd order FDTD program and the 1st order one are presented.

The developed computer program has been interfaced with EMTP96. A brief description of such an implementation, which allows to deal with voltages induced by nearby lightning electromagnetic fields (LEMP) on distribution systems characterized by complex configurations, is also given.

II. FDTD 2nd ORDER INTEGRATION SCHEME FOR TRANSMISSION LINE COUPLING EQUATIONS

A. Case of a single-conductor line above an ideal ground

The second order finite difference technique used in this paper is based on the Lax-Wendroff algorithm [5]. In [6], this algorithm is applied to the classical transmission line equations excited by lumped excitation sources. We here present an extension of this algorithm to take into account distributed sources due to the action of an external electromagnetic field, using the Agrawal et al. coupling model.

First, let us consider the simple case of a single-conductor overhead line above an ideal ground. For this case the *Agrawal* et al. field-to-transmission line coupling equations read

$$\frac{\partial v^s(x,t)}{\partial x} + L'\frac{\partial i(x,t)}{\partial t} = E_x^e(x,h,t) \qquad (1)$$

$$\frac{\partial i(x,t)}{\partial x} + C'\frac{\partial v^s(x,t)}{\partial t} = 0 \qquad (2)$$

where:

- $E_x^e(x,h,t)$ is the horizontal component of the incident electric field along the *x* axis at the conductor's height *h*;
- *L'* and *C'* are respectively the inductance and the capacitance per unit length of the line;
- $i(x,t)$ is the induced current;

- $v^s(x,t)$ is the *scattered voltage*, related to the total voltage $v(x,t)$, by the following expression

$$v(x,t) = v^s(x,t) + v^e(x,t) = v^s(x,t) - \int_0^h E_z^e(x,z,t)dz \tag{3}$$

where:
- $E_z^e(x,z,t)$ is the exciting (or inducing) vertical electric field that can be considered as unvarying in the height range *0<z<h*;
- $v^e(x,t)$ is the incident voltage.

If we differentiate with respect to the x and t variables, the system of equations (1) and (2) can be rewritten as

$$\frac{\partial^2 i(x,t)}{\partial x^2} - L'C'\frac{\partial^2 i(x,t)}{\partial t^2} = -C'\frac{\partial E_x^e(x,h,t)}{\partial t} \tag{4}$$

$$\frac{\partial^2 v^s(x,t)}{\partial x^2} - L'C'\frac{\partial^2 v^s(x,t)}{\partial t^2} = \frac{\partial E_x^e(x,h,t)}{\partial x} \tag{5}$$

Expanding the line current and the scattered voltage using Taylor's series applied to the time variable, and truncating after the second order term yields

$$v^s(x,t) = v^s(x,t_0) + \frac{\partial v^s(x,t)}{\partial t}\Delta t + \frac{\partial^2 v^s(x,t)}{\partial t^2}\frac{\Delta t^2}{2} + \mathrm{O}(\Delta t^3) \tag{6}$$

$$i(x,t) = i(x,t_0) + \frac{\partial i(x,t)}{\partial t}\Delta t + \frac{\partial^2 i(x,t)}{\partial t^2}\frac{\Delta t^2}{2} + \mathrm{O}(\Delta t^3) \tag{7}$$

where $\mathrm{O}(\Delta t^3)$ is the reminder term, which approaches zero as the third power of the temporal increment.

If we substitute the time derivatives in (6) and (7) with the corresponding expressions using equations (1), (2), (4), and (5), we obtain the following second order differential equation

$$v^s(x,t) = v^s(x,t_0) - \frac{\Delta t}{C'}\frac{\partial i(x,t)}{\partial x} + -\frac{\Delta t^2}{2L'C'}\left(\frac{\partial E_x^e(x,h,t)}{\partial x} - \frac{\partial^2 v^s(x,t)}{\partial x^2}\right) + \mathrm{O}(\Delta t^3) \tag{8}$$

$$i(x,t) = i(x,t_0) - \frac{\Delta t}{L'}\left(\frac{\partial v^s(x,t)}{\partial x} - E_x^e(x,h,t)\right) + \frac{\Delta t^2}{2L'C'}\left(\frac{\partial^2 i(x,t)}{\partial x^2} + C'\frac{\partial E_x^e(x,h,t)}{\partial t}\right) + \mathrm{O}(\Delta t^3) \tag{9}$$

In order to represent equations (8) and (9) using an FDTD scheme, we will proceed with the discretization of time and space as follows

$$v^s(x,t) = v^s(k\Delta x, n\Delta t) = v_k^n \tag{10}$$

$$i(x,t) = i(k\Delta x, n\Delta t) = i_k^n \tag{11}$$

$$E_x^e(x,h,t) = E_x^e(k\Delta x, h, n\Delta t) = Eh_k^n \tag{12}$$

where:
- Δx: spatial integration step;
- Δt: time integration step;
- $k = 1,2,\dots, K_{\max}$
- $n = 1,2, \dots, N_{\max}$

In the integration scheme, the scattered voltage and current at time step n, are known for all spatial nodes. Therefore equations (8) and (9) allow us to compute the scattered voltage and current at the time step $n+1$.

The spatial derivatives of the scattered voltage, line current, and horizontal electric field can be written respectively as

$$\left.\frac{\partial v^s(x,t)}{\partial x}\right|_{t=n\Delta t} = \frac{v_{k+1}^n - v_{k-1}^n}{2\Delta x} + \mathrm{O}(\Delta x) \tag{13}$$

$$\left.\frac{\partial i(x,t)}{\partial x}\right|_{t=n\Delta t} = \frac{i_{k+1}^n - i_{k-1}^n}{2\Delta x} + \mathrm{O}(\Delta x) \tag{14}$$

$$\left.\frac{\partial E_x^e(x,h,t)}{\partial x}\right|_{t=n\Delta t} = \frac{Eh_{k+1}^n - Eh_{k-1}^n}{2\Delta x} + \mathrm{O}(\Delta x) \tag{15}$$

On the other hand, the time derivative of the horizontal electric field reads

$$\left.\frac{\partial E_x^e(x,h,t)}{\partial t}\right|_{t=n\Delta t} = \frac{Eh_k^{n+1} - Eh_k^{n-1}}{2\Delta t} + \mathrm{O}(\Delta t) \tag{16}$$

The second order spatial derivatives can be written as

$$\left.\frac{\partial^2 v^s(x,t)}{\partial x^2}\right|_{t=n\Delta t} = \frac{v_{k+1}^n + v_{k-1}^n - 2v_k^n}{\Delta x^2} + \mathrm{O}(\Delta x) \tag{17}$$

$$\left.\frac{\partial^2 i(x,t)}{\partial x^2}\right|_{t=n\Delta t} = \frac{i_{k+1}^n + i_{k-1}^n - 2i_k^n}{\Delta x^2} + \mathrm{O}(\Delta x) \tag{18}$$

Inserting equations (10)-(18) into (8) and (9), we obtain the following 2nd order FDTD scheme

$$v_k^{n+1} = v_k^n - \frac{\Delta t}{C'}\left(\frac{i_{k+1}^n - i_{k-1}^n}{2\Delta x}\right) + -\frac{\Delta t^2}{2L'C'}\left(\frac{Eh_{k+1}^n - Eh_{k-1}^n}{2\Delta x} - \frac{v_{k+1}^n + v_{k-1}^n - 2v_k^n}{\Delta x^2}\right) \tag{19}$$

$$i_k^{n+1} = i_k^n - \frac{\Delta t}{L'}\left(\frac{v_{k+1}^n - v_{k-1}^n}{2\Delta x} - Eh_k^n\right) + \frac{\Delta t^2}{2L'C'}\left(\frac{i_{k+1}^n + i_{k-1}^n - 2i_k^n}{\Delta x^2} + C'\frac{Eh_k^{n+1} - Eh_k^{n-1}}{2\Delta t}\right) \tag{20}$$

It is worth noting that, as opposed to the 1st order point centered scheme, the current and voltage nodes in the 2nd order scheme are coincident. Fig. 1 shows a schematic representation of the spatial discretization of the line.

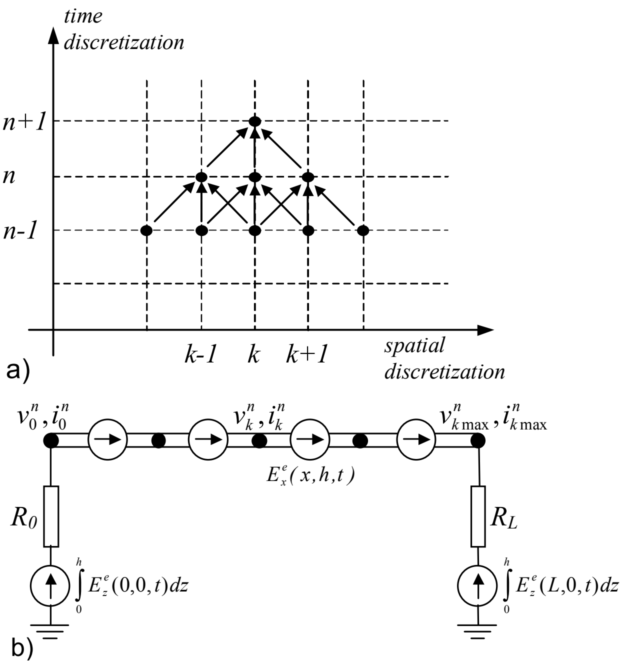

Fig. 1. FDTD 2nd order integration scheme applied to the case of a single-conductor lossless overhead line above a perfectly conducting ground illuminated by an external electromagnetic field. 1a: time and spatial discretization; 1b: schematic representation of the spatial discretization along the line.

The boundary conditions for resistive terminations, can be expressed as follows:

$$v_0^n = -R_0 i_0^n + \int_0^h E_z^e(0,0,t)dz \tag{21}$$

$$v_{k\max}^n = R_L i_{k\max}^n + \int_0^h E_z^e(L,0,t)dz \tag{22}$$

B. Case of a single-conductor line above a lossy ground

We now extend the 2nd order integration scheme previously developed, in order to take into account the presence of a uniform lossy ground, characterized by its conductivity σ_g and its relative permittivity ε_{rg}.

In this case, the *Agrawal* et al. coupling equations become [3]

$$\frac{\partial v^s(x,t)}{\partial x} + L'\frac{\partial i(x,t)}{\partial t} + \int_0^t \xi'_g(t-\tau)\frac{\partial i(x,\tau)}{\partial \tau}d\tau = \\ = E_x^e(x,h,t) \tag{23}$$

$$\frac{\partial i(x,t)}{\partial x} + C'\frac{\partial v^s(x,t)}{\partial t} = 0 \tag{24}$$

in which $\xi'_g(t)$ is the transient ground resistance [7], which can be evaluated using the following analytical expression [8]

$$\xi'_g(t) = \min\left\{ \frac{1}{2\pi h}\sqrt{\frac{\mu_o}{\varepsilon_o \varepsilon_{rg}}}, \right. \\ \left. \frac{\mu_o}{\pi \tau_g}\left[\frac{1}{2\sqrt{\pi}}\sqrt{\frac{\tau_g}{t}} + \frac{1}{4}\exp(\tau_g/t)\mathrm{erfc}\left(\sqrt{\frac{\tau_g}{t}}\right) - \frac{1}{4} \right] \right\} \tag{25}$$

in which ε_0 and ε_{rg} are the air and ground permittivity respectively, μ_0 is the air permeability, $\tau_g = h^2\mu_0\sigma_g$ (where σ_g is the ground conductivity) and *erfc* is the complementary error function.

In equations (23) and (24), the contributions from the wire impedance and the ground admittance, which have shown to be negligible for typical power lines [3], are deliberately disregarded.

Following a procedure similar to the case of a lossless line, equations (23) and (24) become

$$\frac{\partial^2 i(x,t)}{\partial x^2} - L'C'\frac{\partial^2 i(x,t)}{\partial t^2} - C'\frac{\partial v'_g(x,t)}{\partial t} = \\ -C'\frac{\partial E_x^e(x,h,t)}{\partial t} \tag{26}$$

$$\frac{\partial^2 v^s(x,t)}{\partial x^2} - L'C'\frac{\partial^2 v^s(x,t)}{\partial t^2} + \frac{\partial v'_g(x,t)}{\partial x} = \\ \frac{\partial E_x^e(x,h,t)}{\partial x} \tag{27}$$

in which

$$v'_g(x,t) = \int_0^t \xi'_g(t-\tau)\frac{\partial i(\tau)}{\partial \tau}d\tau \tag{28}$$

Expanding the current and the scattered voltage using Taylor's series, and replacing the time derivative of the current and of the scattered voltage in equations (26)-(27), we obtain the following second order differential equation

$$v^s(x,t) = v^s(x,t_0) - \frac{\Delta t}{C'}\frac{\partial i(x,t)}{\partial x} + \\ -\frac{\Delta t^2}{2L'C'}\left(\frac{\partial E_x^e(x,h,t)}{\partial x} - \frac{\partial^2 v^s(x,t)}{\partial x^2} - \frac{\partial v'_g(x,t)}{\partial x} \right) + \\ + \mathrm{O}(\Delta t^3) \tag{29}$$

$$i(x,t) = i(x,t_0) - \\ + \frac{\Delta t}{L'}\left(\frac{\partial v^s(x,t)}{\partial x} - E_x^e(x,h,t) + v'_g(x,t) \right) + \\ + \frac{\Delta t^2}{2L'C'}\left(\frac{\partial^2 i(x,t)}{\partial x^2} + C'\frac{\partial E_x^e(x,h,t)}{\partial t} - C'\frac{\partial v'_g(x,t)}{\partial t} \right) + \\ + O(\Delta t^3) \tag{30}$$

Following a similar procedure as in the previous paragraph, we obtain the 2nd order FDTD scheme

$$v_k^{n+1} = v_k^n - \frac{\Delta t}{C'}\left(\frac{i_{k+1}^n - i_{k-1}^n}{2\Delta x} \right) + \\ - \frac{\Delta t^2}{2L'C'}\left(\frac{Eh_{k+1}^n - Eh_{k-1}^n}{2\Delta x} - \frac{v_{k+1}^n + v_{k-1}^n - 2v_k^n}{\Delta x^2} \right) + \\ + \frac{\Delta t^2}{2L'C'}\left(\frac{v'^{\,n}_{g_{k+1}} - v'^{\,n}_{g_{k-1}}}{2\Delta x} \right) \tag{31}$$

$$i_k^{n+1} = i_k^n - \frac{\Delta t}{L'}\left(\frac{v_{k+1}^n - v_{k-1}^n}{2\Delta x} - Eh_k^n + v'^{\,n}_{g_k} \right) + \\ + \frac{\Delta t^2}{2L'C'}\left(\frac{i_{k+1}^n + i_{k-1}^n - 2i_k^n}{\Delta x^2} + C'\frac{Eh_k^{n+1} - Eh_k^{n-1}}{2\Delta t} \right) + \\ - \frac{\Delta t^2}{2L'}\left(\frac{v'^{\,n}_{g_k} - v'^{\,n-1}_{g_k}}{\Delta t} \right) \tag{32}$$

C. Extension to the case of a multiconductor line

The Agrawal et al. field-to-transmission line coupling equations for a multiconductor line above a lossy ground are given by [7]:

$$\frac{\partial}{\partial x}\left[v_i^s(x,t)\right] + \left[L'_{ij}\right]\frac{\partial}{\partial t}\left[i_i(x,t)\right] + \left[\xi'_{gij}\right] \otimes \frac{\partial}{\partial t}\left[i_i(x,t)\right] = \\ = \left[E_x^e(x,h_i,t)\right] \tag{33}$$

$$\frac{\partial}{\partial x}\left[i_i(x,t)\right] + \left[C_{ij}'\right]\frac{\partial}{\partial t}\left[v_i^s(x,t)\right] = 0 \tag{34}$$

In which
- $\left[L'_{ij}\right]$ and $\left[C'_{ij}\right]$ are the matrices of line inductance and capacitance;
- $\left[\xi'_{gij}\right]$ is the matrix of transient ground resistance;
- $\left[i_i(x,t)\right]$ and $\left[v_i^s(x,t)\right]$ are the vectors of line current and scattered voltage;
- $\otimes$ denotes the convolution product.

Following the same procedure described in II.1 we obtain the following second-order differential equation:

$$\left[v_i^s(x,t)\right] = \left[v_i^s(x,t_0)\right] - \Delta t\left[C_{ij}'\right]^{-1}\frac{\partial\left[i_i(x,t)\right]}{\partial x} + \\ - \frac{\Delta t^2}{2}\left[\left[L_{ij}'\right]\left[C_{ij}'\right]\right]^{-1}\left(\frac{\partial\left[E_x^e(x,h_i,t)\right]}{\partial x} - \frac{\partial^2\left[v_i^s(x,t)\right]}{\partial x^2} \right) + \\ + \frac{\Delta t^2}{2}\left[\left[L_{ij}'\right]\left[C_{ij}'\right]\right]^{-1}\left(\frac{\partial\left[v_{gi}'(x,t)\right]}{\partial x} \right) \tag{35}$$

$$\left[i_i(x,t)\right] = \left[i_i(x,t_0)\right] - \\ + \Delta t\left[L_{ij}'\right]^{-1}\left(\frac{\partial\left[v_i^s(x,t)\right]}{\partial x} - \left[E_x^e(x,h_i,t)\right] + \left[v'_{gi}(x,t)\right] \right) + \\ + \frac{\Delta t^2}{2}\left[\left[C_{ij}'\right]\left[L_{ij}'\right]\right]^{-1}\left(\frac{\partial^2\left[i_i(x,t)\right]}{\partial x^2} + \left[C_{ij}'\right]\frac{\partial\left[E_x^e(x,h_i,t)\right]}{\partial t} \right) + \\ - \frac{\Delta t^2}{2}\left[\left[C_{ij}'\right]\left[L_{ij}'\right]\right]^{-1}\left(\left[C_{ij}'\right]\frac{\partial\left[v'_{gi}(x,t)\right]}{\partial t} \right) \tag{36}$$

in which

$$\left[v'_{gi}(x,t)\right] = \left[\xi'_{gij}\right] \otimes \frac{\partial}{\partial t}\left[i_i(x,t)\right] \tag{37}$$

And finally, the 2nd order FDTD representation of (35) and (36) read

$$\left[v_i\right]_k^{n+1} = \left[v_i\right]_k^n - \Delta t\left[C_{ij}'\right]^{-1}\left(\frac{\left[i_i\right]_{k+1}^n - \left[i_i\right]_{k-1}^n}{2\Delta x} \right) + \\ - \frac{\Delta t^2}{2}\left[\left[L_{ij}'\right]\left[C_{ij}'\right]\right]^{-1}\left(\frac{\left[Eh_i\right]_{k+1}^n - \left[Eh_i\right]_{k-1}^n}{2\Delta x} - \frac{\left[v_i\right]_{k+1}^n + \left[v_i\right]_{k-1}^n - 2\left[v_i\right]_k^n}{\Delta x^2} \right) + \\ + \frac{\Delta t^2}{2}\left[\left[L_{ij}'\right]\left[C_{ij}'\right]\right]^{-1}\left(\frac{\left[v'_{gi}\right]_{k+1}^n - \left[v'_{gi}\right]_{k-1}^n}{2\Delta x} \right) \tag{38}$$

$$\left[i_i\right]_k^{n+1} = \left[i_i\right]_k^n - \Delta t\left[L_{ij}'\right]^{-1}\left(\frac{\left[v_i\right]_{k+1}^n - \left[v_i\right]_{k-1}^n}{2\Delta x} - \left[Eh_i\right]_k^n + \left[v'_{gi}\right]_k^n \right) + \\ + \frac{\Delta t^2}{2}\left[\left[C_{ij}'\right]\left[L_{ij}'\right]\right]^{-1}\left(\frac{\left[i_i\right]_{k+1}^n + \left[i_i\right]_{k-1}^n - 2\left[i_i\right]_k^n}{\Delta x^2} \right) + \\ + \frac{\Delta t^2}{2}\left[\left[C_{ij}'\right]\left[L_{ij}'\right]\right]^{-1}\left(\left[C_{ij}'\right]\frac{\left[Eh_i\right]_k^{n+1} - \left[Eh_i\right]_k^{n-1}}{2\Delta t} \right) + \\ - \frac{\Delta t^2}{2}\left[\left[C_{ij}'\right]\left[L_{ij}'\right]\right]^{-1}\left(\left[C_{ij}'\right]\frac{\left[v'_{gi}\right]_k^n - \left[v'_{gi}\right]_k^{n-1}}{\Delta t} \right) \tag{39}$$

D. Treatment of periodical groundings along the line

The equations relevant to the 1st order FDTD scheme for the treatment of the periodical groundings of the line conductors, if any (e.g. grounding of shielding wires) have been presented in [4]. In the proposed 2nd order scheme, the voltages and currents nodes are coincident, which allows simplifying the equations for the treatment of the periodical groundings. The treatment of a shunt impedance representing one of the conductor groundings for an overhead line illuminated by an external electromagnetic field is schematically illustrated in Fig. 2.

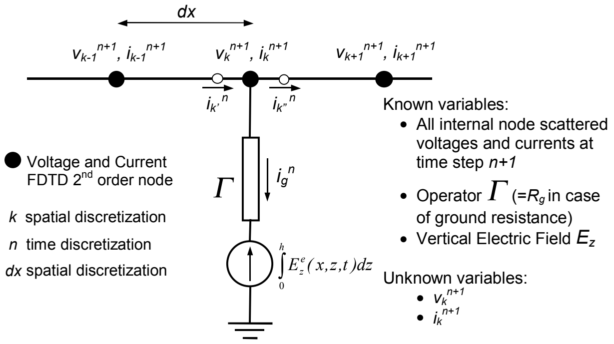

Fig. 2. Shunt impedance at a generic point along an overhead line illuminated by an external exciting electromagnetic field.

The node voltage v_k^{n+1} (see Fig. 2) can be expressed as follows:

$$v_k^{n+1} = \Gamma\,(i_g^{\,n+1}) + \int_0^h E_z^e(x,z,t)dz \tag{40}$$

where

- i_g^{n+1} is the current flowing in the grounding impedance;
- Γ is an integral-differential operator, which describes the voltage drop across the shunt impedance as a function of current i_g. ($\Gamma = R_g \cdot i_g$ for the simple case of a resistance).

Current i_g^{n+1} can be expressed as function of the currents $i_{k'}^{n+1}$, $i_{k''}^{n+1}$ applying Kirchhoff's law on the currents at the grounding node

$$i_g^{\,n+1} = i_{k'}^{n+1} - i_{k''}^{n+1} \tag{41}$$

Currents $i_{k'}^{n+1}$, $i_{k''}^{n+1}$ can be expressed as a function of the adjacent current nodes assuming the following linear spatial interpolation:

$$i_{k'}^{n+1} = 2i_{k-1}^{n+1} - i_{k-2}^{n+1} \tag{42}$$

$$i_{k''}^{n+1} = 2i_{k+1}^{n+1} - i_{k+2}^{n+1} \tag{43}$$

By introducing (41) (42) and (43) in (40) we obtain the equation for the scattered voltage at the grounding point. The node current i_k^{n}, needed in equations (19) (20) to compute voltages and currents at nodes *k-1* and *k+1*, must be substituted with $i_{k'}^{n}$ and respectively with $i_{k''}^{n}$ in the equations written for the node *k-1* and *k+1*.

E. Treatment of corona

The *Agrawal* et al. model was already adapted in [9,10] to deal with corona originated by voltages induced by nearby lightning. In those papers, the corona process was described macroscopically by a charge-voltage diagram. Both 1st order FDTD and 2nd order Gear [11] algorithms were used in [9,10] to solve the coupling equations, and the latter was found to be numerically more stable than the first.

The proposed 2nd order FDTD scheme was also extended to take into account corona effect, using the same model for corona as in [9,10]. The 2nd order scheme results in a considerable reduction of numerical instabilities appearing in the classical 1st order scheme.

III. INTERFACE WITH EMTP96

In order to make it straightforward the analysis of the LEMP response of real distribution systems characterized by a certain topological complexity, the developed program has been interfaced with the Electromagnetic Transient Program (EMTP96). In principle, the developed program based on the FDTD 2nd order scheme could have been suitably enlarged and extended case by case to take into account the specific system configuration, as discussed in [13]. Such an interface is somewhat inspired by a previous one, linking the 1st order FDTD program (called LIOV – lightning-induced overvoltages – code [12,13]) and the EMTP M39 [13]. For sake of simplicity, we shall refer hereafter to the 2nd order FDTD program as LIOV2.

The concept at the basis of the new interface is schematically described in Fig. 3 (single-conductor line). As in [13], the distribution line is considered as consisting of a number of illuminated LIOV lines connected to each other through EMTP. The difference from [13] is that the new interface does not require any modification to the source code of the EMTP: the induced currents at the terminal nodes computed by the LIOV2 code are input to the EMTP via current controlled generators and the voltages calculated by the EMTP are input to the LIOV2 code via voltage sources.

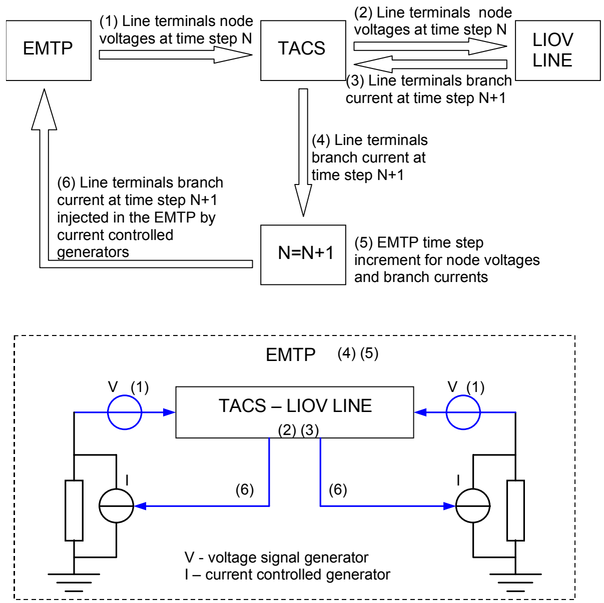

Fig. 3. Interface between LIOV2 and EMTP96

IV. SIMULATIONS

A first comparison of the newly proposed 2nd order integration scheme with the 1st order one has been performed making reference to a 2-km long, 10-m high single-conductor line above a lossy ground shown in Fig. 4. The ground conductivity is 0.001 S/m and its relative permittivity is 10. The stroke location is at 50 m from the left-end line terminal. The lightning channel base current peak value is 60 kA and its maximum time derivative is 120 kA/µs. The return stroke speed is 1.2×10^8 m/s. The LEMP is computed adopting the MTL return stroke model [14] and the value of the spatial and temporal steps adopted for the simulations are 10 m and 10^{-8} s respectively.

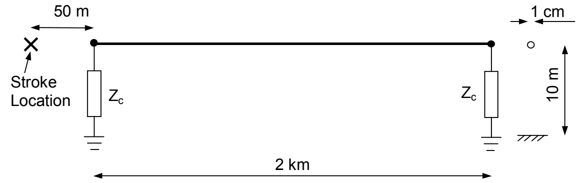

Fig. 4. Line geometry for the comparison between FDTD 1st order and 2nd order in presence of lossy ground

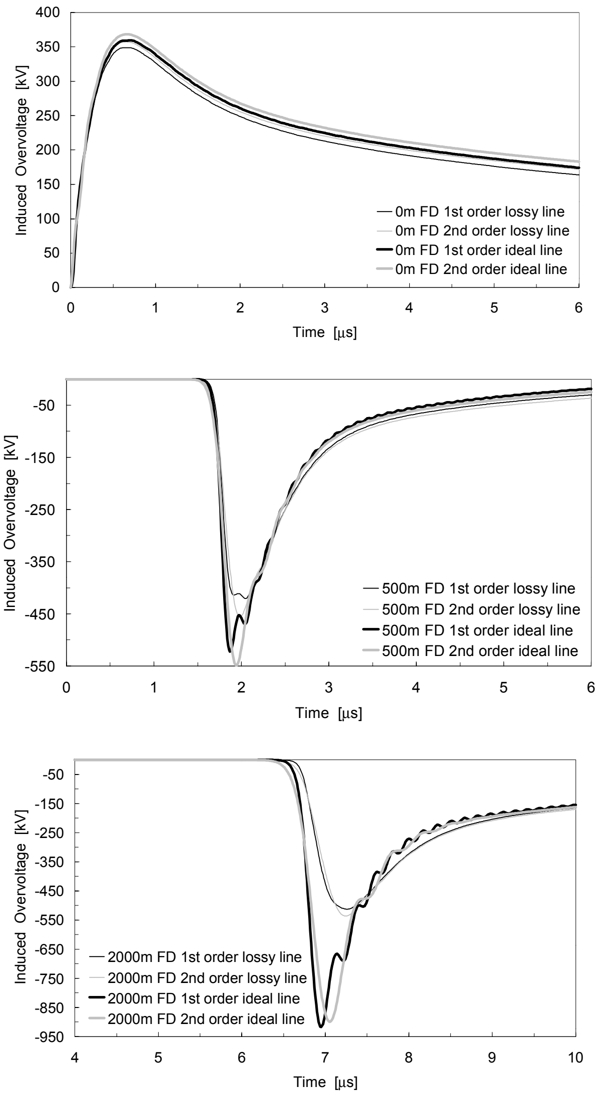

Fig. 5. Lightning induced overvoltages calculated at three different observation points of the line of Fig. 4 (x=0m, x=500m, x=2km) using 1st and 2nd order FDTD scheme. Field calculation: lossy ground (0.001 S/m), line impedance: ideal line and lossy line.

In Fig. 5 we show the results calculated both with the 1st order and 2nd order FDTD algorithms. For both cases, we show the results considering the effect of the ground resistivity both in the electromagnetic field and in the calculation of line parameters ("lossy line") and the results obtained taking into account ground losses only in the electromagnetic field calculation ("ideal line"). It can be seen that the waveshapes computed using the 2nd order FDTD algorithm are less affected by numerical oscillations, especially for observation points approaching the line far-end.

A comparison between the two methods has been performed also for the case of a line with surge arresters. To perform these simulations, we have used the interface between the developed code and EMTP96, as described in the previous section.

Fig. 6 show the geometry of the line used for the simulations and Fig. 7 shows the numerical results. Again, it can be seen that the proposed 2nd order scheme leads to an improvement of the computed results, in terms of numerical stability.

V. SUMMARY AND CONCLUSIONS

The 2nd order FDTD program proposed here allows for the calculation of lightning-induced voltages on multiconductor overhead transmission lines with multiple groundings of shielding wires above a lossy ground. Both the frequency dependence of the ground impedance and the corona effect are also taken into account. The integration scheme is numerically more stable than the 1st order one, without significant increase in the computation time[1], especially for line configurations involving frequency-dependent losses and non linearities.)

A beta version of an interface between the developed program and EMTP96 has been realized; work is in progress to improve its capabilities for the treatment of more complex line geometries and distribution systems. The interface allowed emphasizing the efficiency of the proposed 2nd order scheme in terms of numerical stability for more complex and non-uniform line geometries.

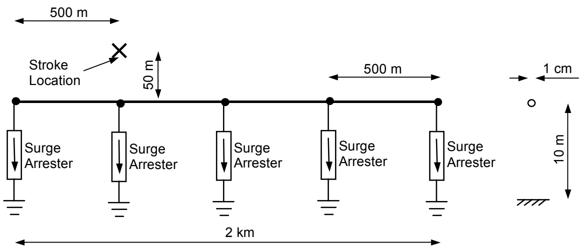

Fig. 6. Line geometry for the comparison between FDTD 1st order ad 2nd order in presence of surge arresters, using the developed interface between LIOV2 and EMTP96.

[1] The calculation of the exciting lightning electromagnetic field representing, for the problem of interest, the bulk of the computation time.

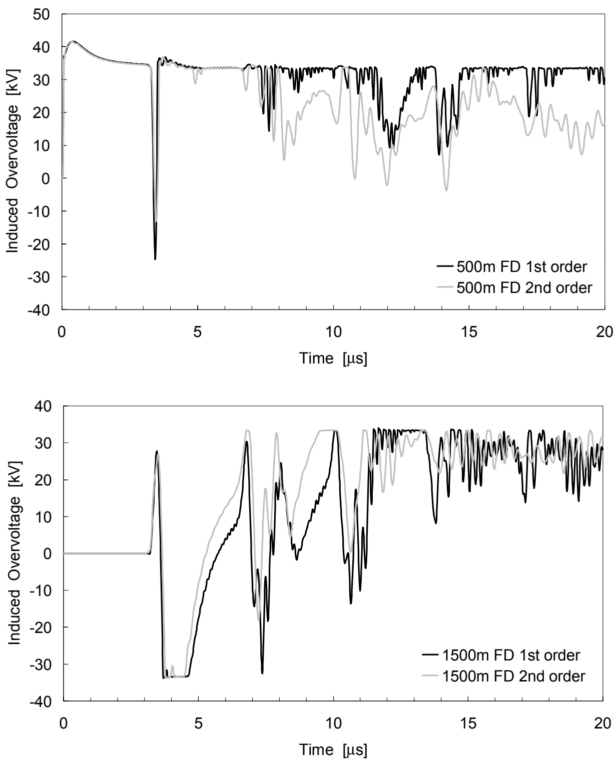

Fig. 7. Lightning induced overvoltage at two observation points (x=500m, x=1500m) of Fig. 6. Comparison between FDTD 1st order and 2nd order in presence of surge arresters.

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